

ULTRASONIC GRAIN SIGNALS CLASSIFICATION USING * AUTOREGRESSIVE MODELS

Jafar Saniie, Tao Wang and Xiaomei Jin

Electrical & Computer Engineering Department
Illinois Institute of Technology
Chicago, IL 60616

$$\alpha(f) = a_1 f + a_2 \bar{D}^3 f^4 \quad (3)$$

ABSTRACT

Autoregressive (AR) analysis is used to characterize the ultrasonic microstructure (i.e., grains) scattering of materials. Grain scattering results in an upward shift in the expected frequency of broadband echoes, while attenuation results in a downward shift. Both the upward and downward shifts have been found to be correlated to grain size distribution. In order to evaluate the spectral shift in grain signals, we have adopted low order AR models and extracted the features such as AR coefficients, resonating frequency and maximum energy frequency. A Euclidean distance classifier, based on these features, is implemented to classify grain scattering characteristics. Using both computer simulated and experimental data, the probability of correct classification is about 75% for the second order AR model and 88% for the third order AR model, when the expected frequency shift is less than 4%.

where a_1 is the absorption constant, a_2 is the scattering constant, and $\alpha_s(f) = a_2 \bar{D}^3 f^4$. In the Rayleigh scattering region multiple reflections between grain boundaries are negligible and $\alpha_s(f)$ shows high sensitivity to the grain size distribution. Also, in this region, high frequency components are backscattered with larger intensity when compared with low frequency components. Consequently, this situation results in an upward shift in the expected frequency of the power spectrum of broadband echoes. Since the spectral shift is grain size dependent, the estimate of the upward shift can be used for grain size characterization. Furthermore, inspection of Equations 1 and 2 reveals that, the term $e^{-2\langle\alpha(f)\rangle z}$ influences the frequency shift in a downward direction. The downward shift is dependent on the position of the scatterers relative to the transmitting/receiving transducer. The two opposing phenomena can potentially be used for grain size evaluation.

INTRODUCTION

While the ultrasonic grain scattering signal conveys microstructure information, this information is often born in such a complex manner that the signal exhibits a great deal of variability in time domain. Thus, spectral analysis is often adopted as an alternative method for signal characterization. The spectrum of measured grain signal can be modeled as [1, 2]:

$$R(f) = \langle U(f) \rangle \sum_{k=1}^N \beta_k e^{-j2\pi f \tau_k} \quad (1)$$

where,

$$\langle U(f) \rangle = U(f) e^{-2\langle\alpha(f)\rangle z} \langle \alpha_s(f) \rangle \quad (2)$$

The random variable N is the number of echoes detected with random intensity, β_k , and random arrival time, τ_k . In the above equations, the term $\langle U(f) \rangle$ is referred to as the transfer function of the expected ultrasonic wavelet in the material including the effects of scattering, $\alpha_s(f)$ and attenuation, $\alpha(f)$. Both the $\alpha_s(f)$ and $\alpha(f)$ are dependent on the grain size and frequency of ultrasonic waves. If the measurement is in the Rayleigh scattering region [1], then,

In this paper we examine the application of the linear predictive technique [3], a method of spectral estimation for grain signal characterization. In particular, linear predictive analysis is applied to grain signals using the second and third order autoregressive processes. Then, the spectral features obtained by this method are used for pattern recognition and grain size characterization.

LINEAR PREDICTIVE THEORY

Assume the measured grain signal $r(n)$ is an AR process with 'p' parameters, then the predictive value of the sampled grain signal $\hat{r}(n)$ is defined as:

$$\hat{r}(n) = -\sum_{i=1}^p a_i r(n-i) \quad (4)$$

where the a_i refers to the AR coefficients and the p is the order of the AR model.

The best estimate of a_i can be obtained by solving the following equations [3]:

$$\Phi(0, j) + \sum_{i=1}^p a_i \Phi(i, j) = 0; \quad 1 \leq j \leq p \quad (5)$$

*This work is supported in part by ONR#N000014-86-K-0520

where the correlation function $\Phi(i, j)$ is:

$$\Phi(i, j) = \sum_n r(n-i)r(n-j) \quad (6)$$

Equation (5) is known as the normal equation and can be used for solving a_i since there are p equations and p unknown AR coefficients.

Theoretically, as the number of predictor parameters increases, the estimation error will decrease. A low guess for model order results in a highly smoothed spectral estimate, and a high order introduces spurious and undesired details into the spectrum. Therefore, choosing the order of the model becomes a key problem in linear predictive analysis. In our study, we found that the low order of the AR process smooths the randomness of the grain signal spectrum and displays the trend of spectral shift [4,5]. In this paper, we will use both the second and the third order model to characterize the spectral-shift in the expected frequencies of returned echoes. A comparison between the classification performance based on the probability of error using both the second and third order model is presented.

