I. Introduction

Ultrasonic pulse-echo imaging of materials to detect flaws is hampered by the presence of interfering and attenuating random scatterers (i.e., grains) associated with the flaw’s environment. The energy loss caused by grain scattering and absorption is frequency dependent. It has been shown [1] that the scattering due to the grain scatterers in the Rayleigh region, filters the broadband pulse and results in an upward shift of the expected frequency. However, this is not the case for flaw echoes since flaws are generally larger in size than the grain and often behave like geometric reflectors. In fact, flaw echoes display a downward shift in their expected frequency caused by the overall filtering effect of attenuation. This downward frequency shift of the flaw is a productive attribute since the bandpass filtering can be used to improve the signal-to-clutter ratio [1]-[3]. The information-bearing frequency bands (those with relatively high signal-to-noise ratio (SNR)) are dependent on both the frequency characteristics of the measuring transducer and scattering properties of materials. The location of these bands must be known a priori in order to properly implement bandpass filtering. Therefore, robust flaw detection techniques which show less sensitivity to the environment are desirable.

Effective techniques for detecting targets in coherent noise are frequency agility and frequency diversity, which have been investigated in radar detection for several decades [4], [5], and more recently in ultrasound [6], [7]. Clutter decorrelation is achieved by simultaneously transmitting with two or more channels centered at different frequencies (frequency diversity), or by shifting the transmitted frequency between pulses (frequency agility). In an ultrasonic imaging system, a practical adaptation of these techniques, referred to as split-spectrum processing (SSP) has been examined [7]. This entails transmitting a broad-band signal into the media and partitioning the received signal in several narrow-band channels as shown in Fig. 1. The observations from the output of these channels are normalized with respect to the power, $\sigma_i$, in each cell, and sent to a detection processor. The detection processor consists of an order statistic (OS) filter and some decision rule. In this paper the decision rule is a simple form of thresholding with respect to amplitude.

Important issues in SSP are the number of observations over frequency, correlation between observations, and statistical information in each observation. From a practical point of view a limited number of useful information-bearing frequency bands exists. Thus, there is an upper bound on the number of partitions (observations) that can be made without a large amount of frequency-band overlap. The correlation between observations can be reduced by reducing the bandwidth overlap of each channel. This will limit the number of observations and the bandwidth of each frequency band. It is necessary to point out that reducing the bandwidth of each filter results in degradation of the resolution that may mask flaw echoes. Correlation is not as critical to the performance of the detector as is selecting the proper frequency range containing significant flaw information, although, this knowledge of the frequency range is not generally known a priori.

An important step in detection is to extract information from the signal that would minimize some error criterion. Several types of processors that extract such information are minimization and averaging [6]-[8]. Minimization has shown to be more effective for the situation in which the flaw signal was present in all of the channels, although, this is not always the case in practice. The flaw signal may not be present in some frequency bands due to sensitivity to frequency shifts and/or significant attenuation caused by the grains of the material. Under these circumstances, a more robust operator is needed and minimization may not perform satisfactorily.

Minimization, median, and maximization processors all fall under the category of order statistic (OS) filters that have been readily developed in the statistics field [9], and have found application in radar, sonar and image processing [10]-[17]. OS filters have the ability to emphasize statistical separation between samples belonging to two different hypotheses [14]. In this paper, the performance of order statistic filters in conjunction with split-spectrum processing is analyzed in the context...
of ultrasonic flaw detection. The split-spectrum processing technique provides a set of observation features corresponding to different frequency bands representing uncorrelated microstructure noise [7]. When some of the observations deviate from the assumed hypothesis, the OS filter will behave differently. In general, the performance of the OS filter will deteriorate with the contamination of unwanted statistical information. In this study, the OS filter is analyzed when the detected flaw echoes have different statistical distributions. Experimental and simulated results are presented to show how effectively the OS filter can utilize information contained in different frequency bands to improve flaw detection.

