# HIGH RESOLUTION PARAMETER ESTIMATION OF ULTRASONIC ECHOES FOR NDE APPLICATIONS

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**ABSTRACT** - Ultrasonic backscattered echoes represent not only the impulse response of the transducer, but they also contain information pertaining to the inhomogeneity of the propogation path, the effect of frequency dependent absorption and scattering, the dispersion effect, and the geometric shape, size and orientation of reflectors. Therefore, a well-defined modeling of the backscattered echo leading to a high resolution parameter estimation of the echo amplitude, arrival time, echo skewness, center frequency and bandwidth is highly desirable for nondestructive evaluation of materials, target detection, object classification, velocity measurement and/or the ranging system. In this paper, a maximum likelihood (ML) model of the backscattered echoes is developed, assuming that all parameters describing the shape of the echo are unknown. The unknown parameters can be estimated by minimizing the mean square error using iterative techniques. Iterative parameter estimation has suffered has suffered from the problem of convergence and/or inaccurate results due to the local minima of the error function. In this investigation, a two-stage iterative estimation process known as the "Expectation/Maximization algorithm" is developed for the optimal parameter estimation of the echoes with low signal-to-noise ratio. In the first stage of the algorithm, we estimate the expected signal, and in the second stage the maximum likelihood criterion is used to estimate parameters. It has been observed that the EM algorithm is less dependent on the initial guess, is computationally efficient and performs well in resolving interfering multiple echoes.

## I. INTRODUCTION

Ultrasonic signals contain information pertaining to the inhomogeneity of the propogation path, the effect of frequency dependent absorption and scattering, the dispersion effect, and the geometric shape, size and orientation of reflectors [1]. In the field of ultrasonic testing/measurement many problems require estimating signal amplitude, arrival time, frequency, bandwidth, and phase. Depending on the nature of the application, one or more of these parameters can be used for material testing, target detection, object classification, velocity measurement and the ranging system.

In this paper, a maximum likelihood (ML) parameter estimation of the backscattered echoes is considered, assuming that all parameters describing the shape of the echo are unknown but deterministic. The backscattered echoes are modeled as the weighted sum of the Gaussian echo wavelets corrupted with white Gaussian noise (WGN). Then the parameter estimation objective becomes fitting a model comprised of a number of superimposed echoes to measured data corrupted by noise. The unknown parameters of the echoes can be estimated by minimizing the mean square error resulting from the differences between the model and the measured signal, using iterative optimization methods. For estimating a large number of unknown parameters, iterative optimization methods suffer from the problems of convergence due to the local minima, dependency on the initial guess and significant growth in computation [2]. In this investigation, a two-stage iterative estimation process is developed. In the first stage we estimate the "expected signal", then, in the second stage we estimate the parameters using the maximum likelihood criterion. This method is known as the Expectation/Maximization (EM) algorithm [3]. It has been observed that the EM algorithm is superior to the conventional least square (LS) iterative optimization techniques. In particular, the EM algorithm is less dependent on the initial guess and performs well in resolving interfering superimposed echoes. The computation requirements for the EM algorithm depend linearly on the number of echoes.

## II. MODEL OF ULTRASONIC ECHOES AND EM ALGORITHM

The impulse response of the ultrasonic transducer can be modeled as a Gaussian shape echo [1].

$$x(t) = e^{-\alpha t^2} \cos(2\pi f_c t) \tag{1}$$

Using this transducer for ultrasonic testing, the model for a measured signal consisting of M backscattered echoes becomes:

$$y(t) = \sum_{m=1}^{M} f(\theta_m; t) + \upsilon(t) \tag{2}$$

where $\upsilon(t)$ is a white Gaussian noise (WGN) process with a variance of $\sigma_\upsilon^2$, and $\theta_m$ is a vector of the parameters,

  

$$\theta_m = [\alpha_m \quad \tau_m \quad f_{cm} \quad \phi_m \quad \beta_m] \tag{3}$$

and $f(\theta_m;t)$ is the general model for a detected Gaussian echo,

$$f(\theta_m;t) = \beta_m \cdot e^{-\alpha_m(t-\tau_m)^2}\cos\{2\pi f_{c_m}(t-\tau_m)+\phi_m\} \tag{4}$$

