PERFORMANCE COMPARISON OF TIME-FREQUENCY DISTRIBUTIONS FOR ULTRASONIC NONDESTRUCTIVE TESTING

M. A. Malik * and J. Saniie
Department of Electrical and Computer Engineering,
Illinois Institute of Technology, Chicago, IL 60616
* EET Department, DeVry Institute of Technology, Chicago, IL 60618

ABSTRACT- In the nondestructive testing of materials ultrasonic backscattered echoes often exhibit critical time and frequency information. The time-frequency (t-f) analysis of ultrasonic signals instantaneously reveals the frequency and time of arrival of target echoes which help to characterize the target. But varying results are obtained by applying different t-f algorithms to ultrasonic signals. Performance evaluation of t-f algorithms for ultrasonic testing is useful for the choice of an algorithm. In this paper we present a performance comparison of t-f algorithms for ultrasonic applications. In particular, we compare the t-f algorithms based on the detection of the time of arrival and frequency of target echoes, estimation of instantaneous frequency of multiple target echoes, and cross-terms attenuation and concentration. These characteristics are studied by applying Wigner-Ville distribution (WVD), exponential distribution (ED), Gabor transform (GT), and wavelet transform (WT) to simulated and experimental ultrasonic data.

I. INTRODUCTION

The backscattered signal information in ultrasonic nondestructive testing is nonstationary due to frequency dependent scattering, attenuation and dispersion. The standard spectral analysis cannot determine the time of arrival of different frequency components in the signal. Therefore, joint time-frequency (t-f) representations of such signals are more revealing. The results of applying different time-frequency representations to backscattered signals would be different. A unified analysis [1] to compare the performance of different t-f representations including Wigner-Ville distribution (WVD), exponential distribution (ED), Gabor transform (GT), and wavelet transform (WT) is presented in this paper. WVD, ED, and GT are members of Cohen's [2] generalized t-f representation (GTFR). A single Gaussian ultrasonic backscattered echo can be truly represented by Wigner-Ville distribution (WVD) as far as optimal t-f concentration is concerned. But WVD, having a weighting factor (the so called kernel function) equal to 1, generates cross-terms for signals containing multiple echoes. These cross-terms sometimes obscure the true energy representation of the signal. Another important but not essential performance criteria for t-f distributions are their time and frequency marginal properties. There is a trade off between the cross-terms elimination and the marginal properties. Gabor transform (GT) is an example of one such distribution in which cross-terms are eliminated at the cost of marginal properties. But the t-f concentration of GT is dependent on its window duration parameter. The optimal concentration of GT is obtained when GT's window parameter matches the duration parameter of the ultrasonic echo. Some distributions, for example exponential distribution (ED) [3], can attenuate to some extent the cross-terms without sacrificing the marginal properties. WT, which does not generate cross-term, is a time-scale distribution and uses a flexible window for analyzing different frequency components in a signal. The elimination or attenuation of cross-terms in Cohen's class of t-f distributions depends on the selection of weighting factor or the so called kernel function. One form of the kernel function eliminates or reduces the cross-terms in the ambiguity plane and the other form localizes the time dependent autocorrelation function. The relationship of GTFR with generalized ambiguity function (GAF) and generalized time dependent autocorrelation function (GTDAF) provide a means of choosing a kernel function with the desirable property of cross-terms attenuation.

II. TIME-FREQUENCY KERNEL FUNCTIONS AND THEIR PROPERTIES

Cohen's class of quadratic/bilinear GTFR is expressed by

$$S_E(t,\omega) = \frac{1}{4\pi^2}\iiint e^{-j\theta t - j\tau\omega + j\theta\mu}\,\Phi(\theta,\tau).\; s^*(\mu-\frac{1}{2}\tau).s(\mu+\frac{1}{2}\tau)\,d\mu\,d\tau\,d\theta \qquad (1)$$

Each member of GTFR is characterized by an arbitrary signal independent kernel function $\Phi(\theta, \tau)$, in which θ is the frequency variable and τ is the time variable. The properties of t-f distributions are dependent on this signal independent kernel function and different kernel functions describe different distributions. Some of the important properties of t-f distributions depending on the constraints on the kernel function are given in Table (1).

