ROBUST TARGET DETECTION USING ORDER STATISTIC FILTERS

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Abstract

In radar, sonar, and ultrasonic detection systems, interference due to clutter can severely deteriorate the quality of the received signal to the point of concealing the target. This paper presents theoretical analysis of order statistic processors for improved target detection. The sort function is used to provide insight into the optimal rank for detection of various targets and clutter environments where observations are independent and identically distributed. When observations do not contain identical statistical information, the analysis becomes more complex. Both simulation and experimental results are used to illustrate the extent of robustness that can be obtained from a particular rank in the presence of observations containing insignificant target information (i.e. null observations).

Introduction

Order statistics (OS) have been studied in many areas of statistical signal processing such as speech, image, and sonar. The OS filter is a nonlinear processor that can be expressed by:

$$x_{r:n} = OS_{r:n}\{x_1, x_2, x_3 \cdots x_n\} \quad \text{for } 1 \le r \le n \tag{1}$$

where x_i is the unordered observed input value from the sequence of size n (window size) and r is the rank of the input from the ordered sequence that becomes the output $x_{r:n}$. This filter is the median detector when $r=(n+1)/2$ (for n odd), the maximum detector when $r=n$, and the minimum detector when $r=1$.

This paper develops a statistical analysis of the OS filter to obtain general input-output relationships so that predictions concerning the performance of the OS filter in various target-clutter situations can be made. The relation between the optimal rank and properties of the target and clutter distributions are discussed. The expected value of the output of the OS filter is examined to establish the OS filter as an estimator of the quantiles of the input signal distribution. Then the performance of the OS filter is analyzed through simulation for various target-clutter distributions using sort function analysis for cases in which the observations are and are not independent and identically distributed. Finally, results from ultrasonic experimental measurements using split-spectrum processing (i.e. a method of obtaining frequency diverse observations) are examined for supporting theoretical predictions.

Detection Properties of OS Filters

In order to determine the expected value of the output of the OS filter, the general expression for the output probability density function, $f_{X_{r:n}}(\cdot)$, of the OS filter is derived in terms of r and n as defined in Equation (1) along with the input probability density function. The input signals, x_i, are assumed to be independent and identically distributed with the distribution function, $F_X(\cdot)$, and the density function, $f_X(\cdot)$. In the following derivation let X represent the random variable for the input of the OS filter and $X_{r:n}$ represent the random variable for the output of the OS filter with rank r.

The distribution function for $X_{r:n}$ is defined as $F_{X_{r:n}} = Pr\{X_{(r:n)} < x\}$. The probability that at least r of the n values are less than x can be found by applying the binomial distribution [4],

$$F_{X_{r:n}}(x) = \sum_{i=r}^{n} \binom{n}{i} F_X^i(x)(1-F_X(x))^{n-i} \quad \text{for } 1 \le r \le n. \tag{2}$$

The density function can be found by taking the derivative of Equation (2),

$$f_{X_{r:n}}(x) = r\binom{n}{r} F_X^{r-1}(x)(1-F_X(x))^{n-r} f_X(x) \quad \text{for } 1 \le r \le n. \tag{3}$$

The density function for $X_{r:n}$ is the product of the probability density function of a single input, $f_X(x)$, and another function given by:

$$W(x) = r\binom{n}{r} F_X^{r-1}(x)(1-F_X(x))^{n-r} \quad \text{for } 1 \le r \le n. \tag{4}$$

While the exact behavior of the output of the OS filter depends on $f_X(x)$, the general behavior of this filter, particularly the mean and variance of the output, can be determined by the properties of $W(x)$.

Before finding the expected value of the output of the OS filter, the notation can be simplified by letting $u=F_X(x)$. If u is substituted into Equation (4), the result is:

$$w_{r:n}(u) = r\binom{n}{r} u^{r-1}(1-u)^{n-r} \quad \text{for } 0 \le u \le 1 \tag{5}$$

where the subscript $r{:}n$ denotes the parameters of the associated OS filter and $w_{r:n}(u)$ will be referred to as the *sort function* [1-3]. The modal point of $w_{r:n}(u)$ occurs when u is equal to a value [2],

$$u_r = \left(\frac{r-1}{n-1}\right) \quad \text{for } 1 \le r \le n \tag{6}$$

For a given n, the set of u_r values corresponding to all possible values of r constitutes a set of quantiles for the input distribution function. The extreme end points of the distribution corresponding to $u=0$ and $u=1$, will be referred to as the 0^{th} quantile and the $(n-1)^{th}$ quantile, respectively. In Figure 1 the plots of the sort function for five different values of r are presented with $n=5$. In each plot note that the set of modal points divides the u domain into four equal intervals. Therefore, these modal points correspond to quantiles through the inverse distribution function.

