Spectral Evaluation of Ultrasonic Flaw Echoes Using Wigner Distribution

K.K. Chin, X.M. Jin and J. Saniie

Department of Electrical and Computer Engineering
Illinois Institute of Technology
Chicago, IL 60616

ABSTRACT

In ultrasonic flaw detection, the ultrasonic signal backscattered from the microstructure of the material results in a complex and random pattern (i.e., clutter) which interacts with and distorts the flaw echoes, making detection of flaws difficult. However, flaw echoes often exhibit different frequency content than those of microstructure noise, and in this paper Wigner distribution (i.e., a time-frequency representation of signal) is studied to resolve this difference. Although Wigner distribution (WD) shows high frequency concentration and accurate frequency estimation in certain instances, it does not necessarily yield high frequency resolution. Furthermore, the interference introduced from the multi-frequency cross terms complicates interpretation. In this paper, the performance of WD is evaluated using several simulated ultrasonic signals. In particular, WD is used for detecting flaw echoes and characterizing dispersive signals backscattered from inhomogeneous materials. These signals were also characterized using short-time Fourier transform (STFT) and results indicate that STFT outperforms WD.

I. INTRODUCTION

Spectral analysis is a useful technique by which certain hidden features in the time domain can be displayed. These features are basically related to the presence or absence of energy in certain frequency bands. With the availability of digital computers, high sampling rate analog-to-digital converters, and fast algorithms for Fourier transform, it becomes quite practical to characterize the backscattered ultrasonic echoes by spectrum analysis. Time-frequency representation and analysis is particularly useful for characterizing dispersive signals in which different frequencies arrive at different times (i.e., nonstationary signals). Wigner distribution (WD) and short-time Fourier transform (STFT) are commonly used in time-frequency evaluation of nonstationary signals.

WD was introduced in 1932 by Wigner with application in quantum mechanics [1]. In recent years, several published papers have dealt with the applications of Wigner distribution in the area of signal processing [2-5], signal synthesis, detection and estimation [6,7], and ultrasonic medical imaging and tissue characterization [8-10]. In this paper we examine the properties and performance of WD for estimating frequency and quantitatively characterizing ultrasonic signals measured in nondestructively evaluating materials. In particular, WD is used for detecting flaw echoes having poor signal-to-noise ratio due to the presence of clutter backscattered from the microstructure of the material.

II. WIGNER DISTRIBUTION PROPERTIES

The cross Wigner distribution of two analog signals f(t) and g(t) is defined as [2]

$$WD_{f,g}(t,\omega) = \int_{-\infty}^{\infty} e^{-j\omega\tau} f(t+\frac{\tau}{2}) g^*(t-\frac{\tau}{2}) d\tau \ . \tag{1}$$

Hence, the auto Wigner distribution of an analog signal f(t) becomes

$$WD_f(t,\omega) = \int_{-\infty}^{\infty} e^{-j\omega\tau} f(t+\frac{\tau}{2}) f^*(t-\frac{\tau}{2}) d\tau \ . \tag{2}$$

WD has certain properties that make it attractive in both theoretical analysis and practical implementation. It is especially advantageous in analyzing dispersive signals in which different frequency contents arrive at different times. But, it also exhibits a unique drawback when new frequency components are introduced. Newly added frequency components will degrade the performance of frequency estimation, hence making the analysis of the signal difficult. The bilinear property can be used to address the effect of cross terms. If a signal is the superposition of two signals f(t) and g(t), then the auto WD of this signal is

$$WD_{f+g}(t,\omega) = WD_f(t,\omega) + WD_g(t,\omega) + 2Re[WD_{f,g}(t,\omega)], \tag{3}$$

where $WD_{f,g}(t,\omega)$ is the cross Wigner distribution of two signals f(t) and g(t). Often, this cross term creates new frequency components which makes the analysis of multi-component signals difficult.

For practical reason, a window function is usually applied to the signal before the WD is evaluated, and this is referred to as the pseudo Wigner distribution (PWD) derived as

$$PWD_{f_m}(t,\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} WD_f(t,\xi) WD_m(0,\omega-\xi) d\xi \ . \tag{4}$$

PWD is equivalent to the original WD of the signal, $WD_f(t,\omega)$, convolves in frequency with the WD of the window evaluated at time zero, $W_m(0,\omega)$, resulting in a smoothed version of the $WD_f(t,\omega)$.

