Model-Based Frequency Estimation for Ultrasonic NDE Applications

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Abstract - In the ultrasonic NDE of materials, spectral analysis of backscattered echoes is a useful tool for flaw detection, frequency-shift estimation, and dispersive echo characterization. In order to evaluate the local information, spectral analysis must be applied to short data segments and must offer high frequency resolution. In this paper three high resolution model-based spectral estimation techniques: the autoregressive (AR) method using the Burg algorithm, Prony's method for exponential signal representation, and MUltiple signal classification (MUSIC) method have been studied for ultrasonic NDE applications. These algorithms have been applied to both simulated data and experimental measurements for frequency estimation and flaw detection. The maximum energy frequency estimates using these methods show excellent sensitivity to changes in the frequency of ultrasonic echoes. The AR method shows a more robust performance for frequency estimation than the Prony or MUSIC methods.

INTRODUCTION

Spectral analysis is a desirable alternative to amplitude evaluation, since the amplitude of the defect echoes can be affected by many factors (some unknown). Often the defect echoes are comparable to the microstructural scattering noise. Spectral analysis is a useful technique by which certain hidden features in the time domain can be displayed. These features are basically related to the maximum energy frequency, or the presence/absence of energy in certain frequency bands. By applying spectral analysis techniques such as the AutoRegressive (AR) method, Prony's and the MUltiple Signal Classification (MUSIC) methods, the frequency information buried in the random pattern of backscattered ultrasonic echoes can be extracted and analyzed. This paper evaluates the above spectral estimation algorithms applied to short data segments of ultrasonic signals for frequency-shift estimation and ultrasonic flaw detection.

The exploration of the frequency content of ultrasonic backscattered signals can give spectral energy profiles corresponding to the grains and the large geometric reflectors (i.e., defects). The research work by Mason and McSkimm [1] shows that scattering by grains contributes to a large portion of ultrasonic attenuation, the remaining attenuation is dependent on absorption which is a linear function of frequency. The model for the overall frequency-dependent attenuation coefficient is defined as:

$$\alpha(z,f) = \alpha_a(z,f) + \alpha_s(z,f) \qquad (1)$$

where $\alpha_s(z,f)$ is the scattering coefficient, $\alpha_a(z,f)$ is the absorption coefficient, z represents the distance that ultrasonic wave traveled and f is the frequency of the ultrasonic wave. The intensity of scattering is a nonexplicit function of individual grain diameters, ultrasonic wavelength, inherent anisotropic character of the individual grains, and the random orientation of the crystallites. In the Rayleigh scattering region (the wavelength, λ , is larger than the average grain diameter, $\overline{D}$), the scattering coefficients vary with the third power of the grain diameter and the fourth power of the frequency, while the absorption coefficient increases linearly with frequency [2]:

$$\alpha_s(z,f) = c_s(z)\overline{D}^3 f^4, \quad \alpha_a(z,f) = c_a(z) f \qquad (2)$$

where $c_a(z)$ is the absorption constant, and $c_s(z)$ is the scattering constant.

In general, the spectrum of defect signals can be modeled as:

$$|R_d(f)| \propto |A(f)|\,|U(f)| \qquad (3)$$

where A(f) is the transfer function corresponding to the attenuation characteristics of the signal propagation path and is proportional to $\exp[-2\int_0^z \alpha(z,f)\,dz]$, where z is the position of the defect. U(f) is the impulse response of the transducer and can be modeled as a bandpass Gaussian-shaped spectrum. In the above equation, it is assumed that the size of the defect is larger than the ultrasonic wavelength and the defect functions like a geometric reflector. The spectrum of the signal received from grain scattering can be modeled as:

$$|R_g(f)| \propto |A(f)|\,|S(f)|\,|G(f)|\,|U(f)| \qquad (4)$$

where, $S(f) = \alpha_s(z,f)$, is the frequency-dependent scattering function, and G(f) is a frequency modulation function representing the sum of small scatterers with random orientation and phases.

In summary, the above equations indicate that grain scattering results in an upward shift in the expected frequency of broadband echoes, while echoes from defects show a downward shift due to attenuation. Therefore, the frequency information can be related to the internal structure of materials. It is the objective of this paper to use the AR, Prony's and the MUSIC methods to obtain the high resolution frequency information for frequency shift estimation and ultrasonic flaw detection.

AUTOREGRESSIVE (AR) METHOD

Assume a measured backscattered signal x(n) to be an AR process with p parameters, then the predicted value of the sampled grain signal $\hat{x}(n)$ can be defined as :

$$\hat{x}(n) = -\sum_{i=1}^{p} a_i\, x(n-i) \qquad (5)$$

Where a_i refers to the AR coefficients and p is the order of the AR model.

