FREQUENCY DIVERSE STATISTIC FILTERING FOR CLUTTER SUPPRESSION

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Abstract

By combining the conventional order statistic filtering concept with the split-spectrum processing (SSP) technique, a new method called frequency diverse statistic filtering is introduced in this paper. Three types of frequency diverse statistic filters, namely weighted mean, median and absolute-minimization are examined. The analysis shows that if the target and the clutter spectra are known individually, the Wiener filter can be realized by frequency diverse statistic filtering using a linear operation (i.e. weighted mean). However, if only the input signal is known, the frequency diverse statistic filter with a nonlinear, order statistic operation (i.e. median or absolute-minimization) can be used resulting in SNR enhancement. Both computer simulation and experimental data have been used to evaluate the performance of the filters and verify the theoretical analyses.

1. Introduction

Although the optimal linear filtering theory was developed almost a half century ago and found wide applications in the areas of signal processing, control and communications, nonlinear and statistic filtering methods are still under investigation in many situations in which the linear filtering is inadequate. More recently, a statistical scheme called median filtering has been used with some success in the area of signal processing [1]. The median filtering is realized by replacing the input signal value at each point by the median of the sample values in a finite neighborhood (n - L/2, n + L/2) about that point

$$y(n) = OS_M[\, x(k),\ k = n\text{-}L/2, \ldots, n\text{+}L/2] \qquad (1)$$

where $OS_M(.)$ is the median operator. It is also possible to apply this concept to an ensemble of signals with different spectral contents. One method which yields such an ensemble is the split-spectrum processing (SSP) technique introduced in the late 70's to enhance target detection in ultrasonic non-destructive testing [2]. This technique creates a frequency diverse ensemble of decorrelated narrowband signals by windowing the spectrum of the received wideband signal and using linear or nonlinear statistic algorithms to form the output signal. In the past, considerable success has been reported in signal-to-noise ratio (SNR) enhancement using this technique [2].

In this paper, a more generalized scheme is introduced called the *frequency diverse statistic filter* shown in Fig.1, which combines the SSP technique with the statistic filtering theory. The filter output can be defined as

$$y(n) = STA\_OP\,[\, x_1(n), \ldots, x_M(n)\,] \qquad (2)$$

where STA_OP represents a statistic operation (i.e. mean, median, minimization, and so on), and $x_i(n)$ are the narrowband signals centered at different frequencies.

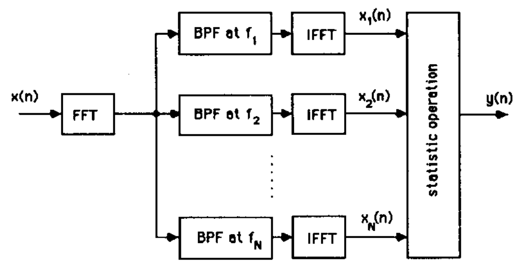

Fig.1 Implementation of frequency diverse statistic filter

2. Theory

Since the theoretical analysis of a generalized frequency diverse statistic filter is difficult and requires numerical methods, several assumptions will be made to permit an analytical solution. However, it can be shown that the basic conclusions presented here remain valid for the simulated and experimental data. The input signal x(n) which consists of the target signal s(n) and the noise term n(n) is assumed to exist over the entire frequency range. The filtered narrowband signals $x_i(n)$ shown in Fig.1 contain only a unique single frequency component (i.e. harmonics of x(n)). Therefore,

$$x_i(n) = \frac{1}{N} A_i \cos[\frac{2\pi}{N} i\,(n - t_i)] \qquad \text{for } i=0, 1, \ldots, N\text{-}1 \quad (3)$$

where A_i and t_i are the amplitude and time delay of the i^{th} frequency component, respectively. Furthermore, it will be assumed that the noise is a bandlimited stationary random process with known spectral terms a_i (i=0, 1, ..., N-1), yielding harmonics

$$n_i(n) = \frac{1}{N} a_i \cos[\frac{2\pi}{N} i\,(n - \tau_i)] \qquad \text{for } i=0, 1, \ldots, N\text{-}1 \quad (4)$$

where τ_i , (i=0, 1, ..., N-1) are assumed to be independent random variables uniformly distributed over the interval (0, 1, ..., N-1). Since the target signal s(n) is assumed to be white with fixed time delay T

$$s_i(n) = \frac{1}{N} \cos[\frac{2\pi}{N} i\,(n - T)] \qquad \text{for } i = 0, 1, \ldots, N\text{-}1 \quad (5)$$