The physical parameters of the backscattered echoes are: $\alpha_m$: bandwidth factor, $\tau_m$: arrival time, $f_{cm}$: center frequency, $\phi_m$: phase, and $\beta_m$: amplitude. Our objective is to estimate these parameters assuming that all of them are unknown. The accuracy of the estimated parameters depends on the SNR which is defined as the ratio of signal energy to noise energy. In particular, the energy for a single echo with amplitude $\beta$ can be represented as,

$$E_s = \frac{\beta^2}{2}\sqrt{\frac{\pi}{2\alpha}}\{1+e^{\frac{-f_c^2}{(\alpha/\pi^2)}}\} \tag{5}$$

Meanwhile, the energy of a zero-mean Gaussian noise process is $E_v = \sigma_v^2$. Hence, the SNR [dB] becomes

$$SNR = 10\log\frac{E_s}{E_v} = 10\log\frac{\frac{\beta^2}{2}\sqrt{\frac{\pi}{2\alpha}}\{1+e^{-\frac{f_c^2}{\alpha/\pi^2}}\}}{\sigma_v^2} \quad \text{[dB]} \tag{6}$$

The above equation has been used to determine the SNR of the simulated signals presented in this paper. Then, the accuracy and efficiency of parameter estimations has been evaluated in terms of SNR.

Up to this point, we have explicitly defined an echo by its parameter vector, $\theta_m$, and the *SNR* in terms of echo parameters and noise variance. The accuracy of parameter estimation is essential in detecting and classifying target echoes. This section presents an algorithm to estimate $M$ superimposed echoes in Gaussian noise.

For computational purposes, the discrete model for the measured backscattered echoes, $y(n)$, is considered,

$$y(n) = \sum_{m=1}^{M} f(\theta_m;n) + \upsilon(n), \qquad n = 0,1,2,\ldots, N-1 \tag{7}$$

where

$$f(\theta_m;n) = \beta_m e^{-\alpha_m(n-\lambda_m)^2}\cos\{2\pi f_{c_m}(n-\lambda_m)+\phi_m\}, \tag{8}$$

and $\upsilon(n)$ is i.i.d, zero mean Gaussian random sequence. In general, the parameter vectors $\theta_m$ can be estimated by minimizing the mean square error function given by,

$$\sum_{n=0}^{N-1}\left|y[n] - \sum_{m=1}^{M} f(\theta_m;n)\right|^2 \tag{9}$$

This is a multiparameter estimation problem and requires a multidimensional search. Gradient-descent algorithms may be used for problems of this type, but they are often sensitive to local minima and computationally inefficient. However, an alternative method, known as the Expectation-Maximization (EM) algorithm, is commonly used to estimate multiple superimposed signals [3]. The essence of this algorithm is that it translates the multiple echo estimation problem into one echo estimation, which offers significant computational efficiency. Hence, the complexity of the problem is not affected by the number of unknown echoes [4]. The EM algorithm for multiple echo estimation is realized using an initial guess for each $\theta_m$, and iteratively implementing the following two computational steps:

**Expectation Step**: For $m=1,2,\ldots,M$ compute

$$\hat{x}_m^{(k)}(n) = f(\theta_m^{(k)};n) + \mu_m\{y(n) - \sum_{i=1}^{M} f(\theta_m^{(k)};n)\}, \tag{10}$$

where $\sum_{m=1}^{M}\mu_m = 1$

**Maximization Step**: For $m=1,2,\ldots,M$

$$\text{Min}\left\{\sum_{n=0}^{N-1}\left|\hat{x}_m^{(k)}(n) - f(\theta_m^{(k)};n)\right|^2\right\} \longrightarrow \theta_m^{k+1} \tag{11}$$