II. THEORY OF ORDER STATISTIC FILTERS

In this section, the order statistic filter is analyzed in an effort to optimize detection in a robust manner. The OS filter is shown to be a quantile estimator of the input density function where the performance of the detector can be improved by choosing a quantile region in which there are large statistical differences between the distributions under the two hypotheses (flaw present or not present).

The order statistic filter is a discrete processor that operates on a set of \( n \) simultaneous sample values, \((z_1, z_2, \cdots, z_n)\), corresponding to the \( n \) channels of the SSP output. These \( n \) values are ordered according to amplitude to produce the sequence, below:

\[
z_{1:n} \leq z_{2:n} \leq z_{3:n} \leq \cdots \leq z_{n:n}
\]  

(1)

A given order or rank, \( r \), is chosen and \( z_{r:n} \) is the output of the OS filter. This filter is the median detector when \( r = (n + 1)/2 \) (for odd \( n \)), the maximum detector when \( r = n \) and the minimum detector when \( r = 1 \).

A fundamental question for optimizing OS filters involves finding the relationship between the input and output statistical behavior of the data. Assuming that the frequency-diverse observations, \( z_i \), are independent and identically distributed (i.i.d.) with the probability distribution function \( F_0(z) \), and the probability density function \( f_0(z) \), the distribution function for the output of the OS filter is given by [9]:

\[
F_{Z_{r:n}}(z) = \sum_{i=r}^{n} \binom{n}{i} F_0^i(z)(1 - F_0(z))^{n-i} \quad \text{for} \quad 1 \leq r \leq n
\]  

(2)

where \( z \) represents the value of the observations of the input sequence and \( z_{r:n} \) is the real value of the ordered input sequence, while \( Z \) is the random variable for the input of the OS filter and \( Z_{r:n} \) is the random variable for the ordered sequence of the OS filter. The density function can be found by taking the derivative of (2):

\[
f_{Z_{r:n}}(z) = r \binom{n}{r} F_0^{r-1}(z)(1 - F_0(z))^{n-r} \cdot f_0(z) \quad \text{for} \quad 1 \leq r \leq n.
\]  

(3)

Equation (3) completely describes the output statistics of a general OS filter when the density function of the observations are i.i.d. This equation demonstrates that the input density function, \( f_0(z) \), is weighted by another function given by

\[
w_{r:n}(u) = r \binom{n}{r} u^{r-1}(1 - u)^{n-r} \quad \text{for} \quad 0 \leq u \leq 1
\]  

(4)

where

\[
u = F_0(z).
\]  

(5)

The function \( w_{r:n}(u) \) is referred to as the sort function [14]. The sort function is the beta probability density function and examples of this are shown in Fig. 2 for five observations \( n = 5 \). Note that the sort function’s modal point occurs at equidistant spaces on the \( u \) axis. The expected value for the output of the OS filter is

\[
E[Z_{r:n}] = \int_0^1 F_0^{-1}(u) w_{r:n}(u) du.
\]  

(6)

The sort function acts as a weighting function to emphasize a particular region of the inverse distribution function over the integration as stated in (6). Fig. 3 shows an example of the sort function with \( r = 7 \) and \( n = 25 \) superimposed on a Weibull inverse distribution function with a shape parameter equal to 1.5. It has been shown through our earlier work that the OS filter with increasing \( n \) approaches a consistent estimator of a specific quantile value of the distribution function [14]. Quantile values are a set of points that divide the distribution function domain into equal probability regions. The OS filter as a consistent quantile estimator can be written in the form

\[
\lim_{n \to \infty} E[Z_{r:n}] = F_0^{-1}(u_r)
\]  

(7)

where

\[
u_r = \frac{r - 1}{n - 1} \quad \text{for} \quad 1 \leq r \leq n.
\]  

(8)

In the limit, both \( r \) and \( n \) approach infinity but \( u_r \) remains a finite ratio of \( n \) and \( r \). The OS filter is an asymptotically unbiased estimator of the quantile corresponding to \( u_r \). With finite observations \( n \), the sort function will have some dispersion about the quantile, \( u_r \), that allows the values of neighboring quantiles to influence the output.