From an implementation point of view, if we digitize the signal in both time and frequency, then the discrete Wigner distribution of a finite discrete signal f(n) becomes [2,3]

$$WD_f(n,m) = 2 \sum_{k=-N+1}^{N+1} e^{-j\frac{km2\pi}{M}} f(n+k) f^*(n-k) \ , \tag{5}$$

where M = 2N - 1. Note that the above equation is just a discrete Fourier transform (DFT) of the signal $f(n+k)f^*(n-k)$ for each time instant n. Hence fast Fourier transform (FFT) can be applied efficiently to reduce the calculation time.
To avoid the aliasing effect resulting from the sampling process, either i) the sampling rate must be at least twice the Nyquist rate or ii) f(n) must be made an analytic signal [2,3].

WD of a signal, real or complex, is always real, and this can be examined using discrete Fourier transform (DFT) of such a function. However, if one uses an FFT algorithm which usually demands the number of samples to be a power of 2, a time shift must be inserted in order to make the signal, $f(n+k)f^*(n-k)$, symmetrically even. This can be achieved by multiplying appropriate phase terms with the result of FFT{$f(n+k)f^*(n-k)$} for each time instant n. Also, if the highest frequency contained in signal f(n) is Δ, then 2Δ is the highest frequency contained in the signal, $f(n+k)f^*(n-k)$, because of the multiplication. Care should be taken when the frequency content of the WD of a signal is evaluated. In addition, to avoid the aliasing problem and to reduce some cross terms introduced from the negative frequencies, analytic signals are used throughout this paper. For signal f(n), its analytic form can be derived from the following operation

$$f_a(n) = f(n) + j\{Hil[f(n)]\}\ , \qquad (6)$$

where *Hil* represents the Hilbert transformation. In effect, its spectrum becomes

$$F_a(\omega) = \begin{cases} 2F(\omega)\ , & \omega>0 \\ F(0)\ , & \omega=0 \\ 0 & ,\ \omega<0 \end{cases} . \qquad (7)$$

in which only positive frequencies survive.

III. APPLICATION OF WD TO ULTRASONIC SIGNALS

In order to examine the properties of WD, several signals are simulated and their WDs are evaluated. The first signal examined here is a Gaussian function modulated by a cosine function with a frequency of 10 MHz and a sampling rate of 100 MHz (Figure 1a). The WD of this signal is represented in Figure 1b from which one can tell that the frequency is highly concentrated and its relationship with the signal in time is apparent. Note that in this figure, O represents origin, T represents time, and F represents frequency. This convention is followed throughout the paper. Figure 1c is just a zoom-in of Figure 1b which indicates that the WD of a Gaussian function is Gaussian in both time and frequency. Also the marginal property can be confirmed from Figure 1d and Figure 1e. The frequency content of this modulated Gaussian function is then estimated using the first moment of WD,

$$\Omega_f = \frac{\frac{1}{2\pi}\int_{-\infty}^{\infty}\omega\, WD_f(t,\omega)d\omega}{\frac{1}{2\pi}\int_{-\infty}^{\infty}WD_f(t,\omega)d\omega} \qquad (8)$$

As shown in Figure 1f, the single frequency component 10 MHz in the signal is reflected as 20 MHz because of the multiplication process in the calculation of WD. The fluctuations at both ends in the time axis ($0<t<0.5\times10^{-6}$ and $2\times10^{-6}<t<2.5\times10^{-6}$) are introduced because there are virtually no signals present. Now if a noise signal