2.1 Weighted Mean and Wiener Filtering

The weighted mean of the input signal harmonics $x_i(n)$ is a basic statistic operation which can be expressed as

$$y(n) = \sum_{i=0}^{N-1} w_i\, x_i(n) \qquad \text{for } n=0, 1, \ldots, N\text{-}1 \qquad (6)$$

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where w_i (i=0, 1, ..., N-1) are the weighting factors. Since each $x_i(n)$ contains a single frequency component, the "optimal linear" weighting factors w_i can be found directly from the Wiener filter transfer function [3]

$$H(f) = \frac{S(f)}{S(f) + N(f)} \quad (7)$$

where S(f) and N(f) are the signal and the noise power spectra, respectively. From Eqs.(4) and (5) it can be shown that S(i)=1 and $N(i)=\frac{a_i^2}{2}$ [4]. Therefore,

$$w_i = H(i) = \frac{1}{1 + \frac{a_i^2}{2}} \qquad \text{for } i=0, 1, \ldots, N-1 \quad (8)$$

Based on the above analysis, Wiener filtering (i.e optimal weighted mean filtering) can be realized by using the system structure of Fig.1 with the weighting factors in Eq.8.

Since the target signal s(n) is assumed to be a δ function (i.e. white), signal-to-noise ratio can only be enhanced if the noise spectrum is non-white. In practice, it has been shown that the target and noise spectra do not completely overlap [5-6]. Therefore, the Wiener filter can be used to enhance target detection.

In this paper, the normalized output signal-to-noise ratio (SNR) will be defined as

$$(SNR)_Y = \frac{E\{y^2(T)\}}{\sum_{n=0}^{N-1} E\{y^2(n)\}} \quad (9)$$

Hence, the maximum output SNR becomes unity for the noise-free case. The SNR for the weighted mean filtering case can be derived as [4]

$$(SNR)_Y = \frac{1}{N} \frac{\left(\sum_{i=0}^{N-1} w_i\right)^2 + \frac{1}{2}\sum_{i=0}^{N-1} w_i^2 a_i^2}{\sum_{i=0}^{N-1} w_i^2 + \frac{1}{2}\sum_{i=0}^{N-1} w_i^2 a_i^2} \quad (10)$$

Substituting Eq.(8) into Eq.(10), yields the SNR for the Wiener filter [4]

$$(SNR)_W = \frac{1}{N} \frac{\left(\sum_{i=0}^{N-1} \frac{1}{2 + a_i^2}\right)^2 + \frac{1}{2}\sum_{i=0}^{N-1}\left(\frac{a_i}{2 + a_i^2}\right)^2}{\sum_{i=0}^{n-1}\left(\frac{1}{2 + a_i^2}\right)^2 + \frac{1}{2}\sum_{i=0}^{N-1}\left(\frac{a_i}{2 + a_i^2}\right)^2} \quad (11)$$

The input SNR can be found from Eq.(10) by setting $w_i=1$

$$(SNR)_X = \frac{1}{N} \frac{N^2 + \frac{1}{2}\sum_{i=0}^{N-1} a_i^2}{N + \frac{1}{2}\sum_{i=0}^{N-1} a_i^2} \quad (12)$$

The SNR enhancement for the Wiener filter is the ratio $(SNR)_W/(SNR)_X$ based on Eqs.(11) and (12), which yields a measure of performance.

2.2 Median and Absolute-Minimization Filtering

The frequency diverse median filter can be defined as

$$y(n) = x_{(N/2+1)}(n) \quad (13)$$

where $\{ x_{(i)}(n) \}$ is obtained from the ordered sequence of N harmonics of the input signal

$$x_{(1)}(n) \le x_{(2)}(n) \le \ldots \le x_{(N/2)}(n) \le x_{(N/2+1)}(n) \le \ldots x_{(N)}(n) \quad (14)$$

The performance of the frequency diverse median filter can only be demonstrated experimentally, since an analytical representation has not been obtained.

