EVALUATION OF EXPONENTIAL PRODUCT KERNEL FOR QUADRATIC TIME-FREQUENCY DISTRIBUTIONS APPLIED TO ULTRASONIC SIGNALS

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ABSTRACT- The display of energy of ultrasonic backscattered echoes simultaneously on a joint time-frequency (t-f) plane reveals critical information pertaining to time of arrival and frequency of echoes. The quadratic t-f distributions play important role in displaying the energy of the signal on a joint t-f plane. The t-f energy distribution of the signal is dependent on a weighting function, kernel, of generalized quadratic t-f distribution. This kernel, a function of product of time lag and frequency lag variables, controls the t-f concentration of the signal and the suppression of artifacts generated by the quadratic t-f distribution. A generalized exponential product (GEP) kernel function is explored in this paper. Exponential (i.e., Choi-Williams) distribution is a special case of this generalized exponential distribution. A whole family of Quadratic exponential distributions can be generated by varying the parameters of the generalized exponential product kernel. We evaluate these parameters on the basis of optimum concentration of the ultrasonic backscattered echoes, resolution of defect echoes, suppression of the cross-terms artifacts, and performance in the presence of noise. These parameters are evaluated by reducing the cross-terms and keeping auto-terms on the ambiguity plane close to the ideal. It is shown that by controlling the parameters of the generalized exponential product kernel we can achieve better performance in the form of time-frequency concentration, and resolution for multiple echoes as compared to exponential distribution. The application of GEP kernel to ultrasonic experimental data, with properly chosen parameters, not only discern the defect echo embedded in grain echoes but diminish the cross-terms generated by the bilinear structure of the t-f distribution.

I. INTRODUCTION

Quadratic (i.e., also known as Bilinear) time-frequency representation is a useful tool for the analysis of ultrasonic signals in nondestructive evaluation of materials. The ultrasonic target echoes embedded in microstructure noise can be detected by displaying the energy of the backscattered ultrasonic signal on a joint time-frequency plane. A quadratic/bilinear generalized time-frequency representation (GTFR) is expressed by the following equation [1].

$$S_{E_\Phi}(t,\omega)=\frac{1}{4\pi^2}\iiint e^{-j\theta t-j\tau\omega+j\theta\mu}\,\Phi(\theta,\tau)\,.s^*(\mu-\frac{1}{2}\tau)\,s(\mu+\frac{1}{2}\tau)\,d\mu\,d\tau\,d\theta \tag{1}$$

where $S_{E_\Phi}(t,\omega)$ is the time-frequency (t-f) energy distributioin, $s(t)$ *is the signal to be analyzed and* $\Phi(\theta,\tau)$ *is an arbitrary* signal independent kernel function. All integrals are from $-\infty$ to ∞ unless otherwise indicated.

Each representation in Equation (1) is characterized by kernel function $\Phi(\theta,\tau)$, in which θ is the frequency-lag variable and τ is the time-lag variable. The properties of time-frequency distributions are dependent on this kernel function and different distributions are obtained by choosing different kernels. It is, therefore, logical and essential to understand the structure of this kernel and to evaluate different constraints put on this kernel function for getting certain desirable properties of the distribution. It is also important to know how to achieve smoothing by using different kernel functions to reduce the artifacts in the case of multicomponent signals. This kernel function, $\Phi(\theta,\tau)$, can be considered as a window function of ambiguity plane. It reduces and in certain cases eliminates the cross-terms artifacts of the ambiguity function (AF) and keeps the auto-terms. Different distributions can be obtained by choosing different kernels. Wigner-Ville distribution (WVD) is obtained by choosing the kernel $\Phi(\theta,\tau)=1$ and this gives

$$S_{WVD}(t,\omega)=\frac{1}{2\pi}\int s^*(t-\frac{1}{2}\tau)\,s(t+\frac{1}{2}\tau)\,e^{-j\tau\omega}d\tau \tag{2}$$

WVD is the ideal t-f distribution as far as the t-f concentration (resolution) of the signal is concerned, but it generates the cross-terms *due to its bilinear structure. In order to achieve desirable* properties of a t-f distribution as well as reducing the cross-terms artifacts a generalized exponential product (GEP) kernel function of the following form can be chosen.