The three properties (3), (4), and (5) tabulated in Table 1 are *the key to the comparison of the* Cohen' class of t-f distributions, WVD, ED, and GT [4]. It is clear from Table (1) that the kernel function has different forms. Property (3) of Table (1) reveal that the cross-terms will be attenuated if the kernel, $\Phi(\theta, \tau)$, of a *t-f* distribution is lowpass in both time and

Table 1. Desirable Properties of TFR and Kernel Constraints

Property: P	Constraint: C
1) Time shift invariance, $s(t) \Rightarrow s_{t_0}(t) = s(t-t_0) \Rightarrow S_E(t-t_0, \omega)$	$\Phi(\theta,\tau)$ is independent of time, t
2) Frequency shift Invariance, $S(\omega) \Rightarrow S_{\omega_0}(\omega) = S(\omega-\omega_0) \Rightarrow S_E(t, \omega-\omega_0)$	$\Phi(\theta,\tau)$ is independent of frequency, ω
3) Cross-terms attenuation,	$\Phi(\theta,\tau)$ is lowpass in (θ,τ)
4) Cross-terms time concentration ("strong" finite time support)	$\phi(t,\tau) = \int \Phi(\theta,\tau) e^{j\theta t} d\theta = 0, \quad \lvert t \rvert \neq \lvert \tau \rvert / 2$
5) Cross-terms frequency concentration("strong" finite frequency support)	$\Phi(\theta,\omega) = \int \Phi(\theta,\tau) e^{-j\omega\tau} d\tau = 0, \quad \lvert \omega \rvert \neq \lvert \theta \rvert / 2$

frequency variable. The ambiguity kernel [2], $\Phi(\theta, \tau)$, for WVD, ED and GT are given by

$$\begin{aligned} \Phi_{WVD}(\theta,\tau) &= 1, \\ \Phi_{ED}(\theta,\tau) &= e^{-\theta^2\tau^2/\sigma}, \\ \Phi_{GT}(\theta,\tau) &= \int g(t+\tau/2) \\ &\quad \cdot g^*(t-\tau/2)\, e^{j(-\theta)t}\, dt \\ &= e^{-\theta^2/8\alpha^2}\, e^{-\alpha^2\tau^2/2} \end{aligned} \tag{2}$$

where σ, an arbitrary signal independent scale factor, controls the amount of cross-terms attenuation and the t-f concentration of the auto-terms in ED and in the limit $\sigma \Rightarrow \infty$, $\Phi_{ED}(\theta,\tau) \Rightarrow \Phi_{WVD}(\theta,\tau)$. The window function $g(t)$ of GT is given by

$$g(t) = \left[2\alpha^2/\pi\right]^{1/4} e^{-\alpha^2 t^2} \tag{3}$$

Equation (2) indicates that the kernel function for GT is lowpass in both time and frequency variables and, therefore, GT will eliminate the cross-terms. WVD has an all pass kernel which will generate cross terms. ED having a product kernel will attenuate the cross-terms and the amount of cross-terms attenuation depends on the signal independent parameter σ. The window parameter, α, of GT also controls the t-f concentration of the signal. The WVD, ED, and GT of a signal s(t) are expressed as .

$$\begin{aligned} S_{WVD}(t,\omega) &= \frac{1}{2\pi}\int s^*\left(t-\frac{1}{2}\tau\right) s\left(t+\frac{1}{2}\tau\right) e^{-j\tau\omega} d\tau, \\ S_{ED}(t,\omega) &= \iint \sqrt{\frac{\sigma}{4\pi\tau^2}}\, e^{\left\{-\frac{(\mu-t)^2}{4\tau^2/\sigma}\right\}} e^{-j\tau\omega} \\ &\quad s^*\left(\mu-\frac{1}{2}\tau\right) s\left(\mu+\frac{1}{2}\tau\right) d\mu\, d\tau \\ S_{GT}(\tau,\omega) &= \int_{-\infty}^{\infty} s(t)\, g^*(t;\tau,\omega)\, dt, \end{aligned} \tag{4}$$

where, $g(t;\tau,\omega) = \left[2\alpha^2/\pi\right]^{1/4} e^{-\alpha^2(t-\tau)^2} e^{j\omega(t-\tau/2)}$ is elementary Gaussian function of GT. Another form of the kernel function is $\phi(t, \tau)$, which according to the property (4) of Table (1) reveals the properties of the t-f distribution in the time domain. The cross-terms will be smoothed out if the kernel function, $\phi(t, \tau)$, has smoothing effects in both t and τ directions. The property (5) of Table (1) indicates the smoothing of cross-terms in the frequency domain using another form of kernel function $\Phi(\theta, \omega)$. This function must have a smoothing property in both θ and ω directions to have strong finite frequency support, i.e., no energy is generated at frequencies other than those present in the signal.