Second and Third Order AR Models

From the basic linear predictive method, the system transfer function of a second order AR model can be written as :

$$H(z) = \frac{1}{1+a_1z^{-1}+a_2z^{-2}} \quad (7)$$

Since $H(z)$ represents the transfer function of an ultrasonic grain signal which is a bandpass signal, the poles must be complex [4,5]. The phase of the complex poles is the resonating frequency of the second order AR process, f_r , and can be presented as:

$$f_r = \frac{1}{2\pi T} \tan^{-1} \left(\frac{\sqrt{4a_2 - a_1^2}}{a_1} \right) \quad (8)$$

where T is the sampling period.

The maximum value of the power spectrum (i.e., maximum energy frequency) can be found in terms of AR coefficients:

$$f_m = \frac{1}{2\pi T} \cos^{-1} \left(\frac{a_1 a_2 + a_1}{-4a_2} \right) \quad (9)$$

In general, the maximum frequency, f_m , is not equal to the resonating frequency, f_r [4-6]. The maximum energy frequency is defined as the phase in which the product of distance from the point at unit circle to the poles is minimum. When the bandwidth of the system decreases, the f_m will approach to f_r . This can be confirmed by an analytical solution of the second order AR model [4]. The numerical evaluation of $f_r - f_m$ indicates that the resonating frequency of a second order AR system can be approximately represented by the frequency of the maximum energy, and can also be correlated to the frequency shift inherent in random grain signals.

A closer spectrum evaluation can be obtained using a third order autoregressive system:

$$H(z) = \frac{1}{A(z)} = \frac{1}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}} \quad (10)$$

The resonating frequency of the third order model is [5]:

$$f_r = \frac{1}{2\pi T} \tan^{-1} \left[\frac{3\sqrt{3}(A-B)}{2a_1 - 3(A+B)} \right] \quad (11)$$

where,

$$A = \left| -\frac{a_3}{2} + \frac{a_1 a_2}{6} - \frac{a_1^3}{27} + \sqrt{\Delta} \right|^{\frac{1}{3}} \quad (12)$$

$$B = \left| \frac{a_3}{2} - \frac{a_1 a_2}{6} + \frac{a_1^3}{27} + \sqrt{\Delta} \right|^{\frac{1}{3}} \quad (13)$$

$$\text{and } \Delta = \left[\frac{a_2}{2} - \frac{a_1^2}{6} \right]^2 + \left[\frac{a_3}{3} - \frac{a_1 a_2}{9} + \frac{2a_1^3}{81} \right]^3 \quad (14)$$

The maximum energy frequency of a third order system can be obtained by taking a derivative of the power spectrum with respect to f , and setting it equal to zero.

After a number of algebraic steps, solutions for the maximum energy frequency can be obtained [5]:

$$f_m = \frac{1}{2\pi f T} \cos^{-1} \left\{ \frac{1}{2a_3} \left[-\frac{1}{3}(a_1 a_3 + a_2) + \sqrt{\frac{1}{9}(a_1 a_3 + a_2)^2 - \frac{1}{3}(a_1 a_2 a_3 + a_2 a_3^2 + a_1 a_3 - 3a_3^2)} \right] \right\} \quad (15)$$

Similar to a 2nd order AR model, the 3rd order model results in different maximum energy and resonating frequencies. Through the analysis of a large number of computer simulated data, the normalized difference of these two parameters ($|f_r - f_m|$) were found to be less than 20%, and the estimated values for both f_r and f_m were also found to be highly correlated with the actual center frequency of ultrasonic wavelets.

COMPUTER SIMULATION

In our earlier study [1,2], it was shown that grain scattering influences the frequency content of detected echoes. Echoes backscattered from specimens with different grain sizes result in different values for the resonating frequency, the maximum energy frequency, and AR coefficients [5]. Therefore, these parameters are examined in order to find their correlation to the grain size distribution. In order to classify the specimens with different grain sizes, a Euclidean distance classifier, D_j , which measures the distance between the class j with the clustering center, \bar{a}_j , and the estimated parameters a_j [4], is used:

$$D_j = \left[\sum_{i=1}^p (a_i - \bar{a}_{ij})^2 \right]^{\frac{1}{2}} \quad (16)$$

Where p is the number of parameters, and, if AR coefficients are used, p also represents the order of the AR model. In this study, the clustering center, \bar{a}_{ij} , is computed using a set of training signals.