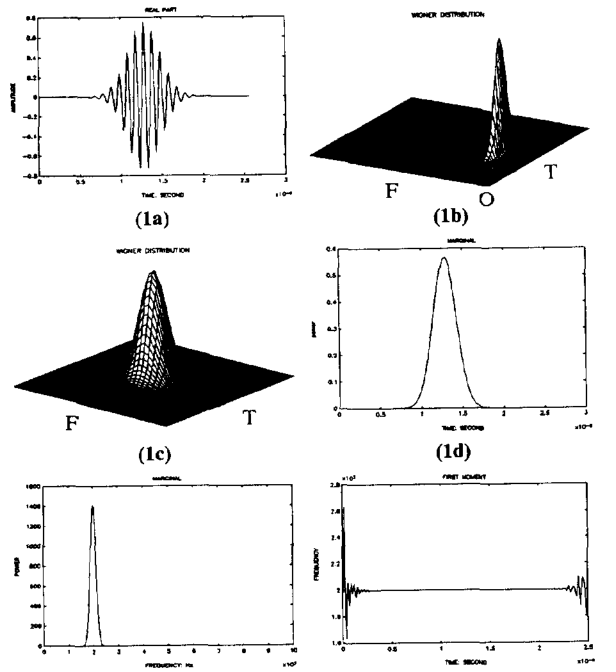

Figure 1 WD of an ultrasonic echo of Gaussian shape, **(a)** ultrasonic echo **(b)** WD **(c)** zoom-in WD **(d)** time marginal **(e)** frequency marginal **(f)** frequency estimate.

(uniformly distributed between -A/2 and A/2) is added to the signal (Figure 2a), then the WD will degrade gradually as the noise power increases (Figure 2b). But, it soon reaches a point when the effect becomes so detrimental that the signal component is buried and renders the WD almost useless (Figure 2c). In addition, the frequency estimation using the first moment method fails even when a small noise power is added to the signal (Figure 2d).

The effect of the cross term is examined using a signal (Figure 3a) consisting of two Gaussian functions separated

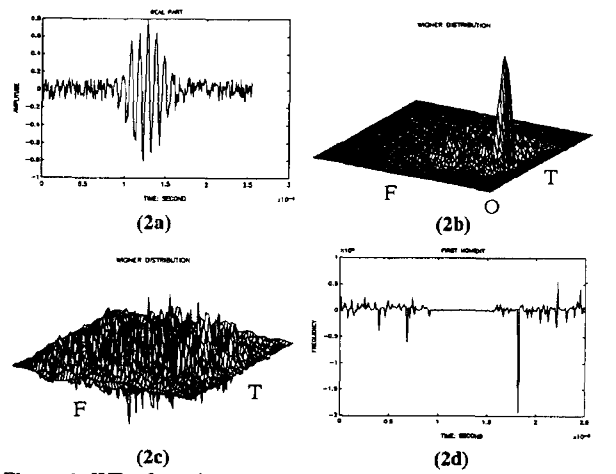

Figure 2 WD of an ultrasonic echo corrupted by noise, **(a)** echo plus noise (A=0.25) **(b)** WD (A=0.25) **(c)** WD (A=2) **(d)** frequency estimate (A=0.25).

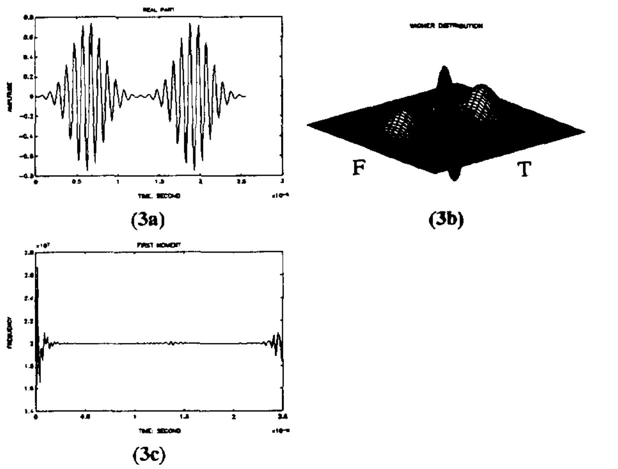

Figure 3 WD of two ultrasonic echoes representing the effect of cross term, (a) ultrasonic echoes having the same frequency (b) WD (c) frequency estimate.

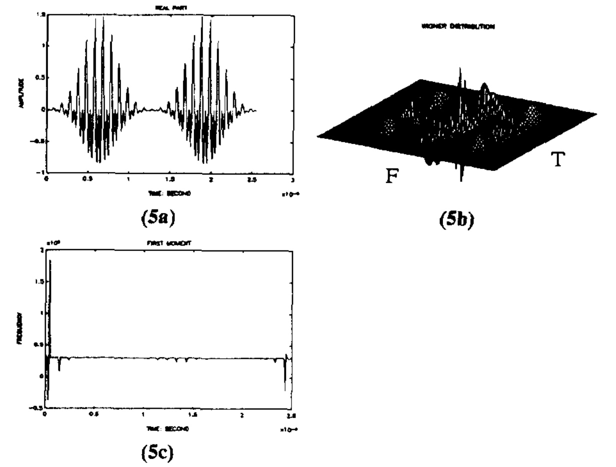

Figure 5 WD of two composite ultrasonic echoes showing the effect of cross terms both in time and frequency, (a) ultrasonic echoes (b) WD (c) frequency estimate.

in time and modulated by a 10 MHz cosine function. Figure 3b is the WD of this signal in which the cross terms are significant, although the frequency contents are still well concentrated. This implies that if two Gaussian functions are too close they will not be distinguishable in the WD due to the cross term. In this case, the frequency estimate is nevertheless very accurate except in those regions where there are no signals (Figure 3c).