III. WAVELET TRANSFORM

The time-frequency analysis of ultrasonic signals backscattered from inhomogeneous materials can be achieved by another effective method (signal decomposition method) using wavelet transform [5, 6]. Unlike GT , in this method of constant-Q analysis using wavelet transform (WT) the shape of the window changes with frequency, keeping the relative bandwidth constant. It is desirable in certain applications to have better time resolution at higher frequencies than at lower frequencies. GT, WVD, ED or any other quadratic/bilinear time-frequency representation cannot deal with this situation because none of these have a flexible window function. WT on the other hand provides a flexible window which is shorter for observing high frequencies and broader when studying low frequencies.

The integral wavelet transform of a signal s(t) relative to a basic wavelet $\psi(t)$ is given by :

$$S_{WT}(\tau,a) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} s(t)\,\psi^*\left(\frac{t-\tau}{a}\right) dt, \quad s \in L^2(\Re) \tag{5}$$

where τ is the time shift parameter, a (the dilation parameter) governs the frequency, and $\psi(t)$ is called a basic wavelet. The basis functions called wavelets are derived from this prototype wavelet by dilations and concentrations (scalings) and shifts.

$$\psi_{(\tau,a)}(t) = a^{-\frac{1}{2}}\,\psi\left(\frac{t-\tau}{a}\right) \tag{6}$$

where $\psi(t)$ is a square integrable function satisfying the following admissibility condition for the convergence purposes.

$$c_\psi = 2\pi \int |\Psi(\omega)|^2 \frac{d\omega}{|\omega|} < \infty \qquad (7)$$

where a, $\tau \in \Re$, and $a \neq 0$. If $\Psi(\omega)$ is differentiable (which is assumed), this means $\Psi(0) = 0$ i.e. $\int \psi(t)dt = 0$. We used Morlet wavelet, $\psi(t) = \exp(jct).\exp(-t^2/2)$, (c=5.33), as the basic wavelet. WT due to its flexible window is useful for the detection of lower frequency ultrasonic targets in the presence of higher frequency scattering noise.

IV. PERFORMANCE EVALUATION OF T-F REPRESENTATIONS

The performance of these algorithms is compared by applying them to signals containing one or multiple ultrasonic echoes. One ultrasonic echo can be expressed as:

$$s_1(t) = \left(\frac{2\sigma_1^2}{\pi}\right)^{1/4} e^{-\sigma_1^2(t-t_1)^2} e^{j2\pi f_1 t}, \qquad (8)$$

where, σ_1, t_1, and f_1 are constants which correspond to sharpness or duration of the echo, time of its arrival, and frequency respectively. The time and frequency spread of GT and WVD of one ultrasonic echo can be, respectively, expressed as:

$$\Sigma_{S_{GT}(\tau,f)} = \iint \left[(\tau - \tau_s)^2 + (4\pi)^2(f - f_s)^2\right] . |S_{GT}(\tau, f)|^2 d\tau\, df = \left[\frac{\sigma_1^2 + \alpha^2}{4\sigma_1^2\alpha^2} + \left(\sigma_1^2 + \alpha^2\right)\right] \qquad (9)$$

$$\Sigma_{S_{WVD}(t,f)} = \iint \left[(t - t_s)^2 + (4\pi)^2(f - f_s)^2\right] . |S_{WVD}(t, f)|^2 d\tau\, df = \left[\frac{1}{4\sigma_1^2} + \sigma_1^2\right] \qquad (10)$$

The time-frequency spread of ED does not have a close form solution but can be numerically evaluated. The time frequency spread of ED is dependent on the signal independent scale factor σ. The time spread of ED approaches that of WVD as the scale factor increases towards infinity. Equation (10) shows that WVD is the optimally concentrated distribution for one ultrasonic echo. GT has the optimum time-frequency concentration. But it has more time-frequency spread than WVD due to inherent time-frequency resolution trade-off. The optimum concentration for GT is obtained if its window bandwidth parameter, α, matches the echo bandwidth parameter, σ_1. The time-frequency resolution of WT depends on dilation parameter a, which governs the frequency. The echo of lower frequency has lower resolution than that of higher frequency echo because the WT treats the lower and higher frequencies differently by using flexible window.

