J. Phys. E: Sci. Instrum., Vol. 16, 1983. Printed in Great Britain

# Digital phase detection based on in-phase and quadrature sampling

**Jafar Saniie† and Mauri Luukkala**
Department of Physics, University of Helsinki,
Siltavuorenpenger 20 D, 00170 Helsinki 17, Finland

Received 20 January 1983

**Abstract.** In this report a simple and efficient algorithm of digital phase detection is presented. From data sampled at two instants of time separated by a quarter of the signal period an estimate of phase and magnitude can be obtained. The additional use of coherent averaging improves the accuracy of phase detection in signals with poor signal-to-noise ratio. A system implementation of the algorithm and error caused by the harmonics of the measured signal is discussed.

## 1. Introduction

Phase detection systems used for signal recovery or characterisation have found application in many scientific studies. Analogue phase detectors are capable of recovering a signal with very poor signal-to-noise ratio (SNR) in which an adequate separation of the signal from noise cannot be achieved by means of linear filtering (Blair and Sydenham 1975, Meade 1982).

With the availability of the microcomputer in any laboratory, digital phase detection can be accomplished, even in the presence of random noise. A recent discussion of digital phase detection, comparable to the analogue phase detector used for many applications, was presented by Momo *et al* (1981). In this report we will elaborate on the basic principles of the digital phase detector and present a simple method for vector presentation of the signal. We will also discuss a system implementation of the algorithm and error caused by the harmonics of the measured signal.

## 2. Principle, implementation and discussion

Through the use of any microcomputer, the phase (time delay) between two periodic signals can be determined by locating the occurrence of certain well defined features (e.g. peak detection, threshold detection or zero crossing). In practice, determining useful features requires a sampling rate several times greater than the Nyquist rate and/or excessive computer time for processing. The required high sampling rate reduces the upper bound in the frequency of the signal and also limits any possibility for real-time processing of the data.

The above-mentioned problems in digital phase detection can be avoided by sampling the measured signal at two instants of time separated by a quarter of the signal period, as shown in figure 1. According to the timing diagram of this figure, the sampled data are

$$\text{In-phase: } S_I(m) = s(mT+\tau) \qquad \text{Quadrature: } S_Q(m) = s(mT+\tau+T/4) \tag{1}$$

† Present address: Department of Electrical Engineering, Illinois Institute of Technology, Chicago, Illinois 60616, USA.

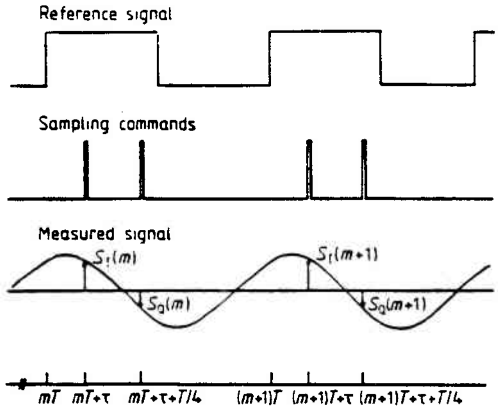


**Figure 1.** Timing diagram of the sampling commands, reference signal and measured signal.

where $m$ represents the $m$th period, $T$ is the period, and $\tau$ is an arbitrary known delay varying between zero and $T$.

Let the measured signal be

$$s(t) = E\cos(\omega t + \varphi) \qquad \omega = 2\pi/T \tag{2}$$

where $E$ and $\varphi$ are the unknown magnitude and phase of the signal respectively. Then, by applying equation (1) to equation (2) the phase lag and magnitude of the measured signal can be determined:

$$\varphi = \tan^{-1}(S_Q(m)/S_I(m)) + (2\pi\tau/T) \qquad E = (S_I^2(m) + S_Q^2(m))^{1/2}. \tag{3}$$

It is important to point out that the above equations produce the desired results only in the absence of noise in the measurement. When the signal is corrupted by additive noise, coherent averaging of the signal will enhance the SNR. Hence, equation (1) can be reformulated to represent the average value over any $N$ periods:

$$S_I(m) = \frac{1}{N}\sum_{n=m}^{N+m-1} s(nT+\tau) \quad \text{for any } m \qquad S_Q(m) = \frac{1}{N}\sum_{n=m}^{N+m-1} s(nT+\tau+T/4). \tag{4}$$

The in-phase/quadrature digital phase detection algorithm can be implemented in a microcomputer system equipped with parallel input/output (PIO) ports and an analogue-to-digital (A/D) converter, as shown in figure 2. The processing speed of the microcomputer and the conversion time of the A/D converter impose an upper limit on the frequency of the measured signal.