$$\Phi(\theta,\tau)=\exp\left\{-\left[\frac{\theta}{\theta_1}\right]^{2k}\cdot\left[\frac{\tau}{\tau_1}\right]^{2n}\right\} \tag{3}$$

where θ is the frequency-lag variable, τ is the time-lag variable, θ_1 is the frequency-lag parameter that controls the shape and amount of cross-terms on $\theta=0$ axis, τ_1 is the time-lag parameter that controls the shape and amount of cross-terms on $\tau = 0$ axis, and paramaters n and k control the concentration of auto-terms in time and frequency, respectively. A whole family of quadratic exponential distributions can be generated by varying the parameters of the GEP kernel. We evaluate these parameters on the basis of optimum concentration of the ultrasonic backscattered echoes, resolution of defect echoes, suppression of the cross-terms artifacts, and performance in the presence of noise. These parameters are evaluated by suppressing the cross-terms and keeping auto-terms on the ambiguity plane close to the ideal. A special case of GEP, exponential kernel (EP), can be obtained by choosing n = k = 1, and $\theta_1{}^2 \tau_1{}^2 = \sigma^2$. Choosing $\Phi(\theta, \tau)$ to be equal to $\exp(-\theta^2 \tau^2/\sigma^2)$ gives

$$S_{ED}(t,\omega) = \iint \sqrt{\frac{\sigma^2}{4\pi\tau^2}}\, e^{\left\{-\frac{(\mu-t)^2}{4\tau^2/\sigma^2}\right\}} e^{-j\tau\omega} \cdot s^*(\mu - \frac{1}{2}\tau) s(\mu + \frac{1}{2}\tau) d\mu\, d\tau \tag{4}$$

as exponential distribution (ED) [2] where σ^2 is an arbitrary scale factor controlling the amount and shape of cross-terms. The cross-terms in ED can be reduced without affecting the marginal properties of the t-f distribution but the auto-terms are also affected [3]. If we choose $\sigma^2 = \infty$, we get the WVD of Equation (2). In order to design the GEP kernel such that the cross-terms are diminished and the auto terms are kept close to the ideal, we utilize the relationship between generalized ambiguity function (GAF) and GTFR.

II. TIME -FREQUENCY REPRESENTATION AND AMBIGUITY FUNCTION

The Woodward ambiguity function has been used to relate range and velocity resolution in analyzing and constructing signals associated with radar [1]. It is expressed by the following equation.

$$S_\chi(\theta, \tau) = \int_{-\infty}^{\infty} s(t + \tau/2) s^*(t - \tau/2) e^{j\theta t} dt \tag{5}$$

Ambiguity function (AF) is important for the design of GEP kernel for t-f distributions because it is characteristic function of WVD [1] as depicted by Equations (6),

$$\begin{aligned} S_{WVD}(t,\omega) &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_\chi(\theta,\tau) . e^{-j(\tau\omega+\theta t)} d\tau d\theta \\ S_\chi(\theta,\tau) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{WVD}(t,\omega) . e^{j(\tau\omega+\theta t)} dt d\omega \end{aligned} \tag{6}$$

and its generalized form, generalized ambiguity function (GAF) $S_{A_\Phi}(\theta, \tau) = \Phi(\theta, \tau) S_\chi(\theta, \tau)$, is characteristic function of GTFR as shown in the following equation.

$$\begin{aligned} S_{A_\Phi}(\theta,\tau) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{E_\Phi}(t,\omega) . e^{j(\theta t + \tau\omega)} dt d\omega \\ S_{E_\Phi}(t,\omega) &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{A_\Phi}(\theta,\tau) . e^{-j(\theta t + \tau\omega)} d\theta d\tau \end{aligned} \tag{7}$$