The key to understanding the operation of the OS filter is (6). From this it is seen how the parameters \( r \) and \( n \) can be used so that the OS filter emphasizes particular regions in the distributions of the input signals. The OS filtering operation censors the signal values outside this quantile region from the decision rule. This property is useful when the classes of signals exhibit a distinctive statistical difference over a limited range of quantiles, such as what may occur with specular reflective targets.
The optimal rank is dependent on the input distributions that are illustrated by the following two examples. In the first example, we assume the number of observations is 25, and the target-plus-clutter observations (i.e., flaw echoes) are Chi distributed with skewness equal to 0.566 in Weibull clutter (i.e., grain echoes) with a skewness equal to 1.05. Their respective inverse distribution functions are shown in Fig. 4. The performance of the OS filter can be seen in Fig. 5 where the probability of detection for all possible rank values \(r = 1, 2, 3, \ldots, 25\) is plotted for 0-dB and 2-dB signal-to-clutter ratios (SCR). The lower ranks perform significantly better since there is greater separation in smaller quantile regions (see Fig. 4, \(u < 0.6\)). Note that the optimal rank occurs at \(r = 2\) for the lower SCR and \(r = 4\) for the higher SCR. For the higher SCR, the optimal choice is less critical, since for any \(r\) value from 1 to 10, the OS filter shows good performance. In the second example, both target-plus-clutter and clutter are Rayleigh distributed with skewness equal to 0.63 where the inverse distributions are as shown in Fig. 6. In Fig. 7, the optimal \(r\) is 19 for the lower SCR (3 dB) and 20 for the higher SCR (6 dB). For the high SCR the robustness of the upper ranks is self-evident from the visual examination of the inverse distributions of Fig. 6, and will increase as the input SCR increases.

III. ROBUST ANALYSIS OF ORDER STATISTIC FILTERS

The discussion in the previous section has been confined to the assumption that input observations are independent and identically distributed, although it may be more appropriate to assume that input observations are independent and stem from different distributions. Under this condition, the output distribution function of rank \(F_{Z,r}(z)\), from a set of independently...
distributed inputs \( F_i(z), F_2(z), \ldots, F_n(z) \) of the respective random variables \( Z_1, Z_2, \ldots, Z_n \) can be determined by the following relationship [9]:

\[
F_{Z_n}(z) = \sum_{i=1}^n \sum_{j_i < \cdots < j_n} \prod_{i=1}^n F_{j_i}(z) \prod_{i=1}^n [1 - F_{j_i}(z)]
\]

(9)

where the summation \( S_i \) extends over all permutations \( (j_1, j_2, \ldots, j_n) \) of the set of numbers, 1, 2, 3, \ldots, \( n \) for which \( j_1 < j_2 < \cdots < j_n \) and \( j_1 + 1 < j_2 + 1 < \cdots < j_n \). Also the output density function can be written in a closed expression in the form of permanents [18]:

\[
f_{Z_n}(z) = \frac{1}{(n - r)! (r - 1)!} \left[ \begin{array}{c} F_1 \ F_2 \cdots F_n \\ F_1 \ F_2 \cdots F_n \\ \vdots \ \vdots \\ f_1 \ f_2 \cdots f_r \\ P_1 \ P_2 \cdots P_n \\ \vdots \ \vdots \\ P_1 \ P_2 \cdots P_n \end{array} \right]^{r - 1 \text{ rows}}^{1 \text{ row}}^{n - r \text{ rows}}
\]

(10)

where \( \begin{pmatrix} \times \end{pmatrix} \) is a permanent evaluated just as a determinant except all terms in the expansion have positive signed cofactors [19]. The terms \( F_i \) and \( P_i \) (i.e., \( P_i = 1 - F_i \)) for \( i = 1, 2, \ldots, n \) correspond to the input distribution and its respective complement; and \( f_{Z_n} \) corresponds to the output density function for rank \( r \) with the inherent dependence on observation \( z \). Note that for all above terms the variable \( z \) is omitted for brevity of expression. Both output distributions and density functions are cumbersome to evaluate and analyze for large \( n \) since the number of terms increases factorially.