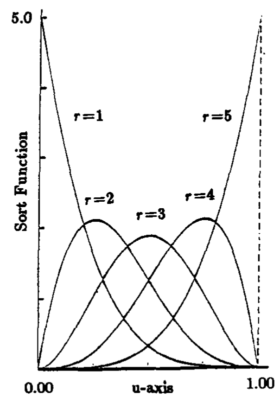

Figure 1. Five different sort functions for window size $n=5$ demonstrate the relationship between the quantiles and the modal points determined by the rank parameter r.

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The expression for the mean of the output of the OS filter for a specific probability distribution is given by [2,3]:

$$E[X_{r:n}]=\int_0^1 F_X^{-1}(u)\,w_{r:n}(u)\,du \qquad (7)$$

In the above equation, note how the sort function acts as a weighting function to emphasize a particular region of the distribution function over the integration. By changing the r parameter of the sort function, the modal point can be set to emphasize different regions of the distribution. For increasing n, with u_r held constant, the sort function is a delta sequence shifted right on the u-axis by an amount equal to u_r. For a constant u_r as n approaches infinity, Equation (7) becomes:

$$\lim_{n\to\infty} E[X_{r:n}]=F_X^{-1}(u_r) \qquad (8)$$

The key to understanding the operation of the OS filter is Equation (7). From this it is seen how the parameters r and n can be used so that the OS filter emphasizes particular regions in the distributions of the input signals. The OS filtering operation censors the actual values of signals outside this quantile region from the decision rule. This property is useful when the classes of signals exhibit a distinctive statistical difference over a limited range of quantiles, such as what may occur with specularly reflective clutter or targets, or when electronically generated interference is present in a frequency diversity system [2].

The optimal rank varies, dependent on the input distributions, and is illustrated by the following two examples (see Figures 2 and 3). In the first example we assume the target is chi distributed with skewness equal to 0.31 in Weibull clutter with a skewness equal to 1.07, and n=25. Figure 2 shows the probability of detection for all possible r values $(1, 2, \ldots 25)$ for 0dB and 2dB signal-to-clutter ratios (SCR). Note that the optimal rank occurs at r=2 for the lower SCR and r=4 for the higher SCR. For the higher SCR, the optimal choice is less critical, since for any r value from 1 to 10, the OS filter shows good performance. In the second example, both target and clutter are Rayleigh distributed. As shown in Figure 3 optimal r is 19 for the lower SCR and 20 for the higher SCR. For high SCR the robustness of the higher ranks is self evident and will increase as the SCR increases.

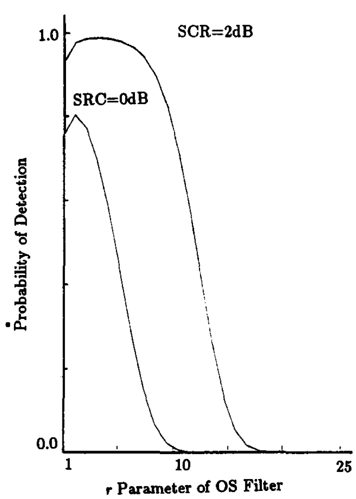

Figure 2. Probability of detection of chi distributed target in Weibull distributed clutter with probability of false alarm fixed at 0.001.