To further examine the performance of WD, a signal is simulated in which the first Gaussian function is modulated by a 20 MHz cosine function and the second by a 10 MHz cosine function (Figure 4a). This signal can be considered to be a simplified dispersive signal in which different frequency components arrive at different times. Since two Gaussians are far apart in time-frequency space (Figure 4b), their cross terms do not interfere with the arrival times of the actual signal. But, there can be a potential problem if the signal components are too close either in time or frequency. In spite of cross terms, the frequency estimate is still held valid in this case as shown in Figure 4c.

To demonstrate the destructive effect of cross terms, a test

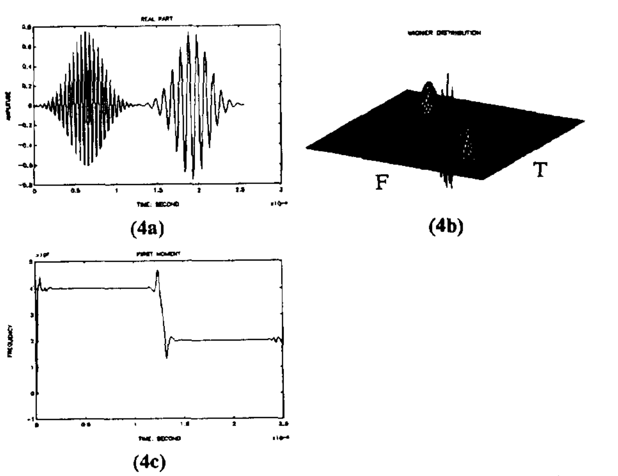

Figure 4 WD of two ultrasonic echoes having two frequencies, (a) ultrasonic echoes (b) WD (c) frequency estimate.

signal is generated which is the sum of two Gaussians modulated separately by two cosine functions of frequency 10 MHz and 20 MHz (Figure 5a). As shown in Figure 5b, the cross terms are prominent and distributed over the time-frequency region which makes the identification of the actual signal components much more difficult. In addition, the frequency estimate does not reveal the actual frequency content because of cross terms (Figure 5c).

After careful examination of the WD of several typical signals discussed above, we further present its application to dispersive signals. In ultrasonic testing, certain materials exhibit dispersive characteristics [11]. Ultrasonic dispersion is often caused by the effect of scattering and absorption. Other factors that contribute to dispersion could be material constants such as density, elastic moduli, and geometry. In ultrasonically testing dispersive material, the pulse is dispersive and does not maintain its original shape. For evaluation purposes, a raised cosine chirp signal is generated (Figure 6a) to represent a dispersive ultrasonic signal. The WD of this signal is shown in Figure 6b, where the frequency increases linearly with time which makes the cross terms indistinguishable because they are buried in the actual signal. So the frequency estimate is still valid which can be confirmed from Figure 6c. This is a rather unusual case in which the cross term effect is of little or perhaps no concern.

In ultrasonic flaw detection, the ultrasonic signal backscattered from the microstructure of materials often results in a random pattern which may mask the flaw echoes. This makes the detection of flaw difficult. However, the time-frequency representation of detected echoes can be used for detecting changes in frequency resulting from the presence of flaw echoes. An ultrasonic grain signal is simulated in order to evaluate the effectiveness and sensitivity of the WD in this particular application. This simulated signal is generated by superimposing multiple echoes with random positions and random amplitudes having the same behavior of random multiple interfering echoes, energy loss, and frequency content as the experimentally measured grain signal. Note that in the simulation process, random positions are uniformly distributed and random amplitudes are Rayleigh distributed. It is assumed that the mean ultrasonic wavelet

