GENERALIZED TIME-FREQUENCY REPRESENTATION OF ULTRASONIC SIGNALS

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ABSTRACT

Time-frequency representation of ultrasonic signals plays an important role in describing the scattering and dispersive effects in materials. Cohen's class of generalized time-frequency representation (GTFR) has been examined for ultrasonic applications. Two special cases of GTFR, Wigner-Ville distribution (WVD) and Choi-William distribution (CWD) are discussed. Due to the bilinear structure, all GTFRs' generate cross-terms for the multicomponent signals. The presence of cross-terms in the WVD of a multicomponent signal obscures the auto-terms. The cross-terms in CWD can be controlled by choosing a proper scale factor which does not effect the marginals. It is shown that the estimation of local time and frequency of ultrasonic echoes corrupted with noise can be accurately performed using CWD. Short-time Fourier transform (STFT) or spectrogram is also discussed as a class of GTFR. STFT or spectrogram does not satisfy the marginals. Simulated results show that CWD outperforms WVD and STFT because it is not marred by severe interference due to cross-terms, still able to satisfy the marginals, and it provides the best resolution in time-frequency plane.

I. INTRODUCTION

Time-frequency representation is a useful tool for simultaneous characterization of ultrasonic signals in time and frequency, e.g., detecting and characterizing dispersive effects and flaw echoes in high scattering materials.

In order to completely specify or represent a signal, in the combined time-frequency plane, the signal energy must indeed have a joint time-frequency distribution. In 1932, Wigner [1] presented a joint probability function for the coordinates and momenta in the study of statistical quantum mechanics. Ville [2] derived the Wigner distribution for analytic signals in 1948, which is known as Wigner-Ville distribution (WVD). In 1946, Gabor [3] presented the idea of an 'information diagram' in which the given signal is expanded into a sum of elementary signals of "minimum" spread in time and frequency, each represented by a rectangle, called a logon. This information diagram gives the approximate description of the energy distribution of the signal in the time-frequency plane. Over the years different time-frequency distributions were derived. Although these distributions are different in behavior and have peculiar properties, each one satisfies the marginals. In 1966, Cohen [4] realized that an infinite number of distributions can be derived from GTFR

$$E(t,\omega)=\frac{1}{4\pi^2}\iiint e^{-j\theta t-j\tau\omega+j\theta\mu}\,\phi(\theta,\tau)\,.\,s^*(\mu-\frac{1}{2}\tau)\,s(\mu+\frac{1}{2}\tau)\,d\mu\,d\tau\,d\theta \tag{1}$$

where s(t) is the signal to be analyzed and $\Phi(\theta,\tau)$ is an arbitrary kernel function. All integrals are from $-\infty$ to ∞ unless otherwise indicated. If $E(t,\omega)$ represents a joint time-frequency energy distribution, then by integrating $E(t,\omega)$ over frequency, we get the instantaneous signal power $|s(t)|^2$, and similarly on integrating over time, we expect to get the energy density spectrum $|S(\omega)|^2$. Also, if the joint energy distribution is integrated over both time and frequency, we expect to get the total energy of the signal. Different distributions can be obtained by choosing different kernels. WVD is obtained by choosing the kernel $\Phi(\theta,\tau)=1$ and this gives

$$WVD(t,\omega)=\frac{1}{2\pi}\int s^*(t-\frac{1}{2}\tau)\,s(t+\frac{1}{2}\tau)\,e^{-j\tau\omega}d\tau \tag{2}$$

Choosing $\Phi(\theta,\tau)$ to be equal to $\exp(-\theta^2\tau^2/\sigma)$ gives

$$CWD(t,\omega)=\iint\sqrt{\frac{\sigma}{4\pi\tau^2}}\,e^{\left\{\frac{-(t-\mu)^2}{4\tau^2/\sigma}\right\}}\,e^{-j\tau\omega}\,.\,s^*(\mu-\frac{1}{2}\tau)\,s(\mu+\frac{1}{2}\tau)\,d\mu\,d\tau \tag{3}$$

as CWD where σ is the arbitrary scale factor. For the time-frequency distribution to satisfy the marginal densities, the kernel function must be chosen to satisfy $\Phi(\theta,0)=\Phi(0,\tau)=1$, and $\Phi(0,0)=1$, if the total energy is to be preserved. A spectrogram is also considered as a member of the GTFR as given by Eq.(1). The total energy of the Spectrogram is the same as that of the signal though it does not satisfy the marginals. The kernel that produces a spectrogram is [6]

$$\phi(\theta,\tau)=\int h^*(t-\frac{1}{2}\tau)\,h(t+\frac{1}{2}\tau)\,e^{-j\theta t}dt \tag{4}$$

where h(t) is the window function. The spectrogram is the squared magnitude of the short time Fourier transform (STFT). The STFT has been most widely used for the analysis of time-varying spectra. But the high resolution can only be achieved with the proper choice of window.