In the expectation step (E-Step), using the current estimate of the parameter vector $\theta_m^{(k)}$ and the observed data $y(n)$, the expected value of the m-th echo, $\hat{x}_m^{(k)}(n)$, is computed. Note that $\hat{x}_m^{(k)}(n)$ is composed of two terms: $f(\theta_m^{(k)};n)$, which is an estimate of the m-th echo and the estimate for $\upsilon(n)$. Therefore, the algorithm decouples the original data set into M separate data sets, with each one consisting of a single echo in WGN. In the maximization step (M-Step), $\theta_m^{k+1}$ is computed as the ML estimate for $\theta_m^k$, using unobserved data $\hat{x}_m^{(k)}(n)$. In other words, the M-Step corresponds to the MLE of a single echo with the estimated data, $\hat{x}_m^{(k)}(n)$. In the next E-Step $\theta_m^{k+1}$ will be used to compute $\hat{x}_m^{(k+1)}(n)$, and so on. This sequence of operations is diagrammed below:

$$\theta_m^{(0)} \xrightarrow{E-Step} x_m^{(0)} \xrightarrow{M-Step} \theta_m^{(1)} \xrightarrow{E-Step} x_m^{(1)} \ldots\ldots$$

Finally, the algorithm alternates between the two steps until convergence. The iteration is terminated when there is no improvement in the estimate of parameter vectors (Ref. [3] presents a discussion on the EM convergence theorem). The flowchart of the algorithm is shown in Figure 1.

In summary, the EM algorithm transforms the $M$ multiple echoes optimization problem given by Eq. 9 into an $M$ separate single echo optimization given by Eq. 11. In the next section the ML estimate algorithm is presented in order to obtain parameters for a single echo.

## III. ML ESTIMATION OF A SINGLE ECHO

This section presents the ML parameter estimation of a single echo modeled as,

$$x(n) = f(\theta;n) + \upsilon(n), \qquad n = 0,1,2,\ldots,N-1 \tag{12}$$

where $f(\theta;n) = \beta e^{-\alpha(n-\lambda)^2}\cos\{2\pi f_c(n-\lambda) + \phi\}$, (13)

$$\theta = [\alpha \quad \lambda \quad f_c \quad \phi \quad \beta]$$

and $\upsilon(n)$ is i.i.d, a zero mean white Gaussian random sequence with a variance of $\sigma_\upsilon^2$. The MLE of $\theta$ is the estimate that maximizes the likelihood function. For any Gaussian random sequence **x**, the likelihood function can be written as;

$$P(x;\theta) = \frac{1}{(2\pi)^{N/2}|C(\theta)|^{1/2}}\exp\{-\frac{1}{2}(x-\mu)^T C^{-1}(\theta)(x-\mu)\} \tag{14}$$

where $C(\theta)$ is the covariance matrix and $\mu$ is the mean vector [5]. For the model given in Eq.16, the mean becomes $f(\theta;n)$ and the covariance matrix is $C(\theta) = \sigma_\upsilon^2 I_N$, where $I_N$ is an $NxN$ identity matrix. Thus, the likelihood function becomes:

$$P(x;\theta) = \frac{1}{(2\pi\sigma^2)^{N/2}}\exp\{\frac{-1}{2\sigma^2}\sum_{n=0}^{N-1}\left[x(n) - f(\theta,n)\right]^2\} \tag{15}$$

Maximizing this function with respect to $\theta$ can be achieved by minimizing the exponential term:

$$J(\theta) = \sum_{n=0}^{N-1}\left[x(n) - f(\theta,n)\right]^2 \tag{16}$$

Note that for a white Gaussian noise (WGN) model, the ML parameter estimation problem becomes a Least-Square (LS) optimization problem (Eq.16). For cases in which the noise is colored or correlated, an inverse covariance matrix estimate is needed to compute the likelihood function (Eq.14).

In fact, even for the Gaussian noise model, we are faced with solving a highly non-linear equation (Eq.16) for five unknown parameters. Iterative LS optimization methods can be applied, although the global convergence may depend on the initial guess and require significant iteration. However, we have developed an efficient optimization procedure, one that estimates one parameter at a time, in part through the explicit solutions, in part through iterations. The following steps outline the procedure for obtaining an MLE parameter vector of a single echo.