AF also generates cross-terms artifacts for multiple components signal due to its bilinear structure. The peculiarity of ambiguity surface, the squared magnitude of ambiguity function, is that for multiple component signals the auto-terms are localized around the origin and the cross-terms are located away from the origin [5]. Therefore, in order to design a GEP kernel and choose its parameter for a certain application we must know the ambiguity function of the signal for that application. We consider a signal which is sum of two ultrasonic echoes and each echo has different parameters, i.e., arrival time t_i, frequency ω_i, and bandwidth related parameter α_i. This signal can be expressed as follows [4] :

$$\begin{aligned} s(t) &= s_1(t) + s_2(t) \\ &= \left(\frac{2\alpha_1^2}{\pi}\right)^{1/4} e^{-\alpha_1^2(t-t_1)^2} e^{j\omega_1 t} \\ &\quad + \left(\frac{2\alpha_2^2}{\pi}\right)^{1/4} e^{-\alpha_2^2(t-t_2)^2} e^{j\omega_2 t} \end{aligned} \tag{8}$$

The sum of auto-terms of square magnitudes of the ambiguity function of the signal s (t) is expressed as follows :

$$\begin{aligned} &\left|S_{\chi_{11}}(\theta,\tau)\right|^2 + \left|S_{\chi_{22}}(\theta,\tau)\right|^2 \\ &= e^{-\alpha_1^2\tau^2} e^{-\frac{\theta^2}{4\alpha_1^2}} + e^{-\alpha_2^2\tau^2} e^{-\frac{\theta^2}{4\alpha_2^2}} \end{aligned} \tag{9}$$

The cross-terms in the AF of the signal s (t) can be derived as follows [4] :

$$\begin{aligned} \left|S_{\chi_{12}}(\theta,\tau)\right|^2 &= \frac{2\sqrt{\alpha_1^2\alpha_2^2}}{\alpha_1^2 + \alpha_2^2} e^{-\frac{2\alpha_1^2\alpha_2^2}{\alpha_1^2+\alpha_2^2}\left[\tau - (t_1 - t_2)\right]^2} \\ &\quad . e^{-\frac{\left[\theta + (\omega_1 - \omega_2)\right]^2}{2(\alpha_1^2 + \alpha_2^2)}} \end{aligned} \tag{10}$$

and

$$\begin{aligned} \left|S_{\chi_{21}}(\theta,\tau)\right|^2 &= \frac{2\sqrt{\alpha_1^2\alpha_2^2}}{\alpha_1^2 + \alpha_2^2} e^{-\frac{2\alpha_1^2\alpha_2^2}{\alpha_1^2+\alpha_2^2}\left[\tau + (t_1 - t_2)\right]^2} \\ &\quad . e^{-\frac{\left[\theta - (\omega_1 - \omega_2)\right]^2}{2(\alpha_1^2 + \alpha_2^2)}} \end{aligned} \tag{11}$$

It is clear from Equations (9-11) that the auto terms of the AF of ultrasonic echoes are centered at the origin and the cross terms are located at a time equal to the difference in time of arrival of two echoes and at a frequency equal to the difference in frequencies of the two echoes. If the two echoes of same frequency arrive at different times then the cross-terms of AF will be located on $\theta = 0$ axis, which is τ-axis. Similarly if the two echoes of different frequencies arrive at the same time, then the cross-terms of AF will be located on $\tau = 0$ axis, which is θ-axis. Knowing these characteristics of AF of ultrasonic signals we can design the GEP kernel function. The parameters of the GEP kernel can be determined by detecting AF cross-terms on τ-axis and θ-axis. The design of GEP kernel is in fact involves designing of a 2-D filter which filters out the cross-terms effectively while keeping the auto-terms. Then the 2-D Fourier transform (FT) of cross-terms deleted AF will generate a 2-D filtered WVD as expressed by Equation (6).

III. DESIGN OF GEP KERNEL FUNCTION FOR ULTRASONIC SIGNALS

The implementation of AF and WVD of a signal and the relationship between AF and WVD is depicted by Figure 1. In order to design a GEP kernel we first find the AF of the signal and then detect the AF cross-terms on $\theta = 0$ axis (i.e., τ-axis) and $\tau = 0$ axis (i.e., θ-axis) excluding the origin on which the AF auto-terms are centered. In this way we have three sets of parameters (i.e., θ_1, τ_1, n, and k) for the GEP kernel function of Equation (3). The three different cases of ultrasonic echoes are as follows:

1. The two backscattered echoes arrive at the same time but *differ in frequencies*. The echoes and the contour plot and surface plot of their AF is depicted in Figure 2. The cross-terms of the AF are separated in frequency and are located on θ-axis. Therefore, we design a GEP kernel such that it is broader along τ-axis in order to emphasize the auto-terms centered at origin and narrower along θ-axis to diminish the cross-terms energy. The width of the kernel along τ-axis depends on the frequency spread of the backscattered echoes. The GEP kernel with parameters $\theta_1 = 2.5$, $\tau_1 = 0.2$, $n = 1$, and $k = 4$ is shown in Figure 3.