Generalized Detection Properties of OS Filters

In the previous section the discussion has been confined to the assumptions that input observations are independent and identically distributed. But, it may be more appropriate to assume that input observations are independent and stem from different distributions. Under this condition, the output distribution function of rank r, $F_{X_{r:n}}(x)$, from a set of independently distributed inputs $F_{X_1}(x), F_{X_2}(x), \cdots, F_{X_n}(x)$ can be determined by the the following relationship [5]

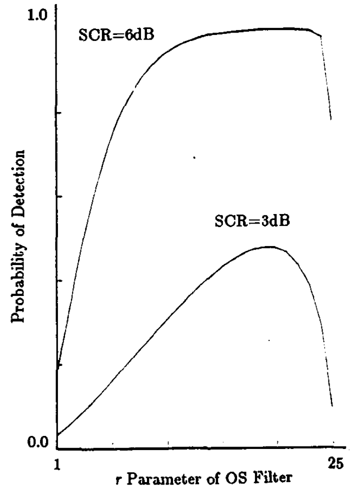

Figure 3. Probability of detection of Rayleigh distributed target in Rayleigh distributed clutter with the probability of false alarm set at 0.001.

$$F_{X_{r:n}}(x) = \sum_{i=r}^{n} \sum_{S_i} \prod_{l=1}^{i} F_{X_{j_l}}(x) \prod_{l=i+1}^{n} [1-F_{X_{j_l}}(x)] \qquad (9)$$

where the summation S_i extends over all permutations $(j_1, j_2, \ldots, j_n)$ of the set of numbers, $1, 2, 3, \ldots, n$ for which $j_1 < \cdots < j_i$ and $j_{i+1} < \cdots < j_n$. These output distribution functions are cumbersome to evaluate and analyze for a large n.

To illustrate the effect of variations in the input distributions, an example for filter of size n=5 is considered which will be used to predict of the general behavior of output densities as a function of observations. Consider the two hypothesis in which target is present, H_1, and target not present, H_0, in each of the observations. In the ideal detection situation, all observations will belong to class H_0 or H_1. However, this may not be the case for some observations. Let us assume in the process of making n observations under the H_1 hypothesis, some of the observations have poor signal-to-clutter ratios (i.e., null observations). Under this constraint, the performance of the order statistic filter will deteriorate depending on the rank of the filter.

For computer simulation, clutter and target-plus-clutter are assumed to be Rayleigh distributed. The performance of the different ranked outputs are evaluated using a fixed threshold with the probability of false alarm set at 0.001 and the probability of detection is calculated over various input signal-to-clutter ratios. The performance of the ranked outputs are shown in Figure 4 where the hypothesis H_1 exist in all observations. The higher rank output (r=4 and r=5) of this case performs better due to good statistical separation in the larger amplitude observations. Also the median output shows improved performance over the lower order ranks. The effect of introducing null observations on the performance of the output ranks can be seen in Figures 5 and 6. For one null observation, the performance is shown in Figure 5 where the minimum rank shows the greatest deterioration and the other ranks are only slightly affected. Inherently, minimization suffers since it searches for the smallest observed values in which the null observations with the lower expected value (confined to H_0 statistics) make up the greatest contributions. Figure 6 shows the performance of the same OS filter with three null channels, the lowest three ranked outputs (median through minimization) have severely diminished in performance which is warranted based on the above discussion.

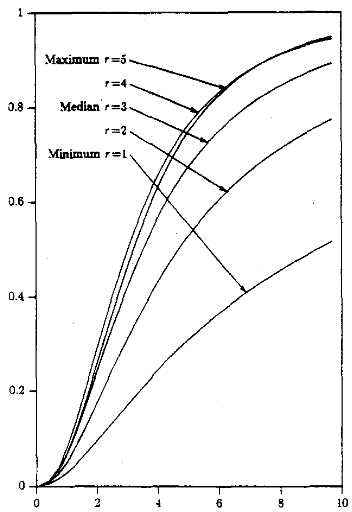

Figure 4. Probability of detection versus various input signal-to-clutter ratios for each output of the OS Filter where there are no null observations.

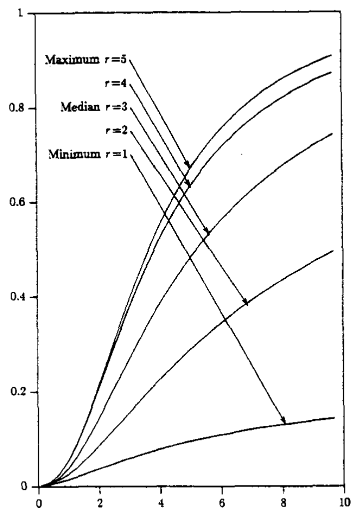

Figure 5. Probability of detection versus various input signal-to-clutter ratios where one null observation exists.