Another type of frequency diverse statistic filter is the absolute-minimization filter, which can be represented as follows

$$y(n) = \min(|x_0(n)|, |x_1(n)|, \ldots, |x_{N-1}(n)|) \quad (15)$$

where $x_i(n)$ (i=0, 1, ..., N-1) are the i^{th} harmonics of the input signal. The probability distribution function for y(n) can be obtained as

$$F_{y(n)}(y) = 1 - \prod_{i=0}^{N-1} [1 - F_{|x_i(n)|}(y)] \quad (16)$$

where, $F_{|x_i(n)|}(\cdot)$ is the distribution function of $|x_i(n)|$.

In the limit that noise contains only the k^{th} harmonic the input signal harmonics become

$$x_i(n) = \begin{cases} \frac{1}{N}\cos[\frac{2\pi}{N} i\,(n - T)] + \frac{C}{N}\cos[\frac{2\pi}{N} i\,(n - \tau_i)] & \text{for } i=k,\ N-k \\ \frac{1}{N}\cos[\frac{2\pi}{N} i\,(n - T)] & \text{elsewhere} \end{cases} \quad (17)$$

where $a_k=a_{N-k}=C$ and τ_i is a random variable. Note that for $i \neq k$ and $i \neq N-k$ (i.e. signal only case), the results are non-random and may be characterized by [4]

$$F_{|x_i(n)|}(\,|x_i(n)|\,) = u(\,|x_i(n)| - |\cos[\frac{2\pi}{N} i\,(n - T)]|\,) \quad (18)$$

The remaining two terms (i=k and i=N-k), which correspond to the signal plus noise case, yield random terms. It can be shown that the non-random terms characterized by Eq.(18) will dominate the output distribution function in Eq.(16), regardless of the statistics of the random terms. Therefore, in general it can be shown that [4]

$$F_{y(n)}(y)\Big|_{\substack{n \neq T \\ n \neq T+N/2}} = u(y) \quad (19)$$

In other words, the output of the absolute-minimization filter y(n) will be zero except for n=T (target location) and $n=T+\frac{N}{2}$ (ghost location). Furthermore, we can prove that $E\{y^2(T+\frac{N}{2})\}=E\{y^2(T)\}$. Therefore, the signal-to-noise ratio at the output of the absolute-minimization filter becomes

$$(SNR)_M = \frac{E\{x^2(T)\}}{E\{x^2(T)\} + E\{x^2(T + N/2)\}} = \frac{1}{2} \quad (20)$$

Note that the SNR is independent of C. Finally, the performance of the absolute-minimization filter $(SNR)_M/(SNR)_X$ can be obtained based on Eqs.(12) and (20).

Since the absolute-minimization filter contains a nonlinear statistic operation, it cannot be characterized by a conventional spectral representation. However, if we examine the filtering process, it is clear that at each time instant n, only one of the harmonics $x_i(n)$ will be selected and the corresponding value will be recorded as the process output y(n). Since each narrowband window corresponds to a specific frequency, the statistical distribution of the narrowband windows selected within a given signal interval N can be used to characterize the frequency response of the absolute-minimization filter in a statistical sense, termed the *spectral histogram*. Thus, the spectral histogram is defined as the distribution (histogram) of each frequency component (i.e. harmonics) of the input signal selected to form the output and may be considered analogous to the Wiener filter transfer function [7]. It can be shown that adaptive frequency diverse statistic filters can be obtained based on the spectral histogram.