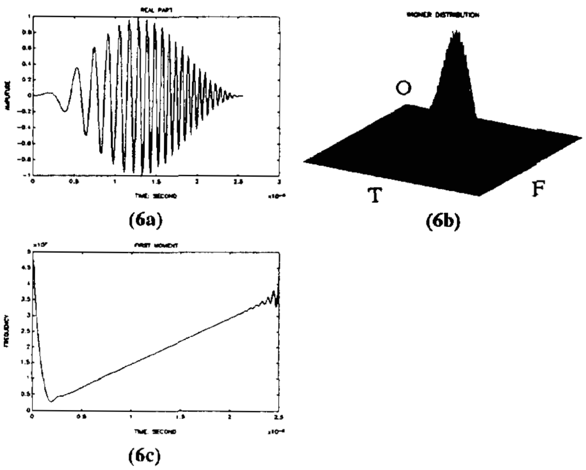

Figure 6 Ultrasonic dispersive echo (raised cosine chirp signal), (a) ultrasonic echo (b) WD (c) frequency estimate.

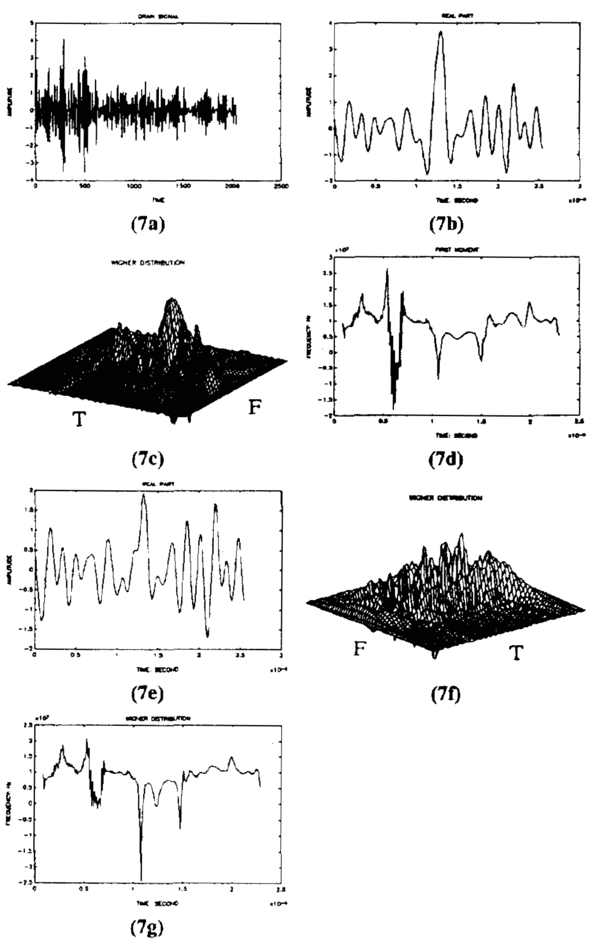

Figure 7 WD of an ultrasonic flaw echo dominated by multiple interfering grain echoes, (a) ultrasonic grain signal (b) a selected portion of grain signal with a flaw echo (A=5) present (c) WD (A=5) (d) frequency estimate (A=5) (e) a selected portion of grain signal with a flaw echo (A=2.5) present (e) WD (A=2.5) (f) frequency estimate (A=2.5).