To illustrate the effect of variations in the input distributions, only examples for filters of size \( n = 3 \) and \( n = 5 \) are considered which enables us to make predictions about the behavior of the output density function for larger \( n \). The output distributions of an OS filter with \( n = 3 \) are given by

\[
F_{Z_{1,3}} = F_1 F_2 F_3 
\]

(11)

\[
F_{Z_{2,3}} = [F_1 F_2 P_3 + F_3 F_2 P_2 + F_3 F_2 P_1] + F_{Z_{1,3}}
\]

(12)

\[
F_{Z_{3,3}} = [F_1 F_2 P_3 + F_3 F_2 P_1 + F_3 F_2 P_1] + F_{Z_{2,3}}
\]

(13)

For the situation in which \( n = 5 \), the output distributions become

\[
F_{Z_{1,5}} = F_1 F_2 F_3 F_4 F_5 
\]

(14)

\[
F_{Z_{2,5}} = [F_1 F_3 F_5 (F_3 F_5 P_5 + F_3 P_5) \\
+ F_1 F_5 (F_3 F_5 P_5 + F_5 P_5)] + P_3 F_1 F_3 F_4 F_5 + F_{Z_{2,5}}
\]

(15)

\[
F_{Z_{3,5}} = [F_1 P_3 (P_3 F_4 F_5 + P_3 F_5 P_3 + P_3 F_4) \\
+ F_3 (F_3 F_5 F_3 + F_3 P_3)] + P_1 [F_1 F_3 P_5 + F_1 F_3 P_3] \\
+ P_2 (F_3 F_4 F_5 + P_3 F_4 P_3) + F_{Z_{3,5}}
\]

(16)

\[
F_{Z_{4,5}} = [P_1 P_2 (F_3 F_5 + F_3 P_5) \\
+ P_2 P_3 (F_3 F_5 + F_3 P_5)] + P_3 [F_1 F_3 P_5 + F_4 P_5] \\
+ P_2 (F_3 F_4 F_5 + P_3 F_4 P_3) + F_{Z_{4,5}}
\]

(17)

\[
F_{Z_{5,5}} = P_1 P_2 P_3 (F_3 F_5 + F_3 P_5) \\
+ P_2 P_3 (F_3 F_5 + F_3 P_5) + F_{Z_{5,5}}
\]

(18)

To evaluate the performance of the ordering operation for detection, consider the two hypotheses where a flaw is present, \( H_1 \), and flaw not present \( H_0 \) in each of the output channels of SSP (observations). Let us assume that in the process of making \( n \) observations under the composite hypothesis \( H_1 \), some of the observations, do not provide target information (i.e., null observations). Under this constraint, the performance of the order statistic filter will deteriorate depending on the rank of the filter. In this study, the effect of null observations is examined as a function of rank and number of observations. The basis for our analysis is (10) and computer simulation is implemented for \( n = 3 \) and \( n = 5 \).

For computer simulation, clutter is assumed to be Rayleigh distributed. The flaw plus clutter distribution can vary due to reflective properties and orientation of the flaw. The performance evaluation is examined for two input distributions for the flaw-plus-clutter, where model 1: the \( H_1 \) hypothesis is modeled as a Rayleigh distribution corresponding to a complex flaw with many reflected echoes, and model 2: \( H_1 \) hypothesis is modeled as a Rayleigh distribution shifted by a constant amplitude when the flaw echo is large in magnitude and small variations of the flaw exist over observations. The preformance of the different ranked outputs are evaluated using a fixed threshold with the probability of false alarm (i.e., false detection of flaw) set at 0.001 where the probability of detection is calculated over various input signal-to-clutter ratios.