The SNR enhancement for the Wiener and the absolute-minimization filters are plotted in Fig.2 for N=64. It can be shown that the best performance for the Wiener filter occurs under the single frequency noise assumption described by Eq.(17) [4]. Figure 2 shows that the performance of the absolute-minimization filter is lower than the Wiener filter due to the distortion (ghost echo) effect described above. However, the distortion can be removed by further processing. For example, a modified absolute-minimization filter can be achieved using the additional samples obtained by linearly combining the neighborhood harmonics of the input signal [4]. As seen in Fig.2, the performance of this modified absolute-minimization filter is slightly superior to the performance of the Wiener filter. Note that in the above analysis, the noise spectrum is assumed to be a single frequency, which is the ideal case. Therefore, the results obtained above give the upper bound for the filter performance. Furthermore, it can be shown that the optimal SNR enhancement values are given by $\lim_{C\to\infty}[(SNR)_W/(SNR)_X] = N - 2$ and $\lim_{C\to\infty}[(SNR)_M/(SNR)_X] = N$, for the Wiener and the modified absolute minimization filters, respectively [4]. However, it is important to note that the major difference between the two filters is that the Wiener filter requires the the knowledge of the signal and noise spectra, while the absolute-minimization filter can be achieved without this information.

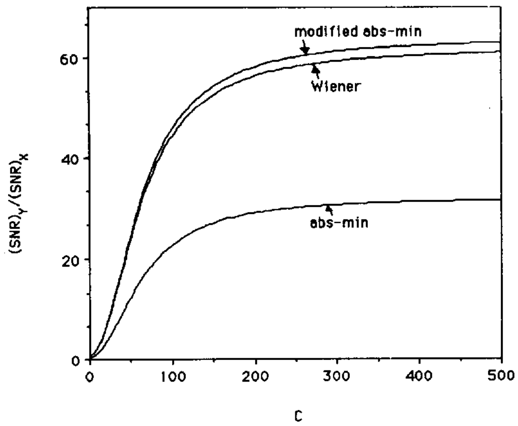

Fig.2 SNR enhancement for the Wiener and absolute-minimization filters

3.Simulation Results

Simulation results for the frequency diverse statistic filters with N=64 are examined in this section. The time domain signals and the corresponding spectra are shown in Figs.3 and 4, respectively. Note that in the simulation example, the noise power spectrum is not limited to a single harmonic but covers a range of frequencies as shown in Fig.4. From Fig.3 it is clear that SNR enhancement can be achieved by all three filters with the modified absolute-minimization filter resulting in the best performance. Note that the Wiener filter is obtained based on the signal and noise power spectra shown in Fig.4, while the frequency diverse median and absolute-minimization filters do not require this information. In addition, Fig.4 shows that the spectral histograms of the median and absolute-minimization filters have shapes similar to the Wiener filter transfer function.

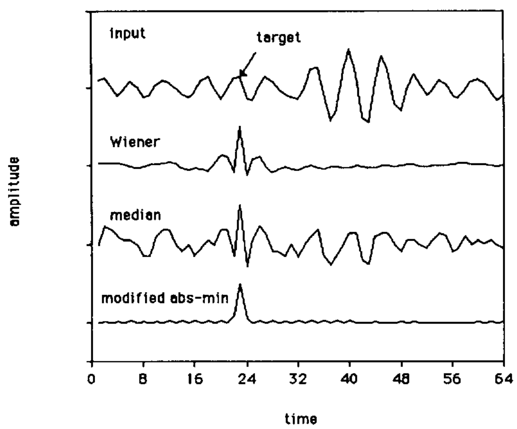

Fig.3 Input signal and filter outputs for simulated data

4. Experimental Results

The frequency diverse statistic filters have also been tested using ultrasonic flaw detection data as shown in Figs.5 and 6. The input signal is obtained using a 5 MHz transducer and corresponds to a cylindrical shaped heat treated stainless steel sample of average grain size 160 μm. A flat-bottom hole inside the sample simulates the flaw. Note that in this case, the input signal is bandlimited to frequencies between 3-8 MHz as shown in Fig.6, but cannot be separated into signal (target echo) and noise components. Figure 5 shows that the target echo