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It must be noted, due to their bilinear structure all the GTFRs produce the so called cross-terms for multicomponent signals.The amount and shape of these cross-terms depend on the particular kernel function identifying the specific distribution. The cross-terms in the WVD of ultrasonic signal [5] makes the interpretation of the signal difficult without further smoothing of WVD. The CWD of ultrasonic signal suppresses the cross-terms with the proper choice of signal independent scale factor.

II. WIGNER-VILLE DISTRIBUTION

We have considered the application of WVD for analyzing ultrasonic signals. The ultrasonic echo can be represented by the following equation

$$s_1(t) = \left(\frac{1}{\pi}\right)^{1/4} e^{-\alpha(t-t_0)^2} e^{\{j\omega_c t\}} \tag{5}$$

where α is a constant inversely proportional to the bandwidth of the signal, ω_c is the center frequency and t_0 is the location of the echo. The WVD of the analytic signal of Eq.(5) is given by

$$WVD_{s1}(t,\omega) = \frac{1}{\sqrt{2\alpha\pi}} e^{\{-2\alpha(t-t_0)^2\}} e^{\left\{-\frac{(\omega-\omega_c)^2}{2\alpha}\right\}} \tag{6}$$

which has the same shape in both the time and frequency direction as shown in Fig. 1.

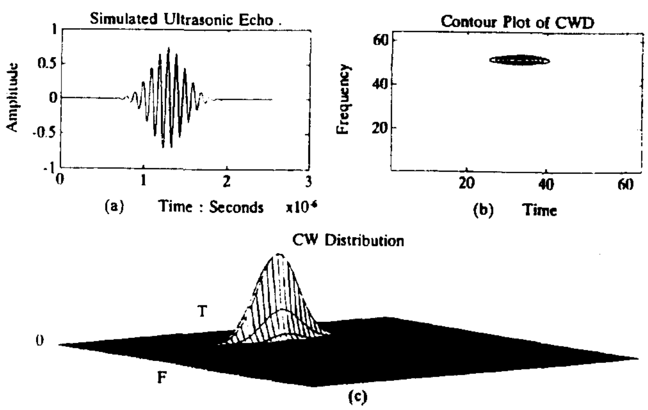

Figure 1. **WVD of an ultrasonic echo**

The monocomponent signals are highly concentrated in WVD. But, if there are multicomponents of the same or different frequencies, the WVD generates auto and cross-term. The interference due to cross-terms in the WVD makes it difficult to represent the signal.

Let us consider the following signal with two ultrasonic echoes with center frequencies ω_1 and ω_2 .

$$s_2(t) = \left(\frac{1}{\pi}\right)^{1/4} \left[e^{\{-2\alpha(t+t_0)^2\}} e^{j\omega_1 t} + e^{\{-2\alpha(t-t_0)^2\}} e^{j\omega_2 t}\right] \tag{7}$$

The WVD of this signal is given by

$$WVD_{s2}(t,\omega)_{(auto-terms)} = \frac{1}{\sqrt{2\alpha\pi}}$$

$$\cdot\left[e^{-2\alpha(t+t_0)^2} e^{-\frac{(\omega-\omega_1)^2}{2\alpha}} + e^{-2\alpha(t-t_0)^2} e^{-\frac{(\omega-\omega_2)^2}{2\alpha}}\right]$$

$$WVD_{s2}(t,\omega)_{(cross-term)} = \frac{1}{\sqrt{2\alpha\pi}}$$

$$\cdot\left[e^{-\alpha t^2} e^{-\frac{\left(\omega-\frac{\omega_1+\omega_2}{2}\right)^2}{2\alpha}} 2\cos((\omega_1-\omega_2)t+\omega t_0)\right] \tag{8}$$

Eq.(8) shows that cross-term is at the frequency in the middle of the two original frequency components and cosine modulated by the difference of frequencies. If the signal is corrupted with noise, then the cross-terms of WVD are all over the t-f plane. Thus WVD cannot represent the true energy distribution of the multicomponent ultrasonic signal with background noise without further processing.