The performance of model 1, the Rayleigh distributed flaw-plus-clutter, for \( n = 3 \) is shown in Fig. 8(a) where the observations belong to hypothesis \( H_1 \) in all channels corresponding to the case of independently and identically distributed inputs. The maximum rank output of this case performs better due to good statistical separation in the large amplitude observations. The performance with \( n = 5 \) can be seen in Fig. 9(a), where ranks \( r = 4 \) and \( r = 5 \) perform best. Also the median output shows improved performance for \( n = 5 \) compared to lower order ranks.

For model 1, the effect of introducing null observations on the performance of the output ranks, \( n = 3 \), can be seen in Figs.
Fig. 8. Probability of detection versus input signal-to-clutter ratio of model I for each output of the OS filter (n = 3) where: (a) No null observations exist. (b) One null observation exists. (c) Two null observations exist.

Fig. 8. Probability of detection versus input signal-to-clutter ratio of model I for each output of the OS filter (n = 3) where: (a) No null observations exist. (b) One null observation exists. (c) Two null observations exist.

(a)–(c). In the presence of one null observation, the performance is shown as in Fig. 8(b) where the minimum rank shows the greatest deterioration and the other ranks are only slightly affected. Inherently minimization suffers since it searches for the smallest observed values in which the null observations with the lower expected value (H₀ statistics) make up the greatest contribution. Fig. 8(c) shows the performance of the same OS filter (n = 3) with two null observations, the lowest two ranked outputs (median and minimization) have severely diminished in performance that is warranted based on the above discussion.

For n = 5, similar outcomes can be seen in the presence of null observations. Fig. 9(b) shows the effect of introducing one null observation where the performance of the minimum rank is deteriorated, but not as severely as in the case where n = 3. Intuitively, the probability that the null observation will become the minimum output value is smaller for n = 5 than for n = 3. As shown in Fig. 9(c), for the presence of three null observations, ranks 1 (minimum), 2, and 3 (median) outputs falter in performance. Thus, showing the degree of deterioration of each rank is dependent on the number of channels confined to null observations. Therefore, for this model the maximum and/or higher order ranks are most robust under the existence of null observations.

Fig. 9. Probability of detection versus input signal-to-clutter ratio of model I for each output of an OS filter with n = 5. (a) No null observations exist. (b) One null observation exists. (c) Three null observations exist.

Analyzing the second flaw model where underlying distribution of the clutter is shifted by a constant amplitude representing the mean of the flaw observations, the performance of the output ranks are shown for n = 5 in Fig. 10. This model is valid when the flaw produces a dominant backscattered echo and the distribution of the observation is due only to clutter echoes superimposed on the flaw echoes. Fig. 10(a) presents an example where there are no null observations, a case in which the minimum detector performs best. The minimum detector focuses on lower amplitude observations in which the statistical separations are significant. This explains the vast improvement has been observed in ultrasonic flaw detection utilizing the minimum output [8].

The performance in the midst of null observations is shown in Figs. 10(b) and (c), where the deterioration of lower ranks is still the most severely affected. When there is only one null channel for n = 5, Fig. 10(b) shows that the minimum detector performance is limited by the parameters of the distribution and becomes independent of the flaw-to-clutter ratio. Of course this is sensible since we are censoring all channels but the one that is dominated by clutter information. In the presence of a null channel, the median and maximum detectors show good performance overall. Fig. 10(c) presents the performance of the OS filter when there are three null channels, the maximum detector still maintains good performance while the median detector and lower ranks are limited in performance. Thus, when the i.i.d. assumption is violated, minimization and other low ranked outputs will be more sensitive to a deteriorated signal in one of the channels and will exclude all information in other channels. These variations in the underlying density function of observations can be dealt with by concentrating on higher-order ranks that exhibit the property of inclusion whereas all the strong amplitude information is passed to the output. In other words, higher order ranks are robust compared to lower order ranks when the underlying assumptions are violated. It is important to mention that with this operation only limited improvement
IV. EXPERIMENTAL RESULTS