**Step 1.** Use an initial guess for $f_c, \lambda, \alpha$, and form the matrix-vector quantities shown below:

$$c = [\cos 2\pi f_c(0-\lambda) \quad \cos 2\pi f_c(1-\lambda) \quad . \quad . \quad \cos 2\pi f_c(N-1-\lambda)]^T$$

$$s = [\sin 2\pi f_c(0-\lambda) \quad \sin 2\pi f_c(1-\lambda) \quad . \quad . \quad \sin 2\pi f_c(N-1-\lambda)]^T$$

$$e = \exp\begin{bmatrix} -\alpha(0-\lambda)^2 & 0 & 0 & 0 \\ 0 & -\alpha(1-\lambda)^2 & 0 & . \\ 0 & 0 & . & . \\ 0 & . & . & -\alpha(N-1-\lambda)^2 \end{bmatrix}$$

**Step 2.** Use the quantities from Step 1 and compute

$$I(f_c) = \frac{\|c^T ey\|^2}{c^T e^2 c} + \frac{\|s^T ey\|^2}{s^T e^2 s},$$

Then, find $f_c$ that maximizes $I(f_c)$.

**Step 3.** Recompute the quantites **c**, **s** and **e** in Step 1 and estimate $\beta$ and $\phi$,

$$\hat{\beta} = \sqrt{\left(\frac{c^T ey}{c^T e^2 c}\right)^2 + \left(\frac{s^T ey}{s^T e^2 s}\right)^2} \quad \text{and} \quad \hat{\phi} = -\arctan\left(\frac{\left(\frac{s^T ey}{s^T e^2 s}\right)}{\left(\frac{c^T ey}{c^T e^2 c}\right)}\right)$$

**Step 4.** Use estimates of $\beta$ and $\phi$, and $f_c$, construct the echo model

$$s(n) = \beta e^{-\alpha(n)^2}\cos\{2\pi f_c(n) + \phi\}$$

and estimate $\lambda$ which maximizes the cross correlation function,

$$\lambda = \arg \quad \max\left\{\sum_{n=0}^{N-1} y(n)s(n-\lambda)\right\}$$

**Step 5.** Use all of the estimates above, find the optimum $\alpha$ which minimizes the term:

$$\alpha = \arg \quad \min\left\{\sum_{n=0}^{N-1}\left[y[n] - \beta e^{-\alpha(n-\lambda)^2}\cos\{2\pi f_c(n-\lambda) + \phi\}\right]^2\right\}$$

In summary, steps 1-5 allow us to obtain ML estimates which are needed in the maximization step of the EM algorithm (for clarity see the flowchart of the algorithm shown in Figure 1).

## IV. PARAMETER ESTIMATION RESULTS

The EM algorithm developed in this investigation has been applied to detect and estimate three non-interfering echoes and three interfering echoes in additive WGN (see Fig. 2 and Fig. 3). For high SNR, the parameter estimation is robust regardless of the initial guess. The parameter estimation of a noise free echo is achieved with a 100% accuracy. However, in the presence of noise (e.g., SNR=5-10 dB), the estimation of the parameters is reasonably accurate with a moderate increase in the number of iterations. In particular, for the cases of three non-interfering and interfering echoes(see Fig. 2 and Fig. 3), we have obtained a surprisingly accurate estimate of arrival time, center frequency and amplitude, all of which are critical parameters for material evaluation.

## V. CONCLUSION

In this investigation, ultrasonic backscattered echoes are modeled as the superpositon of several Gaussian echoes under WGN, assuming that all parameters (bandwidth factor, arrival time, center frequency, phase and amplitude) describing the echo are unknown. Then, the EM algorithm has been applied to obtain the *maximum likelihood estimates* of echo parameters. It has been observed that the EM algorithm is far superior to conventional LS optimization methods. It is less dependent on the initial guess, is computationally efficient and performs well in resolving superimposed echoes. Overall, the results above indicate that the EM algorithm is capable of detecting and characterizing ultrasonic backscattered echoes by their parameters.