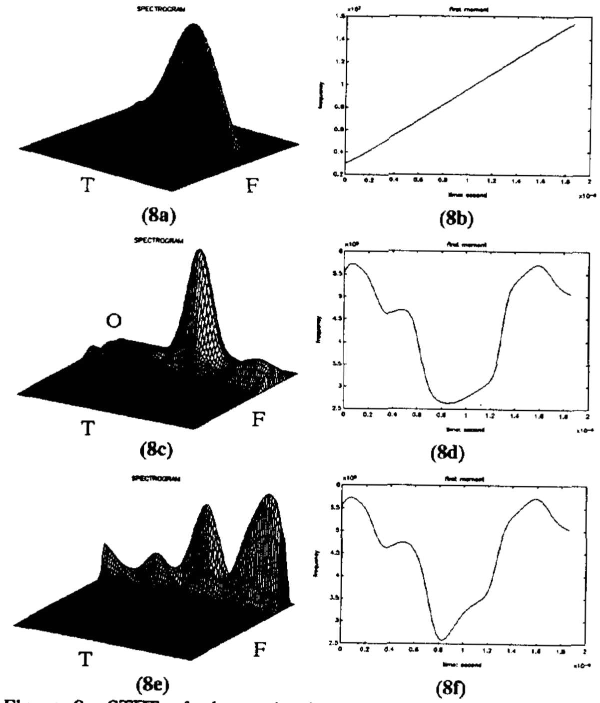

Figure 8 STFT of ultrasonic signals (a) STFT of an ultrasonic dispersive echo (raised cosine chirp signal) (b) frequency estimate of (a) (c) STFT of a selected portion of grain signal with a flaw echo (A=5) present (d) frequency estimate (A=5) (e) STFT of a selected portion of grain signal with a flaw echo (A=2.5) present (f) frequency estimate (A=2.5).

is Gaussian in shape with a center frequency of 5 MHz and a 3dB bandwidth of 2 MHz. The entire simulated grain signal consists of 2048 points with a 100 MHz sampling rate (Figure 7a). The flaw signal with a center frequency of 3 MHz is superimposed on the grain signal. In order to save computation time, PWD is used with a square window 256 samples in length. In doing so, the reduced resolution and the smoothing effect of PWD in the frequency domain may result in some degradation in the detection process. The example presented here is a grain signal with a length of 256 which is superimposed on a flaw signal of amplitude 5 (Figure 7b, A=5). The WD of this signal is presented in Figure 7c in which the flaw signal is present and cluttered by the interfering cross terms. The frequency estimate shown in Figure 7d varies so significantly that the information revealed is obscured to an extent that is very difficult to interpret. If the amplitude of the flaw signal is reduced to half of the previous one (Figure 7e, A=2.5), the flaw signal components in the WD will then be crowded by the cross terms to such an extent that it becomes almost impossible to determine the presence of flaw echo (Figure 7f). For the same reason, the frequency estimate does not reveal the true frequency content of the flaw echo (Figure 7g).

The overall performance of WD for ultrasonic signals appears to be limited. As an alternative method, short-time Fourier transform (STFT) is also studied with respect to the performance of WD. STFT is the most widely used technique in the analysis of nonstationary signals, although, it often suffers from poor resolution. For a window function h(t) of duration Δ centered at time t, the STFT of the signal f(t) is defined as,

$$STFT_{f,h}(t,\omega) = \int_{t-\Delta/2}^{t+\Delta/2} f(\tau)h(\tau-t)e^{-j\omega\tau}d\tau \ . \qquad (9)$$

Consequently, the spectrogram becomes

$$SP_{f,h}(t,\omega) = |STFT_{f,h}(t,\omega)|^2 \ , \qquad (10)$$

which describes the energy density distribution in the time-frequency space. Note that STFT has an inherent trade off between window length and frequency resolution. In general, it performs better than WD when an appropriate window is carefully selected. In this paper, a Hamming window with a length of 64 is used in evaluating STFT. The first example presented is a raised cosine chirp signal which is the same as the one shown in Figure 6a. As shown in Figure 8a & b, STFT performs with an accuracy equal to that of WD for this signal. The STFT of ultrasonic grain signal (Figure 7b) is presented in Figure 8c and the frequency estimate is shown in Figure 8d. In contrast to the WD result (Figure 7d), the low frequency information associated with the flaw echo can be recognized using STFT. When the amplitude of the flaw echo is reduced by half, the low frequency content representing the flaw echo is still recognizable in the STFT (Figure 8e & f). This simply indicates that STFT for ultrasonic flaw detection outperforms WD because it does not introduce new frequency components.

IV. CONCLUSION

In this paper properties of WD and several types of ultrasonic signals were examined with the emphasis being placed on the application of ultrasound in nondestructive evaluation. Overall performance of WD is appealing, especially in the case of dispersive signals (i.e., raised cosine chirp) and modulated Gaussian functions which all yield accurate estimation of the frequency content. However, WD of the ultrasonic grain signal fails to reveal satisfactory results due to the cross terms, and, the detection of flaw is not a trivial task. Consequently, further processing of WD is necessary in order to reach a valuable conclusion. Perhaps, a singular value decomposition (SVD) technique [8-10,12] may prove to be more effective. Also, other modified WD techniques which can minimize the cross terms [13,14] might perform better when applied to the problem of ultrasonic flaw detection. In this study, WD was compared to STFT which was found to be the more effective technique for detecting flaw.

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