Experimental results were obtained using two different types of transducers, one broad-band (10-MHz, Panametric) and one narrow-band (5 MHz, KB-Aerotech), to examine a steel specimen with a constructed flaw embedded within the sample. The two different transducers are used to demonstrate their ability in retaining flaw information. The flaw is formed by drilling a flat-bottom hole with a 1.5 mm diameter and 2.5 cm depth into the sample. The pulse-echo mode measurements were made using the contact technique and data was acquired using 100MHz sampling rate. The ultrasonic backscattered signals and amplitude spectrums using both transducers, broad-band and narrow-band, are shown in Figs. 11(a) and (b) respectively. The peak amplitude flaw-to-clutter ratios for both measurements are slightly less than 0 dB. The narrow-band spectra of Fig. 11(b) shows information in the approximate frequency range of 1-9 MHz and the broad-band spectra of Fig. 11(a) shows a frequency range of 0-15 MHz.

Both the narrow-band and broad-band measurements were processed with a SSP processor using 9 bandpass filters with a bandwidth of 0.75 MHz and frequency steps of 0.5 MHz between adjacent filters starting at 1 MHz. For the narrow-band measurements, the rectified channel outputs are shown in Fig. 12(a), which indicate the presence of the target in only three
channels ranging from 2.5–3.5 MHz. There is also little fluctuation in the flaw pattern as with other channels in which the noise exceeds the flaw in amplitude. The resulting ranked outputs are shown in Fig. 12(b) where no significant flaw enhancement is present in any channel, although higher rank outputs have better flaw-to-clutter ratio when compared with lower ranks. This is due in part to both the sufficient number of existent null channels and high intensity clutter echoes. In fact, the output of channels 2.5, 3, and 3.5 MHz give better flaw/clutter discrimination than any one of the ranked outputs. On the other hand, with the broad-band measurements, the flaw can be seen in every channel in Fig. 13(a). This corresponds to the ideal case of i.i.d. assumption. As shown in Fig. 13(b), all resulting ranks display strong flaw information as predicted in the case of model 2 where it was assumed that the distribution of the flaw-plus-clutter had a small variance.

The previous example was tailored to a specific frequency range (i.e., 1–5 MHz) ensuring no null channels that may not be possible in practice since such a priori knowledge is required. To be more general in analysis, the broad-band signal is processed using nine filters with a 3-dB bandwidth of 1.5 MHz and a frequency range of 3–11 MHz. Fig. 14(a) shows the channel outputs in which only a few display significant flaw information and over half have clutter information only (i.e., null channels). The ranked outputs can be seen in Fig. 14(b) where the ranked outputs above the median show noticeable improvement in flaw resolution in contrast to the lower ranks which relay very little information. These results suggest that all ranks are potentially useful in flaw enhancement, although lower ranks are more vulnerable and lack robustness for practical applications. In general, the application of this detector for complex targets would depend on the validity of the statistical
modeling of flaw echoes, suggesting the use of higher ranks for robust performance.

V. Conclusion

In this paper, we have presented a theory and application of an OS filter in ultrasonic flaw detection problems. The theory suggests that an optimal rank can be found with the knowledge of the distribution of flaw and grain echoes. Although when statistical information in the observations deteriorates (e.g., null observations), the higher ranked outputs perform better when compared with lower ranked outputs. In this study, it has been confirmed through simulated and experimental results that the higher-order output (maximum, median) detectors perform robustly in various situations where a priori knowledge of the distribution is incomplete or inadequate to adapt to varying testing environments.

REFERENCES


Jafar Saniee (S'80-M'81) for a photograph and biography, please see page 124 of the Transactions.

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