III. CHOI-WILLIAMS DISTRIBUTION

According to Cohen [4], one method of deriving the time-frequency distribution is by utilizing the relationship between the spectrum and autocorrelation,

$$E(t,\omega) = \frac{1}{2\pi}\int R_t(\tau) e^{-j\omega\tau} d\tau \tag{9}$$

where $R_t(\tau)$ is the time-dependent autocorrelation function given by:

$$R_t(\tau) = \frac{1}{2\pi}\int e^{\{j\theta(u-t)\}} \phi(\theta,\tau)$$

$$. s^*(u-\frac{1}{2}\tau) s(u+\frac{1}{2}\tau)\, du\, d\theta \tag{10}$$

In order for the symmetry property of autocorrelation to be maintained for the time-dependent autocorrelation function , i.e.

$$R_t(\tau) = R_t^*(-\tau) \quad . \tag{11}$$

the kernel $\Phi(\theta,\tau)$ should satisfy the following condition

$$\phi(\theta,\tau) = \phi^*(-\theta,-\tau) \tag{12}$$

This is a necessary and sufficient condition for the time-frequency distribution to be real. This can be shown by taking the complex conjugate of Eq.(10) and comparing it with the original.

The well known ambiguity function in radar is a modified form of the characteristic function of Wigner distribution. Cohen proposed a relationship between a generalized ambiguity function(AF) and a time-frequency distribution.

$$A(\theta,\tau:\phi) = \iint E(t,\omega:\phi) e^{\{j(\theta t+\tau\omega)\}} dt\, d\omega \tag{13}$$

where $\Phi(\theta,\tau)$ is the kernel specifying the distribution and AF. By combining Eq.(1) and Eq.(13), the generalized AF can be written as

$$A(\theta,\tau:\phi)=\phi(\theta,\tau)\int e^{j\theta\mu} \cdot s^*(\mu-\frac{1}{2}\tau)s(\mu+\frac{1}{2}\tau)d\mu \qquad (14)$$

As depicted by Eq.(14), the bilinear structure of AF will generate auto-terms and cross-terms for a multicomponent signal. These auto-terms and cross-terms are the counterpart of those of the GTFR. A very interesting and important feature of AF is that while its auto-terms are centered at the origin of the ambiguity plane (θ,τ) its cross-terms do not touch the origin and are located away from it. Therefore, in order to decrease the interference due to the cross-terms in GTFR, the AF must be weighed large when it is close to the origin of the ambiguity plane and small weight be given to AF when it is away from the origin of ambiguity plane. This can be achieved by selecting a kernel $\Phi(\theta,\tau)$ which will assign a large weight when θ and τ are close to the origin and a small weight when they are far away from the origin. These ideas of time dependent autocorrelation and AF led Choi and Williams to devise a new time-frequency distribution[8] that they named exponential distribution This distribution satisfies the following properties of GTFR as described by Classen and Mecklenbrauker [6,7].

1. The CWD is real since its kernel satisfies the condition

$$\phi(\theta,\tau) = \phi^*(-\theta,-\tau) = e^{\left\{\frac{-\theta^2\tau^2}{\sigma}\right\}} \qquad (15)$$

2. CWD is time and frequency shift invariant, i.e.

$$CWD_{t_0}(t,\omega) = CWD(t-t_0,\omega)$$

$$CWD_{\omega_0}(t,\omega) = CWD(t,\omega-\omega_0) \qquad (16)$$

3. The marginals in time and frequency are satisfied

$$\frac{1}{2\pi}\int CWD_s(t,\omega)dt = |S(\omega)|^2 \qquad (17)$$

$$\text{since } \phi(0,\tau) = 1 \text{ for all } \tau$$

where $|S(\omega)|^2$ is the energy spectral density of the signal at frequency w.

$$\frac{1}{2\pi}\int CWD_s(t,\omega)d\omega = |s(t)|^2 \qquad (18)$$

$$\text{since } \phi(\theta,0) = 1 \text{ for all } \theta$$

where $|s(t)|^2$ is power of the signal at time t.