The performance of the bilinear distributions deteriorates when there are multiple echoes in a an ultrasonic backscattered signal. Figure (1) shows the application of WVD, ED, GT and WT to a simulated signal which contains six ultrasonic (grain) echoes with their frequency decreasing with time and two more echoes (one with lower frequency and one with higher frequency and partially overlapped) fully overlapped with the last two of the five echoes. The results indicate that the WVD of this signal is not interpretable due to the presence of cross-terms. The ED of this signal is marred lesser with cross-terms as compared to WVD but the multiple echoes in the signal can be discerned. GT can discern the multiple echoes due to inherent time-frequency trade off the echoes with the bandwidth in the vicinity of the bandwidth of GT window function are prominent. The WT discerns the multiple echoes with varying resolution, higher frequency echoes with higher resolution and lower frequency echoes with lower resolution.

We also applied these distributions to the experimental ultrasonic data which contains a flaw echo embedded in grain echoes. The frequency of the flaw echo is very close to that of the grain echoes. The results are shown in Figure (2). The WVD of this signal depicts that the flaw echo in the presence of microstructure noise cannot be completely characterized by WVD. The results of applying ED to this experimental data produced better results than WVD, but the flaw echo is still not completely characterized. GT of the experimental data revealed the flaw echo completely while the grain echoes are smoothed out. WT revealed the flaw echo of lower frequency as well as higher frequency grain echoes.

V. CONCLUSIONS

Simulation results indicate that GT works the best in the presence of Gaussian noise. Experimental results show that GT discerns the low frequency flaw echoes embedded in high frequency microstructure noise more effectively than other methods. GT's cross-terms are located at the intersection of GT's of individual echoes and therefore are eliminated. But the time-frequency concentration of GT of an ultrasonic echo is dependent on the bandwidth parameter of GT's window function. The optimal concentration of GT is obtained when the GT window bandwidth parameter matches the bandwidth parameter of the ultrasonic echo.

ED can discern echoes closely located and also reduces the cross-terms and is therefore suitable for the analysis of experimental data for ultrasonic application though ED is computationally complex.

WVD is computationally efficient but due to the inability of reducing the cross-terms cannot detect echoes in the presence of noise. It is suitable for monocomponent or single ultrasonic echo signals.

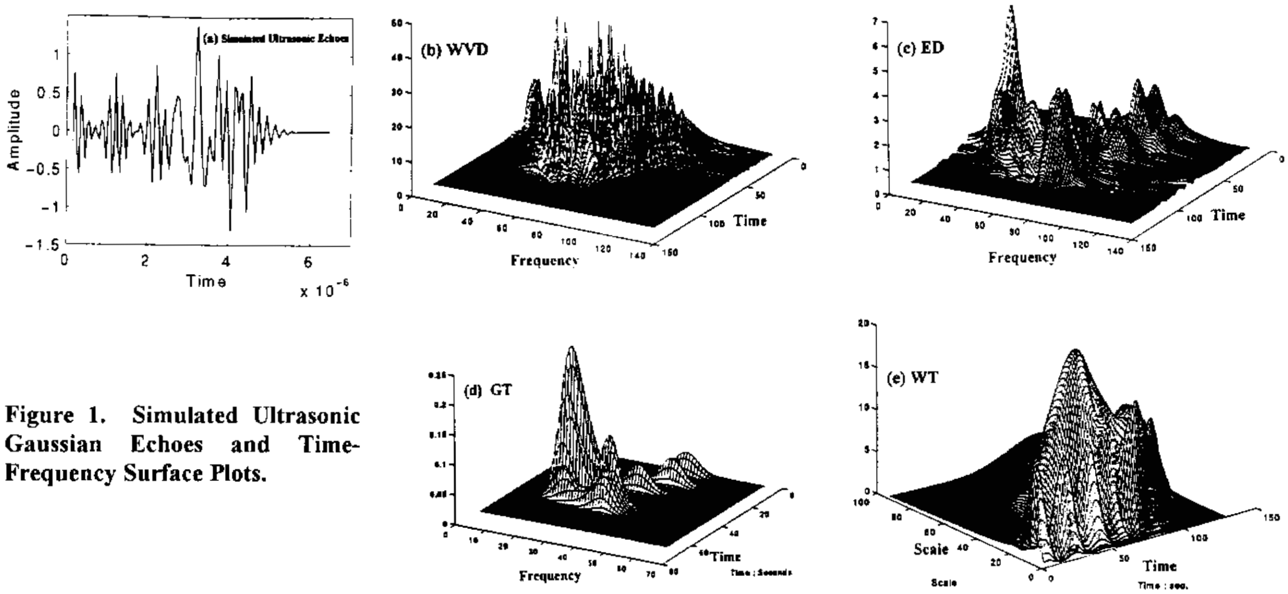

Figure 1. Simulated Ultrasonic Gaussian Echoes and Time-Frequency Surface Plots.