Computer simulations were used in evaluating the performance of the second and third order linear predictive models for classifying the grain signals representing different grain sizes. It is assumed that the transfer function of the expected ultrasonic wavelets, $\langle U(f) \rangle$, are Gaussian in shape with center frequencies of 5.0, 5.2, and 5.4 MHz, and 3 dB bandwidth of 2.5 MHz. To depict the intensity of the detected echoes, a random number generator with a Rayleigh probability distribution is used [4]. In addition, a uniformly distributed random number generator is used for determining the position of the scatterers. The Euclidean distance classifier utilized in this study measures the distance between two sets of linear predictive parameters or the differences between the resonating frequencies, or between the frequencies of maximum energy. The decision is made based on the nearest neighbor rule. The cluster centers are formed by using fifteen sets of training data (i.e., five sets for each cluster center).

Tables 1A and 1B present the estimated mean values of the resonating and the maximum energy frequencies for 2nd and 3rd order AR models along with their standard deviations. As shown in this table, both estimated resonating and maximum energy frequencies are closely related to the actual value of simulated center frequencies, although consistently biased. For example, the estimated resonating frequencies and maximum energy frequencies of a 2nd order AR model are consistently higher than the actual center frequencies with nearly the same variances. However, estimated resonating frequencies and maximum energy frequencies of a 3rd order AR model have a different tendency. The estimated resonating frequencies show a low bias with the actual center frequencies, while the maximum energy frequencies show a higher bias for all three classes of signals with very small variances. It is evident from Table 1A and 1B that an estimate of resonating frequencies or maximum energy frequencies tracks or correlates with actual simulated center frequencies. Consequently, these estimates can be used as features representing the spectral properties of the random grain signal. Furthermore, the estimation of both resonating and maximum energy frequencies for a 2nd order AR model follows the actual center frequency much more closely and consistently than that of a 3rd order AR model. This observation implies that in order to estimate the center or maximum energy frequencies, the second order model would be a better choice, and is consistent with the work of Kuc and Li [6] who used a second order AR model for estimating center frequency.

Table 2 shows the classification results based on AR coefficients, f_r and f_m extracted from 2nd and 3rd order AR models for three classes of signals with center frequencies 5, 5.2, and 5.4 MHz. The results shown in the table confirm that low order linear predictive models (AR) can effectively characterize the random grain signals. A comparison of the results for 2nd and 3rd order AR models indicates that the probability of correct classification of the 3rd order AR model is higher than that of the 2nd order

AR model. That is expected since the 3rd order AR model extracts more frequency information from the random

spectrum than the 2nd order AR model.

The computer simulation shows that misclassification is most likely to occur in the region between two adjacent frequency classes, (e.g., 5.0 and 5.2 MHz or 5.2 and 5.4 MHz). This observation indicates that, when the frequency difference between the two adjacent signal classes increases, the misclassification will decrease.

Results shown in Table 2 suggest that the classification performance with AR coefficients, in both resonating and maximum energy frequencies, are nearly the same for the 2nd order AR model, with the slight difference being caused by an estimation error or by the relatively small number of training patterns used for this study. The probabilities of correct classification when AR coefficients, resonating frequency and maximum energy frequency are used are significantly different for a 3rd order AR model. This is not surprising since both the resonating and maximum energy frequencies are functions of AR coefficients and represent only a reduced feature vector of the estimated AR coefficients. As a result, classification performance will degrade, although this degradation is not that significant. In summary, the classification results shown in Table 2 reveal that the feature vector formed by AR coefficients can effectively characterize the frequency difference of the grain signals, and the correct classification can be as much as 88% for the situation in which the frequency shift is less than 4%.

EXPERIMENTAL RESULTS

Experimental studies were conducted using a Panametric broadband transducer with 6.22 MHz center frequency and a 3-dB bandwidth of 2.75 MHz. Two specimens examined in this study are type 1018 steel blocks. One steel block has an average grain size of 14 μm (referred to "Steel") while the other was heat-treated at 2000 °F, resulting in an average grain size of 50 μm (referred to "Steel-2000"). The specimens were placed at the far fields of the transducer and data was acquired with a 100 MHz sampling frequency. Experiments with the same equipment setting (i.e. the same sampling rate, trigger level, damping and gain.) were repeated at 20 different locations for each specimen. Each measurement was obtained by averaging 256 measurements in order to eliminate system noise, and each data string consisted of 2048 samples.