4. The centroid time of CWD at each frequency is equal to group delay because

$$\frac{d}{d\theta}\phi(\theta,\tau)|_{\theta=0} = 0 \qquad \text{for all } \tau \qquad (19)$$

and the centroid frequency of CWD at each time is equal to the instantaneous frequency of the signal since its kernel satisfies the condition

$$\frac{d}{d\tau}\phi(\theta,\tau)|_{\tau=0} = 0 \qquad \text{for all } \theta \qquad (20)$$

CWD can be derived from the time-dependent autocorrelation. The time-dependent autocorrelation function, as given by Eq.(10), can be written as

$$R_t(\tau) = \frac{1}{2\pi}\int r(\mu-t) \cdot s^*(\mu-\frac{1}{2}\tau)s(\mu+\frac{1}{2}\tau)d\mu \qquad (21)$$

where

$$r(\mu-t) = \int e^{\{j\theta(\mu-t)\}}\phi(\theta,\tau)d\theta \qquad (22)$$

and can be calculated, using a CWD kernel, as

$$r(\mu-t) = \sqrt{\frac{\pi}{\tau^2/\sigma}}\,e^{\left\{\frac{-(\mu-t)^2}{4\tau^2/\sigma}\right\}} \qquad (23)$$

then the time-dependent autocorrelation function of CWD can be written as

$$R_t(\tau)=\int\frac{1}{\sqrt{4\pi\tau^2/\sigma}}e^{\left\{\frac{-(\mu-t)^2}{4\tau^2/\sigma}\right\}} \cdot s^*(\mu-\frac{1}{2}\tau)s(\mu+\frac{1}{2}\tau)d\mu \qquad (24)$$

This is a time-indexed autocorrelation function that can be calculated numerically. CWD is the Fourier transform of this function according to Eq.(9). This time-indexed autocorrelation function has a large weight when μ and t are in close proximity and a small weight when they are farther apart.

We examined the CWD of signal s_1 (t) of Eq.(5) with $\alpha=1/2$. The CWD of this signal is

$$CWD_{s_1}(t,\omega) = \frac{1}{\sqrt{\pi}}\int e^{\{-j(\omega-\omega_1)\tau\}}\sqrt{\frac{\sigma}{\sigma+4\tau^2}} \cdot \exp\left\{-\frac{\tau^2}{4}-\frac{\sigma(t-t_0)^2}{\sigma+4\tau^2}\right\}d\tau \qquad (25)$$

CWD of one ultrasonic echo is not as concentrated as WVD because the weight factor in CWD spreads out the cross-terms at the expense of smoothing the auto-components. The auto-terms of the CWD of the signal s_2 (t) of Eq.(7) with $\alpha = 1/2$ are given by

$$CWD_{s_2}(t,\omega)_{(auto-terms)} = \frac{1}{\sqrt{\pi}}\int\sqrt{\frac{\sigma}{\sigma+4\tau^2}} \cdot \left[exp\left\{-\frac{\tau^2}{4}-\frac{\sigma(t-t_0)^2}{\sigma+4\tau^2}\right\}e^{\{-j(\omega-\omega_1)\tau\}} + \exp\left\{-\frac{\tau^2}{4}-\frac{\sigma(t+t_0)^2}{\sigma+4\tau^2}\right\}e^{\{-j(\omega-\omega_2)\tau\}}\right]d\tau \qquad (26)$$

and the cross-terms are as follows:

$$CWD_{s_2}(t,\omega)_{(cross\text{-}terms)} = \frac{1}{\pi}\int\sqrt{\frac{\sigma}{\sigma+4\tau^2}}$$

$$.\exp\left\{j\frac{(\omega_1+\omega_2)}{2}\tau\right\}.\exp\left\{-\frac{\tau^2(\omega_1-\omega_2)^2}{\sigma+4\tau^2}\right\}$$

$$.\left[\exp\left\{\frac{-\sigma t^2}{\sigma+4\tau^2}-\frac{(\tau-2t_0)^2}{4}\right\}.e^{\left\{j\frac{\sigma(\omega_1-\omega_2)t}{\sigma+4\tau^2}\right\}}\right.$$

$$\left.+\exp\left\{\frac{-\sigma t^2}{\sigma+4\tau^2}-\frac{(\tau+2t_0)^2}{4}\right\}.e^{\left\{j\frac{\sigma(\omega_2-\omega_1)t}{\sigma+4\tau^2}\right\}}\right].d\tau \qquad (27)$$

Eq.(26) and Eq.(27) show that by carefully selecting the scale factor σ the cross-terms can be diminished without sacrificing much of the resolution of the auto-terms.