In this paper, second-order AR models are used to estimate the maximum energy and resonance frequencies of ultrasonic signals. It has been shown in our earlier study, that the second-order AR model provides the best center frequency estimate of multiple random interfering echoes. Consider $p = 2$, and assume the predictive error is a white noise process, then, the transfer function, $H(z)$, for a second-order AR model can be written as:

$$H(z) = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}} \qquad (6)$$

The maximum energy frequency is defined as the frequency at which the power spectrum of the AR process reaches the maximum value. This frequency can be found as [3]:

$$f_m = \frac{1}{2\pi T}\cos^{-1}\left(\frac{a_1 a_2 + a_1}{-4a_2}\right) \qquad (7)$$

The maximum energy frequency, f_m can be correlated to the frequency shift inherent to ultrasonic scattering echoes.

Figure 1 shows the simulated ultrasonic signal with a center frequency linearly decreasing from 5.5 MHz - 3.5 MHz. By just viewing the signal, it is difficult to detect the built in frequency shift. The second order AR model is applied to this data, and the resulting AR coefficients from Burg's algorithm are used to calculate the maximum energy frequency. The data segment is obtained by applying a Hamming window with a timewidth of 128 sample points. Figure 2 shows the estimated maximum energy frequency. It is clear that the maximum energy frequency estimate correlates to the center frequency-shift in the simulated ultrasonic signal.

An important application of the AR method is ultrasonic NDE for flaw detection. In general, a flaw echo contains a lower frequency component than that of grain echoes. If defects occur in materials, the ultrasonic backscattered echoes will include the defects information. But due to random interference of grain echoes, the presence of flaw information is often unclear. The second-order AR model is used with a Hamming window to estimate the maximum energy frequency and to reveal the frequency information of the flaw echo. In the simulated signal (Figure 3) the flaw echo has a center frequency of 3.5 MHz and is assumed to arrive around 10.24 μs. Figure 4 displays the result of the estimated maximum energy frequency of the second-order AR model. This figure indicates that the maximum energy frequency can effectively separate the flaw echo from the grain scattering noise (note the big frequency dips occurring in Figure 4). Figure 5 shows a backscattered signal from a heat treated steel sample with a flat bottom hole with a radius of 1.5 *mm*. Figure 6 presents the maximum energy frequency of the second-order AR model for the signal shown in Figure 5. It is clear that the second-order AR model performs very well in the detection of flaw echoes.

PRONY'S METHOD

Prony's method is based on the idea that any randomly sampled data, including ultrasonic backscattered echoes, can be modeled as a linear combination of a limited number of exponentials [4]. Although this method is not a spectral estimation technique, it can be used to estimate the power spectrum through the computation of the energy spectral density of the exponential model.

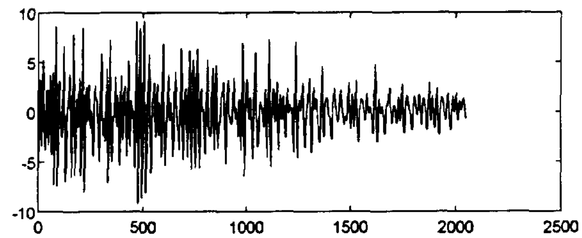

Figure 1. Simulated ultrasonic signal with a simulated frequency-shift.

Figure 2. Frequency-shift estimation (vertical axis) using maximum energy frequencies of the 2nd order AR model.

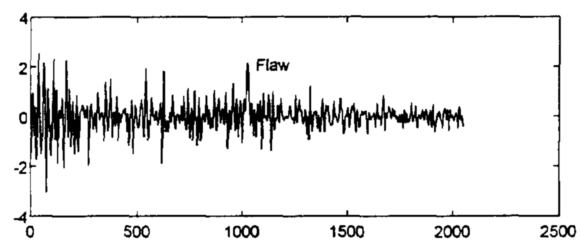

Figure 3. Simulated ultrasonic flaw signal.

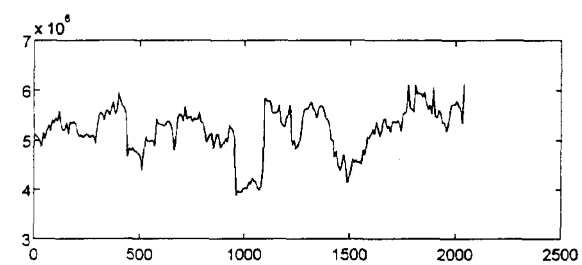

Figure 4. Flaw detection using maximum energy frequency (vertical axis) of the 2nd order AR model (applied to simulated data shown in Figure 3).