Probabilities of correct classification were computed using the Euclidean distance and the final results were tabulated in Tables 3 and 4. An inspection of results given in Table 3 reveals that the difference between the estimation of resonating frequency and the maximum energy frequency for a second order AR model is smaller than that of a third order AR model. This is not surprising since the real pole in a 3rd order model contributes to the estimation of maximum energy frequency. Another important observation one can make from the values given in Table 3 is that both the resonating frequency and maximum energy frequency are shifted down as the grain size increased from 14 μm (Steel) to 50 μm (Steel-2000). This observation is consistent with our previous work performing spectral analysis using homomorphic processing [2].

Consistent estimates of AR coefficients, resonating

frequencies and maximum energy frequencies, make them suitable for grain scattering feature vectors to be used for classification. The results given in Table 4 indicate that resonating and maximum energy frequencies for both second and third order AR systems can classify the grain scattering signals effectively with the probability of correct classification higher than 62%. Better classification performance can be obtained using AR coefficients. The probability of correct classification with a third order model is 85% which is higher than that of the second order model. This implies that, by increasing the order of the AR model, the frequency information extracted from the random signal is increased and therefore, a better classification result can be obtained.

CONCLUSION

In this paper, we have developed a mathematical basis for second and third order autoregressive models to evaluate the spectral shift in grain signals. Grain signal classification is performed utilizing a Euclidean distance classifier based on AR coefficients, resonating and maximum energy frequencies. Comparisons of performances between two AR models have also been provided. Results obtained from both computer simulated and experimental data are very encouraging, and the probability of correct classification is found to be as high as 88% for the third order model under the condition in which the expected frequency shift is less than 4%.

REFERENCES

- [1] J. Sanjie and N. M. Bilgutay, "Quantitative Grain Size Evaluation Using Ultrasonic Backscattered Echoes", *Journal of the Acoustical Society of America*, Vol. 80, pp. 1816-1824, Dec. 1986.
- [2] J. Sanjie, T. Wang and N. M. Bilgutay, "Analysis of Homomorphic Processing for Ultrasonic Grain Signal Characterization", *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, Vol. 36, pp. 365-375, May, 1989.
- [3] J. Makhoul, "Linear Prediction: A Tutorial Review", *IEEE Proceedings*, Vol. 63, pp. 561-580, 1975.
- [4] J. Sanjie, T. Wang, X. Jin and N. M. Bilgutay, "Application of Linear Predictive Analysis in Ultrasonics Grain Signal Characterization", *Review of Progress in Quantitative Nondestructive Evaluation*, edited by D. O. Thompson and D. E. Chimenti, Vol. 8, Plenum, 1989.
- [5] T. Wang, *Ultrasonic Signal Processing and Pattern Recognition in Evaluating the Microstructure of Materials*, Ph.D. Dissertation, Illinois Institute of Technology, 1987.
- [6] R. Kuc and H. Li, "Reduced-order Autoregressive Modeling for Center Frequency Estimation", *Ultrasonic Imaging*, pp. 244-251, July, 1985.

TABLE 1A. The Estimated Resonating & Maximum-Energy Freqs. (Simulated Data, 2nd Order Model)

Center Freq. (MHz)	f_r (MHz)	f_m (MHz)
5.0	5.12 ± 0.0053	5.14 ± 0.0053
5.2	5.31 ± 0.0058	5.35 ± 0.0058
5.4	5.52 ± 0.0062	5.55 ± 0.0062

TABLE 1B. The Estimated Resonating & Maximum-Energy Freqs. (Simulated Data, 3rd Order Model)

Center Freq. (MHz)	f_r (MHz)	f_m (MHz)
5.0	4.67 ± 0.0041	5.60 ± 0.0074
5.2	4.82 ± 0.0042	5.82 ± 0.0065
5.4	4.97 ± 0.0042	6.04 ± 0.0065

TABLE 2. Classification Results Using Simulated Data

Parameters	AR Coeff.	f_r	f_m
Probability of Correct Decision	74.5%	76.5%	78.3% (2nd order)
Probability of Correct Decision	88.3%	81.7%	81.7% (3rd order)

TABLE 3. The Estimated Resonating & Maximum-Energy Freqs. (Experimental Data)

Samples	f_r (MHz)	f_m (MHz)
Steel	6.32 ± 0.0215	6.31 ± 0.0220 (2nd order)
	6.13 ± 0.0072	6.97 ± 0.0197 (3rd order)
Steel-2000	5.68 ± 0.0362	5.63 ± 0.0388 (2nd order)
	6.03 ± 0.0177	6.87 ± 0.0494 (3rd order)

TABLE 4. Classification Results Using Experimental Data

Parameters	AR Coeff.	f_r	f_m
Probability of Correct Decision	72.5%	67.5%	67.5% (2nd order)
Probability of Correct Decision	85.0%	62.5%	67.5% (3rd order)