Our laboratory microcomputer is a Nascom with a Z80A central processing unit which operates at a 4 MHz clock frequency. This system can be used for the digital phase detection of signals up to 32 kHz frequency. The computer program is written in a combination of BASIC and assembly language for maximum processing speed and programming convenience. With the BASIC program, the parameters $T$, $N$ and $\tau$ are initialised and stored in the proper memory locations. The real-time data processing program is written in assembly language. This portion of the program includes generating the reference signal and sampling commands, reading the sampled data and averaging. Then, the calculation of the phase and magnitude given in equation (3) is written in BASIC.

Equation (3) was derived for the ideal situation in which the measured signals are assumed to be sinusoidal functions.

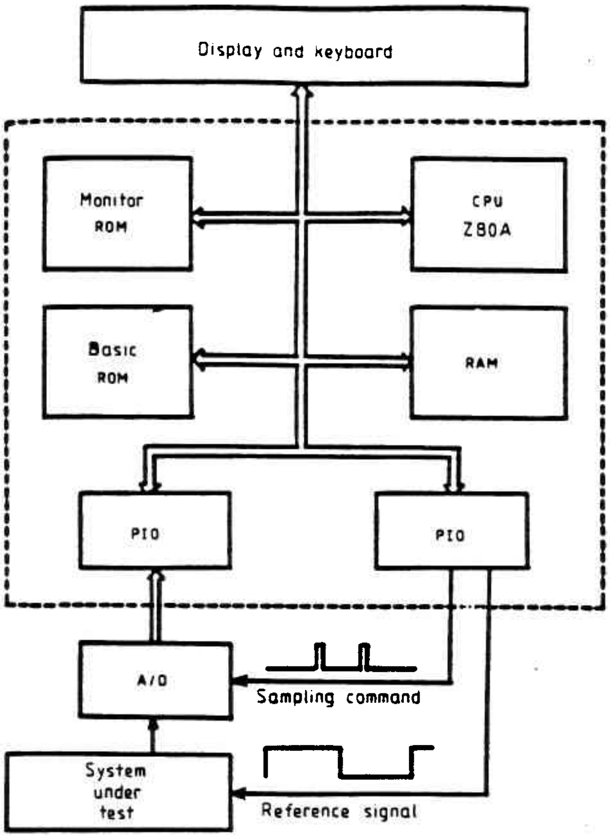


**Figure 2.** Typical block diagram of a microcomputer system with PIO ports and A/D converter to be used as a digital phase detector.

Nevertheless, in many measurements the signal contains harmonics which result in errors in both magnitude and phase estimation. The value of the error can be predicted by considering the Fourier expansion of the signal:

$$s(t) = E_1 \cos(\omega t + \varphi_1) + \sum_{k=2}^{\infty} E_k \cos(k\omega t + \varphi_k). \qquad (5)$$

The first term on the right side of equation (5) is of interest for the phase and magnitude measurements. But, if the above equation is applied to equation (1), an error is introduced in the in-phase/quadrature sample values, governed by the intensities and phases ($E_k$ and $\varphi_k$) of the undesired harmonics. For example, if a triangular wavefunction has a delay which varies between zero and $T$ (one period), the error in phase measurement will be within $\pm 4$ degrees and the magnitude variation may be as much as $\pm 15\%$ when this signal is regarded as a sinusoidal function. Larger errors can be anticipated when the coefficients of the harmonics are significant in comparison to the fundamental. Therefore, it is recommended that the harmonics be filtered out prior to sampling for phase detection. In practice any possible error due to harmonics can be examined by varying the known arbitrary delay $\tau$ and evaluating the reproducibility of the phase and magnitude measurements.

In summary we have described a useful phase detection algorithm based on in-phase/quadrature sampling. This technique is efficient and only requires averaging when the signal-to-noise ratio is inadequate. By introducing a known delay and sampling the measured signal at two instants of time separated by one quarter of the signal period, vector presentation of the signal can be obtained. Finally, the algorithm can be implemented in any microcomputer which contains PIO ports and an A/D converter.


**Acknowledgments**

We wish to thank T Tiusanen for his assistance in setting up the microcomputer system, and the Ministry of Trade and Industry (KTM) for financial support.


**References**


Blair D P and Sydenham P H 1975 Phase sensitive detection as a means to recover signals buried in noise
*J. Phys. E: Sci. Instrum.* **8** 621–7

Meade M L 1982 Advances in lock-in amplifiers
*J. Phys. E: Sci. Instrum.* **15** 395–403

Momo F, Ranieri G A, Sotgiu A and Terenzi M 1981 Microcomputer based phase sensitive detector
*J. Phys. E: Sci. Instrum.* **14** 1253–6