III. SHORT-TIME FOURIER TRANSFORM

Spectrograms have been extensively used for time varying spectrum analysis. They are considered to represent the joint energy distribution of the signal in time-frequency plane. The short-time Fourier transform of a signal is obtained by sliding a window and taking the Fourier transform of the windowed signal. In doing so it is assumed that the signal is stationary during the duration of the window. STFT of a signal s(t) is obtained by calculating the spectrum of $s(\tau).h(\tau\text{-}t)$,where $h(\tau\text{-}t)$ is the window function h(t) centered at t

$$STFT_s(t,\omega) = \int s(\tau)h(t-\tau)e^{-j\omega\tau}d\tau \qquad (28)$$

and the spectrogram is

$$SP_s(t,\omega) = |STFT_s(t,\omega)|^2 \qquad (29)$$

The relative weight given to different parts of the signal in spectrogram depends on the window function. The assumption of short-time stationarity of the signal makes it difficult for the spectrogram to discern muticomponents. The spectrogram only satisfies the realness and shift properties and does not satisfy the marginals as described by Classen and Mecklenbrauker[7]. To estimate both the instantaneous frequency and time delay accurately using a spectrogram, different windows have to be used [4]. The spectrogram can be considered as the time-frequency distribution of the signal smoothed by the time-frequency distribution of the window function.

IV. RESULTS AND DISCUSSION

In Fig.2(a) we have a simulated ultrasonic echo. Its CWD is shown in (b) and the time and frequency marginals are compared in (c),(d) and (e), (f) respectively. These figures show that CWD keeps the marginals intact while smoothing the signal. The frequency marginal shows double the original frequency because of the multiplication process in the calculation of CWD. Fig.2(g) and (h) are WVD and spectrogram of signal. These figures depict that WVD has the highest concentration in both time and frequency for the single ultrasonic echo and the spreading of the CWD depend on the scale factor σ and that of the spectrogram depends on the effective length of the window chosen.

Fig.3(a) is the simulated signal with two ultrasonic echoes of same frequency and corrupted by Gaussian noise. Fig.3(b) is the CWD of (a) using a scale factor σ = 0.2. Fig.3(c)-(f) show that CWD keeps the time and frequency marginals intact. WVD of signal in (a) is shown in (g) and is unusable for discerning the frequency distribution of two echoes.

The contour plots of CWD, WVD, and spectrogram of two ultrasonic echoes with same (figures in the right column) and different (figures in the left column) frequencies, placed near to each other, are shown in Fig.4. These plots indicate that CWD discerns the two echoes, in both time and frequency, better than the WVD and the spectrogram.

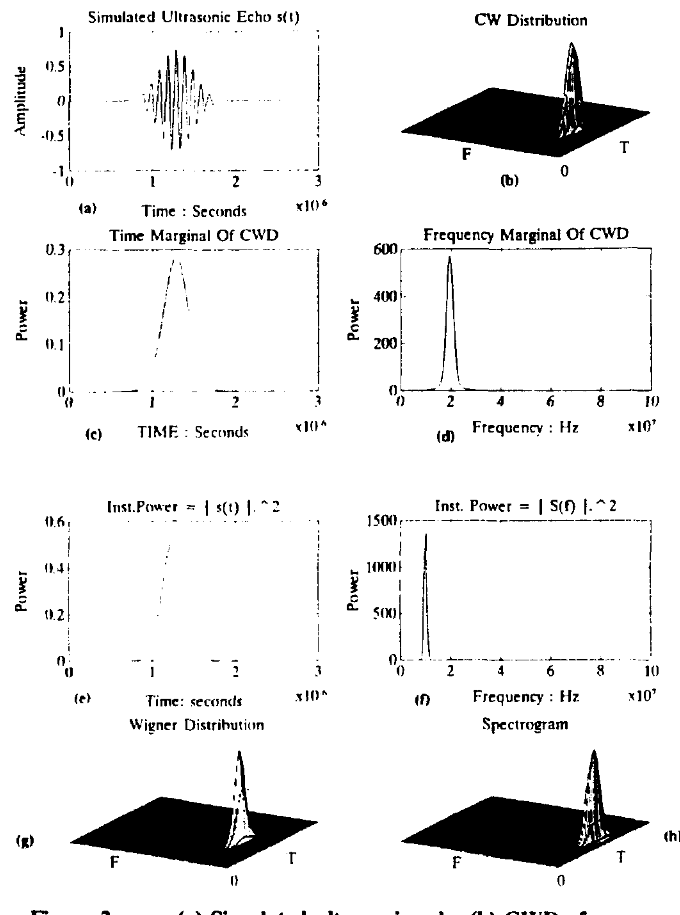

Figure 2. (a) Simulated ultrasonic echo,(b) CWD of the echo,(c),(d) marginals of CWD, (e),(f) signal power and energy density spectrum,(h),(g) WVD and spectrogram of echo