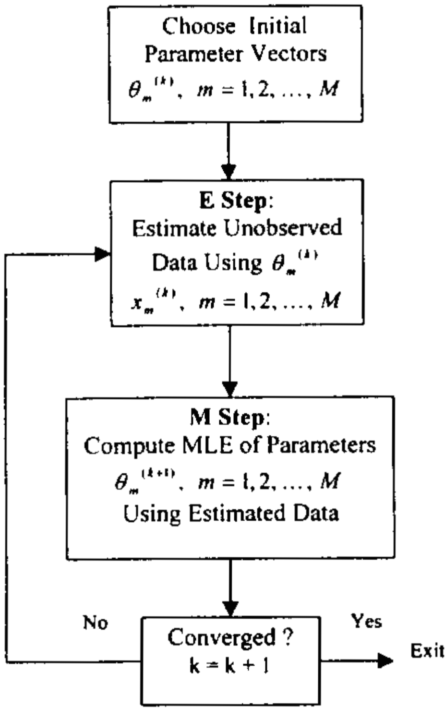


Figure 1. The Flowchart of the EM Algorithm

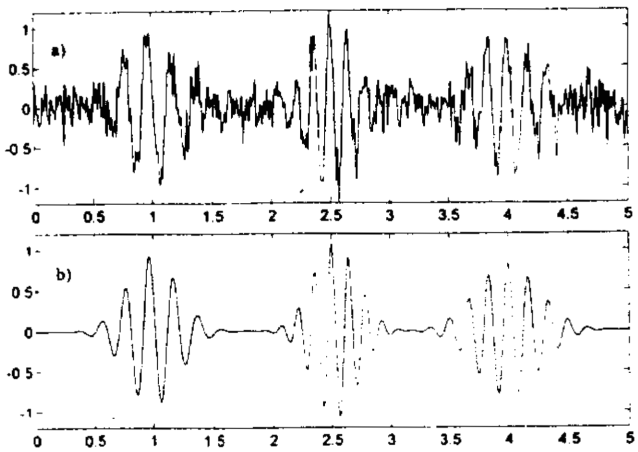


Figure 2. a) Three separate echoes with SNR=5 dB,
b) Estimated echoes for (a).

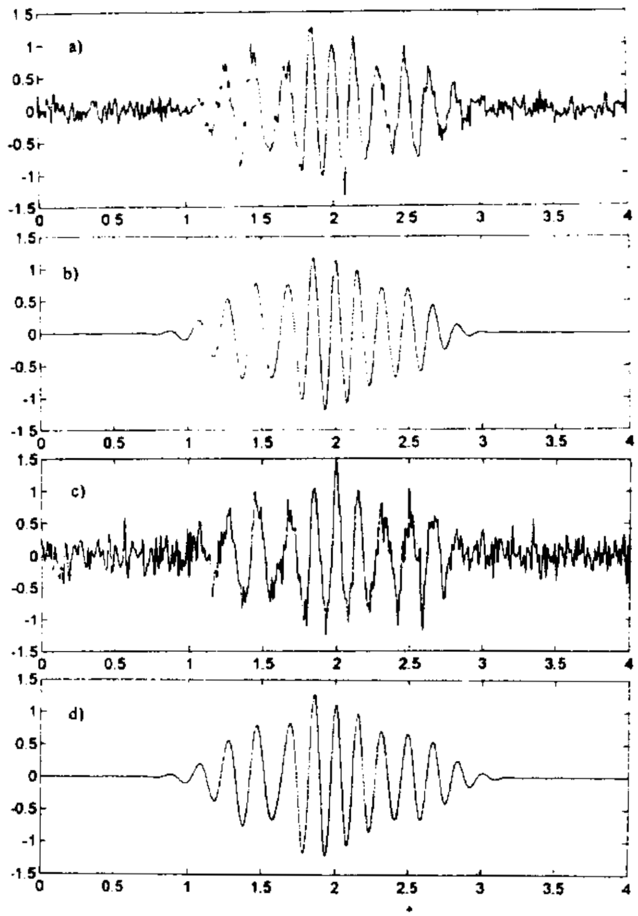


Figure 3. a) Three interfering echoes with SNR=10 dB;
b) Estimated echoes for (a);
c) Three interfering echoes with SNR=5 dB;
d) Estimated echoes for (c).

## VI. REFERENCES


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