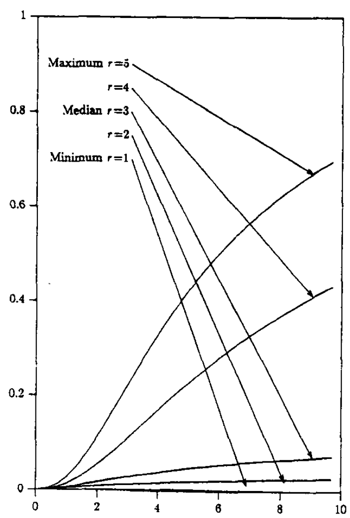

Figure 6. Probability of detection versus various input signal-to-clutter ratios where three null observations exists.

Thus, when null observations exists, minimization and other low ranked outputs will be more sensitive to a deteriorated observation and will exclude information in the other observations. These variations in the underlying density functions of the observations can be dealt with by concentrating on higher ranks which exhibit the property of inclusion where all the strong amplitude information is passed to the output.

Experimental Results

To illustrate the effectiveness of the OS filter in improving the signal-to-clutter ratio and overall detection capabilities, an example of ultrasonic flaw detection has been used. In larged grained materials the ultrasonic flaw signal is oftened masked by echoes returned from the microstructure to the point where detection of the defects becomes very difficult. In this study, multiple observations are obtained through a frequency diverse system in which the backscattered broadband signal is divided into several narrowband channels as shown in Figure 7. The output of these channels is then sent to the OS processor for detection. This technique is referred to as split-spectrum processing (SSP) and has been shown to be effective for detecting targets in coherent clutter [6].

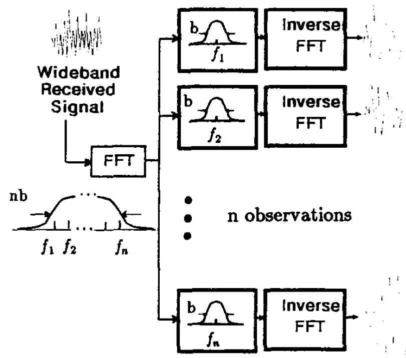

Figure 7. Block diagram of split-spectrum processing.

Experimental data is acquired using a broadband ultrasonic transducer in order to examine a constructed flaw embedded within a steel block. The backscattered signal using the pulse-echo method is shown in Figure 8 along with its amplitude spectrum. In this measurement, the peak amplitude flaw-to-clutter ratio is slightly less than 0dB. The measurements were split-spectrum processed using 9 bandpass filters with bandwidths of 0.75 MHz and 0.5 MHz spacing between adjacent filters starting at 1MHz. The signal corresponding to each frequency band is shown in Figure 9, and in all of the signals the flaw echoes can be observed. This is an ideal situation in which no null observations exist. As shown in Figure 10, all of the resulting ranks display strong flaw information as predicted in theory.

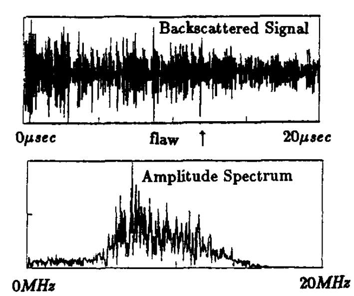

Figure 8. Backscattered grain signal and corresponding amplitude spectrum.

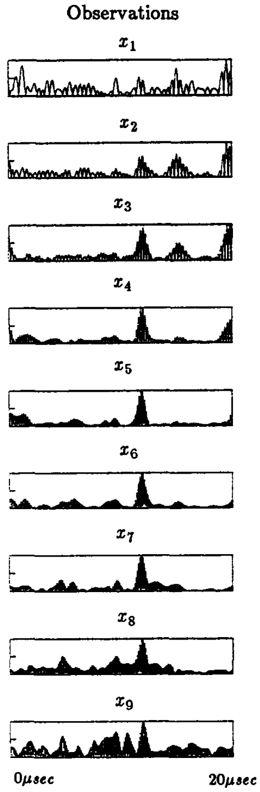

Figure 9. Output of the SSP for the frequency range 1-5 MHz.