The exponential model for *N* complex data samples, $x(1), \cdots, x(N)$, can be expressed as:

$$\hat{x}(n) = \sum_{i=1}^{p} A_i\, e^{[(\alpha_i + j\omega_i)(n-1)T + j\theta_i]} \qquad (8)$$

Where p is the number of exponential terms, A_i is the amplitude of the exponential, T is the time period between adjacent samples, α_i is the damping factor in second^{-1}, ω_i is the sinusoidal angle frequency and θ_i is the sinusoidal initial phase in radians. In order to apply the exponential model to signal spectral estimation, we need to define the Prony spectrum. This method is defined in terms of the estimated data sample $\hat{x}(n)$

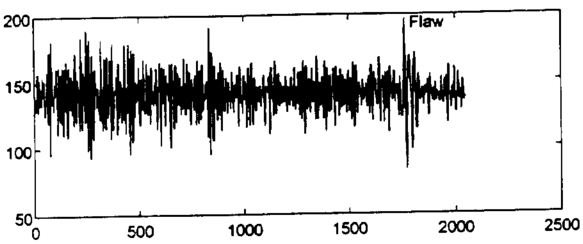

Figure 5. Experimental ultrasonic flaw signal.

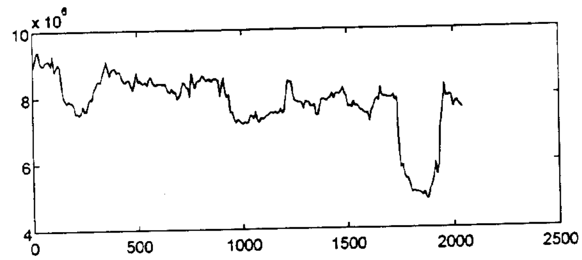

Figure 6. Flaw detection using maximum energy frequency of the 2nd order AR model (applied to experimental data shown in Figure 5).

$$\hat{x}(n+1) = \begin{cases} \sum_{i=1}^{p} C_i Z_i^n & n \ge 0 \\ 0 & n < 0 \end{cases} \qquad (9)$$

where $C_i = A_i e^{j\theta_i}$ and $Z_i = e^{(\alpha_i + j\omega_i)T}$. Then, the Z-transform of the above equation yields:

$$\hat{X}(Z) = \sum_{i=1}^{p} \frac{C_i}{1 - Z_i Z^{-1}} \qquad (10)$$

When $Z = e^{j\omega T}$ is substituted into the above equation, the Prony energy spectrum density can be obtained as:

$$\hat{P}(\omega) = \left| T \sum_{i=1}^{p} \frac{C_i}{1 - Z_i e^{j\omega T}} \right|^2 \qquad (11)$$

In our study, the Prony spectrum is applied to both the computer simulated data and the experimental ultrasonic signals. A sliding Hamming window with a timewidth of 128 data samples is applied to the input sequence. The utilization of the window function is to analyze a short segment of a signal which physically corresponds to a small area of a specimen. The Prony spectrum is calculated for each segment of data, and the maximum energy frequency is estimated. The maximum energy frequency is used for frequency-shift estimation and flaw detection.

Figure 7 presents the results of frequency-shift estimation when it is applied to the signal shown in Figure 1 using Prony's method. This figure shows the estimated maximum energy frequency using a 3-term (i.e., p = 3) exponential model. This figure indicates that the estimated maximum energy frequency using Prony's method can effectively reflect the trend of the downward frequency-shift with a relatively consistent bias. Figure 8 presents the flaw detection achieved from a 5-term Prony's method with a window size of 128, applied to the experimental data shown in Figure 5. This plot illustrates that the low order Prony's method can be used for ultrasonic flaw detection, but its performance is well below that of the AR method.

MUSIC METHOD

The MUSIC method has been developed based on the eigenanalysis of the autocorrelation matrix. This technique is derived from the frequency estimator based on noise subspace, and can reduce the noise contribution and gain better performance in spectral analysis at a lower signal-to-noise level. However, the performance may depend on how well the signal and noise eigenvalues can be separated from the noise-only eigenvalues and may also depend on the selection of the model order and the data length.

In order to develop the frequency estimator based on the eigenanalysis theory, an exponential signal model consisting of p complex sinusoids and corrupted by additive white noise has been adopted. This model can be expressed as:

$$x(n) = \sum_{i=1}^{p} A_i e^{j(2\pi f_i n + \theta_i)} + w(n) \qquad (12)$$

where $x(\cdot)$ is the data samples of a random signal, $\{A_i\}$ and $\{f_i\}$ are unknown amplitudes and frequencies of the i^{th} complex sinusoid. The terms θ_i are a phase of the i^{th} sinusoid and are statistically independent random variables uniformly distributed on $[0, 2\pi]$, and w(n) is a white noise sequence with a spectral density of σ^2. Then, x (n) is a wide-sense stationary random process, and the autocorrelation function of x(n) can be expressed in a matrix form, Φ_{xx}. This autocorrelation matrix is eigen-decomposed into signal subspace and noise subspace:

$$\Phi_{xx} = \underbrace{\sum_{i=1}^{p} (\lambda_i + \sigma^2) v_i v_i^T}_{\text{signal + noise subspace}} + \underbrace{\sum_{i=p+1}^{M} \sigma^2 v_i v_i^T}_{\text{noise subspace}} \qquad (13)$$