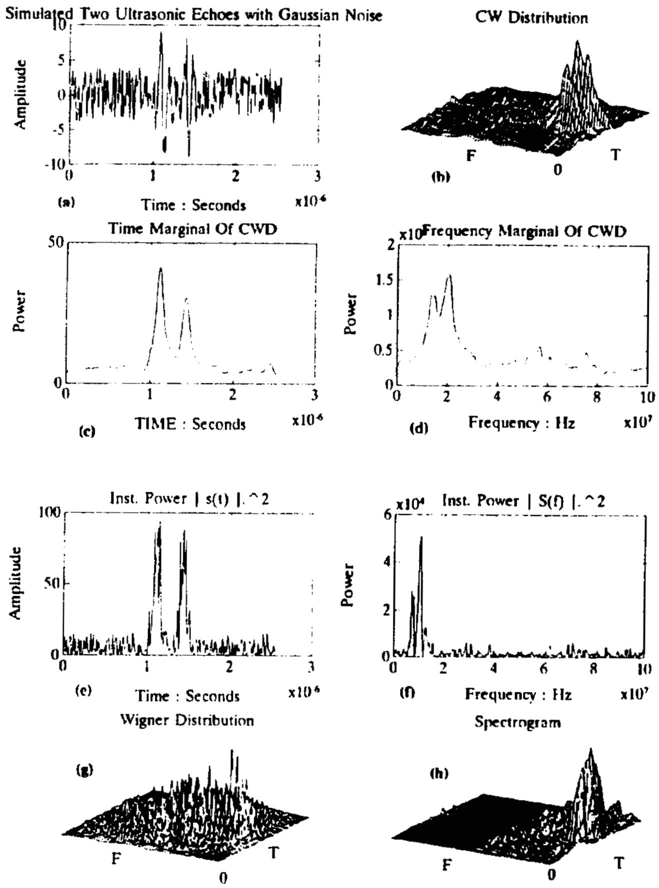

Figure 3. **(a) Two ultrasonic echoes of same frequency corrupted with gaussian noise,(b) CWD of the signal,(c),(d) marginals of CWD,(e),(f) signal power and energy density spectrum, (g),(h) WVD and spectrogram of signal**

V. CONCLUSION

Generalized time-frequency representation for ultrasonic application has been discussed. All GTFRs exhibit difficulties in representation of multicomponent signals due to their bilinear structure. Therefore, in order to display multiple ultrasonic echoes in the time-frequency plane the time-frequency distribution needs to be smoothed or processed further. Results have shown that CWD displays time-frequency information of multiple ultrasonic echoes with reasonable accuracy by suppressing considerably the cross-terms at the expense of minimal loss in resolution. CWD satisfies the marginals to qualify as joint energy distribution and outperforms WVD and STFT in the ultrasonic application of characterizing the dispersive and scattering effects in materials.

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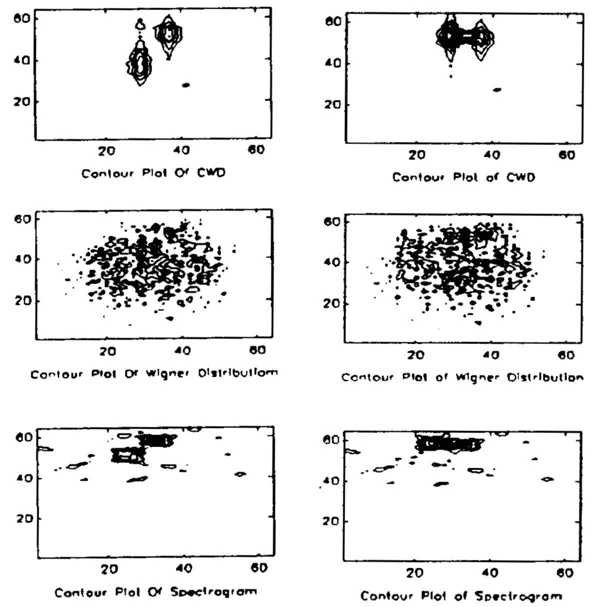

Figure 4. **Contour plots of CWD, WVD and spectrogram of two ultrasonic echoes of same (right column) and different (left column) frequencies, corrupted with Gaussian noise**

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