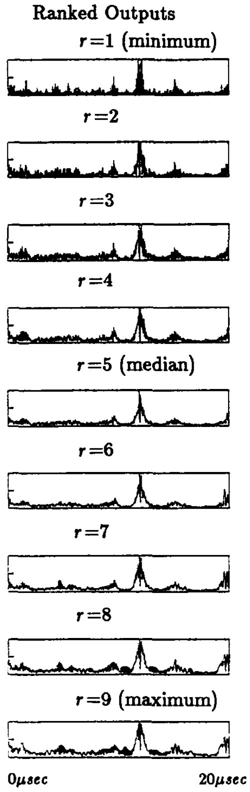

Figure 10. All of the ranked outputs of the OS filters for the 1-5 MHz frequency information.

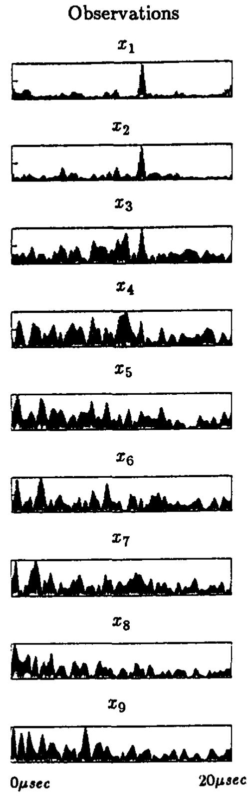

Figure 11. Output of the SSP for the frequency range of 3-11 MHz.

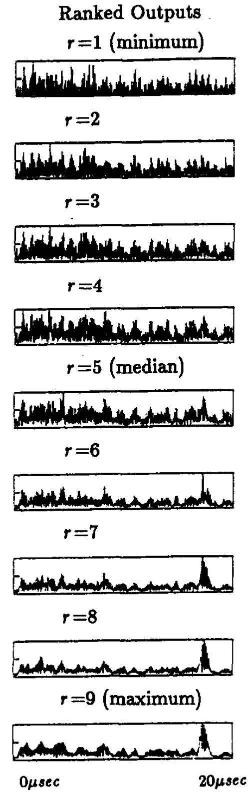

Figure 12. All of the ranked outputs of the OS filters for the 3-11 MHz frequency information.

The above example has been tailored to the specific parameter to ensure no null observations which is not possible in practice since such apriori knowledge may be unobtainable. To be more general in analysis, the broadband signal is processed using 9 filters with a 3dB bandwidth of 1.5 MHz in a frequency span of 3-11 MHz. Figure 11 shows the channel outputs, with only a few channels showing significant flaw information and over half the channels with clutter information only. The ranked outputs are shown in Figure 12 where ranks above the median show significant improvement in flaw resolution, and those below the median relay very little information. These experimental results suggest that all ranks are potentially useful in flaw enhancement, although lower ranks are more vulnerable and lack robustness for practical applications.

Conclusion

In summary, we have presented a theory and applications for OS filters in detection problems. Our theory suggests that the optimal rank can be found when the distribution of target and clutter is known. It has been shown that the performance of higher ranked outputs are theoretically more robust than lower ranks, depending on the number of null observations. This theoretical prediction has been confirmed using experimental results in ultrasonic flaw detection.

Acknowledgements

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References

[1] J. Saniie, K.D Donohue, D.T. Nagle, N.M. Bilgutay, "Frequency Diversity Ultrasonic Flaw Detection Using Order Statistic Filters," *Proc. of IEEE 1988 Ultrasonic Symposium in Chicago, Illinois,* October 3-5, 1988.

[2] K.D. Donohue, *Theoretical Analysis of Order Statistic Filters Applied to Detection,* Doctoral Dissertation, Illinois Institute of Technology, (December 1987).

[3] J. Saniie, K.D. Donohue, N.M. Bilgutay, "Order Statistic Filters as Postdetection Processors," *IEEE Trans. on Acoustic, Speech and Signal Processing,* in press, 1989.

[4] H.A. David, *Order Statistics,* New York: John Wiley and Sons, Inc., 1981.

[5] R.J. Vaughan and W.N. Venables, "Permanent Expressions for Order Statistic Densities," *Journal of the Royal Stat. Soc.,* vol. 34, pp. 308-310, 1972.

[6] N.M. Bilgutay and J. Saniie, "The Effect of Grain Size on Flaw Visibility Enhancement Using Split-Spectrum Processing," *Materials Evaluation,* vol. 42, pp. 808-814, 1984.