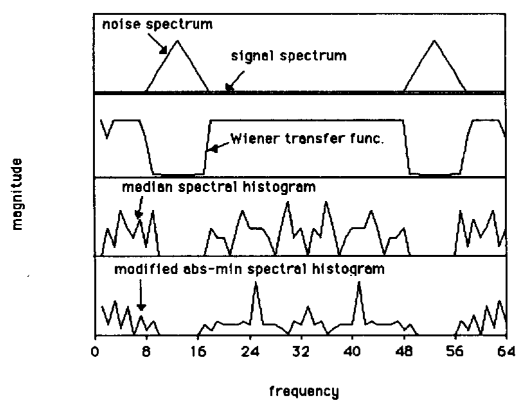

Fig.4 Noise spectrum and filter frequency responses for simulated data

component of the input signal is completely masked by noise. Since the location of the target is known, the optimal bandpass filter parameters can be found by trial and error, yielding the transfer function shown in Fig.6, which approximates the Wiener filter [6]. It is clear that the target can easily be identified at the Wiener filter output. However, the Wiener filter approach is not practical, because it requires *a priori* target information. The frequency diverse median and absolute-minimization filters were also examined using 50 narrowband signals with bandwidth b=0.4 MHz and frequency separation Δf=0.2 MHz in the 0-10 MHz region of the input signal spectrum. It is clear that the median filter provides only limited improvement with high probability of false alarm, while the flaw target can be readily identified by the absolute-minimization filter as shown in Fig.5.

Similar to the simulation case, the spectral histograms for the median and absolute-minimization filters produce high values about the passband of the Wiener filter as shown in Fig.6. Therefore, instead of the trial and error approach, it is possible to design a Wiener filter based on the spectral histograms of the median or absolute-minimization filters. In addition, if the input signal is reprocessed using the median and absolute-minimization filters in the frequency range suggested by the spectral histograms, the target detection can be further improved as shown by the last two curves in Fig.6. These techniques are referred to as the adaptive median and the adaptive absolute-minimization filters, respectively.

5. Summary

By combining the statistic filtering concept with the SSP technique, a class of frequency diverse statistic filters were developed in this paper. Mathematical analysis shows that the Wiener filter can be realized based on the frequency diverse weighted mean filtering. However, the frequency diverse absolute-minimization filtering can produce comparable results to the Wiener filter even when the signal and noise spectra are unknown. Simulation and experimental data presented here support the theoretical analysis and indicate that the frequency diverse median filter can be used as an alternative technique.

Although only three types of frequency diverse statistic filters have been presented in this paper, the work can be extended readily using alternative statistic operations.

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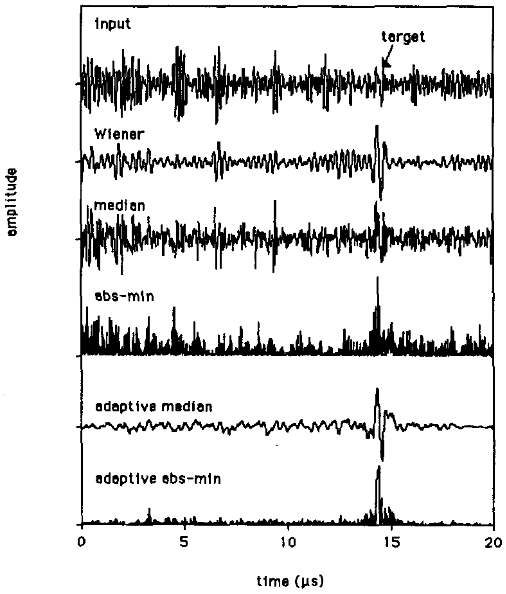

Fig.5 Input signal and filter outputs for the experimental data

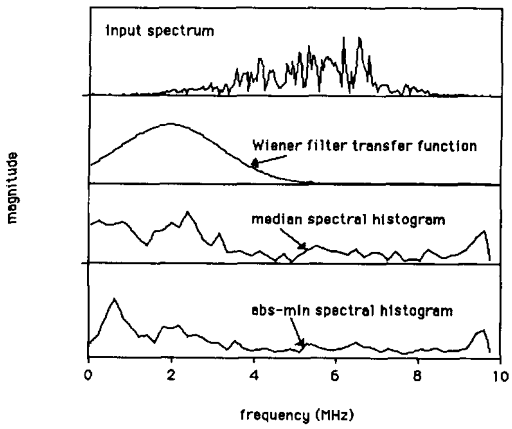

Fig.6 Input signal spectrum and filter frequency responses for the experimental data