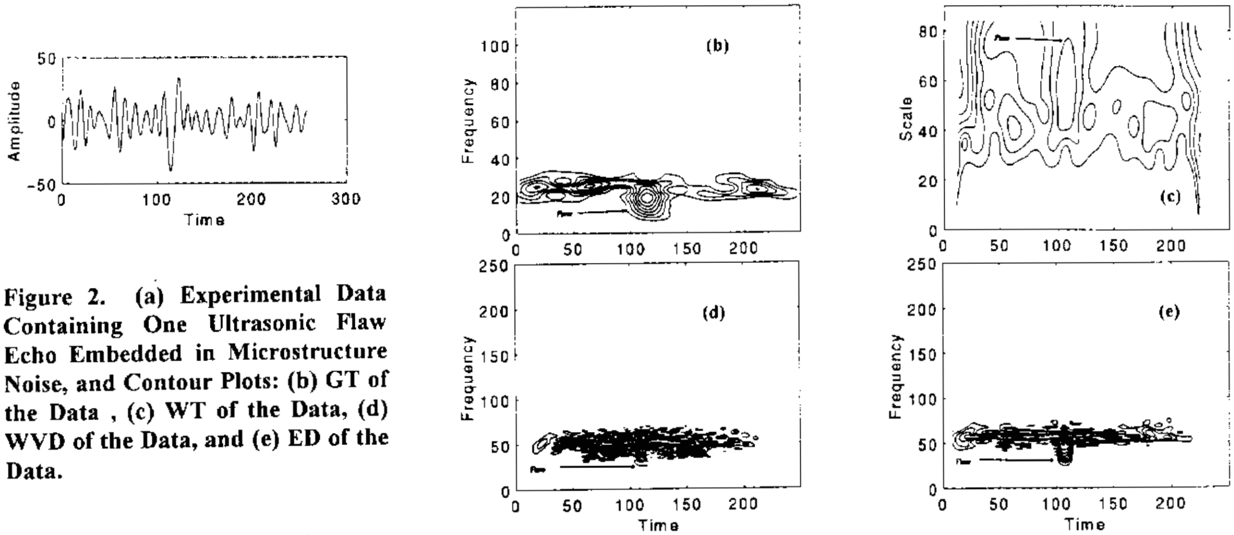

Figure 2. (a) Experimental Data Containing One Ultrasonic Flaw Echo Embedded in Microstructure Noise, and Contour Plots: (b) GT of the Data , (c) WT of the Data, (d) WVD of the Data, and (e) ED of the Data.

WT works best if the signal contains low frequency flaw echoes of longer duration embedded in high frequency grain echoes of shorter duration. It is suitable for experimental data analysis for ultrasonic application. Simulation results also indicate that WT tracks the frequency trend of the grain echoes and detects the flaw echoes better than all the other distributions discussed.

VI. REFERENCES

1. M. A. Malik, " Unified Time-Frequency Analysis of Ultrasonic Signals," Ph.D. Thesis, Illinois Institute of Technology, Chicago, IL, July 1995.
2. L. Cohen, "Time-Frequency Distributions ___A Review," *Proceedings of the IEEE*, vol. 77, no. 7, pp. 941-981 July 1989.
3. H. I. Choi and W. J. Williams, "Improved time-frequency representation of multicomponent signals using exponential kernels," *IEEE transaction on Acoustics Speech, and Signal Processing*, vol. 37, no. 6, pp. 862-871, June 1989.
4. P. J. Loughlin, James W. Pitton, and Les E. Atlas, " Bilinear Time-Frequency Representations: New Insights and Properties," *IEEE Trans. on Signal Processing*, vol. 41, no. 2, pp. 750-767, February 1993.
5. A. Grossmann, R. Kronland-Martinet, and J. Morlet, " Reading and Understanding Continuous Wavelet Transforms," in *Wavelets, Time-Frequency Methods, and Phase Space*, Springer -Verlag, New York, 1988.
6. F. B. Tuteur, " Wavelet Transformation in Signal Detection," in *Wavelets, Time-Frequency Methods, and Phase Space*, Springer -Verlag, New York, 1988.