Where λ_i's are eigenvalues and v_i's are eigenvectors. The MUSIC sinusoidal frequency estimator is defined as:

$$P_{MUSIC}(f) = \frac{1}{\sum_{k=p+1}^{M} \left| s^T(f) v_k \right|^2} \qquad (14)$$

where $s(f) = \left[1, e^{j2\pi f}, e^{j4\pi f}, \cdots, e^{j2\pi(M-1)f} \right]$. The behavior of the MUSIC spectra estimator is largely dependent on the choice of the value p, the number of the principle eigenvectors, the value of M, the order of the eigenanalysis based model, and how well the noise subspace can be separated from the signal plus noise space. Unfortunately, there is no simple way to achieve the best selection of p and M for all applications. In our study, a low order estimation of M = 5, with signal subspace p = 3 has been used because we wanted to consider the maximum energy frequency as a single parameter for signal characterization.

The MUSIC frequency estimator is applied to both computer simulated and experimental ultrasonic data using a sliding Hamming window with a timewidth of 128 data samples. Figure 9 presents the result of frequency-shift estimation applied to Figure 1, with n = 5 and p = 3. This figure indicates that the MUSIC method is an effective way to extract the center frequency information. To evaluate the MUSIC method in ultrasonic flaw detection, the experimental signal shown in Figure 5 is used. Figure 10 shows the flaw detection results using the 5th order (signal subspace 3) MUSIC method. An inspection of these plots demonstrates that the MUSIC method is effective for ultrasonic flaw detection.

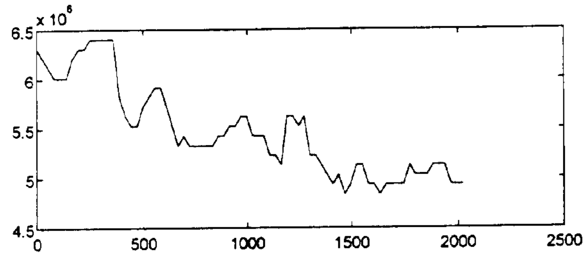

Figure 7. Frequency-shift estimation (vertical axis) using Prony's method (applied to simulated data shown in Figure 1).

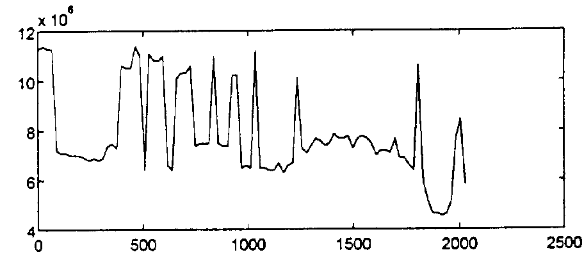

Figure 8. Flaw detection using maximum energy frequency (vertical axis) of 5th order Prony's method (applied to experimental data shown in Figure 5).

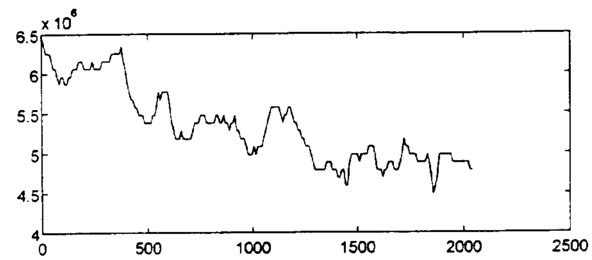

Figure 9. Frequency-shift estimation (vertical axis) using the MUSIC method (applied to simulated data shown in Figure 1).

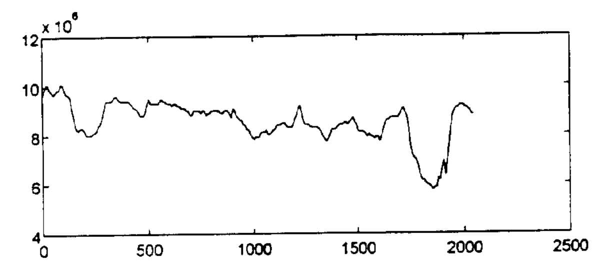

Figure 10. Flaw detection using maximum energy frequency (vertical axis) of 5th order MUSIC method (applied to experimental data shown in Figure 5).

CONCLUSION

In this paper, three model-based high resolution spectral estimation techniques: AR method, Prony's method and the MUSIC method have been studied and applied to ultrasonic scattering signals. It has been shown that the second-order AR model produces better frequency estimation in terms of resolution and accuracy, and also offers better flaw detection capability.

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