2. The two backscattered echoes *with same frequency* and different times of arrival are shown in Figure 4a. In this case the cross-terms of the AF will be separated in time and will be located on τ-axis. This is depicted in contour plot and surface plot of AF of these echoes in Figure 4b and 4c. The GEP kernel in this case needs to be broader along τ-axis and narrower along θ-axis in order to reduce the cross-terms without affecting the auto-terms of the AF. The GEP kernel with these properties is shown in Figure 5. The parameters of this GEP kernel are $\theta_1 = 0.2$, $\tau_1 = 12$, $n = 4$, and $k = 1$.

3. *The third case* in which two echoes not only differ in frequency but also are separated in time is shown in Figure 6a. The cross-terms of AF of this signal are away from both τ-axis and θ-axis at distances depending on the difference in their times of arrival and frequencies as shown in Figure 6b and 6c. The design of GEP kernel for such signal would peak at the origin and time and frequency axes but would taper off

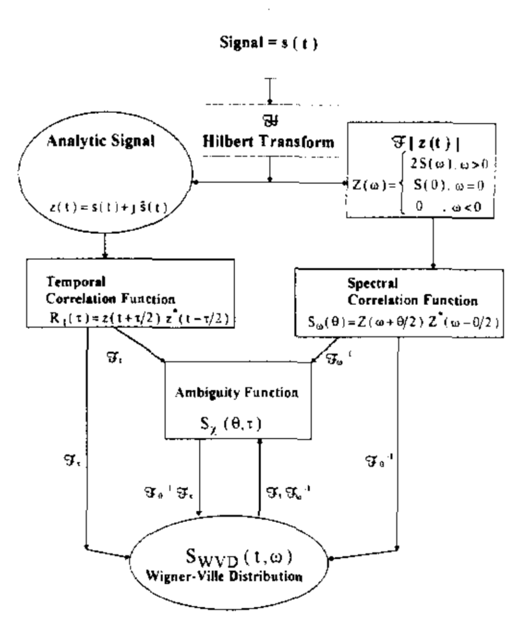

Figure 1. The Implementation of WVD and AF of a Signal s(t) and Their Relationship

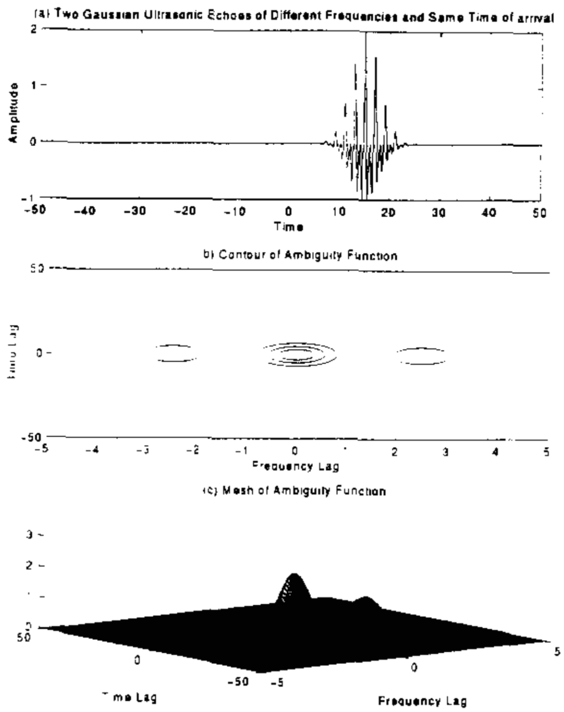

Figure 2 (a) Ambiguity Function(AF) of Two Ultrasonic Gaussian Echoes Located at same Time and with Different Frequencies
(b) Time-Frequency Surface Plot of AF of the Signal in (a)
(c) Contour Plot of AF of the signal in (a).

away from origin and axes. One such kernel is shown in Figure 7. The contour and surface plots of filtered AF shown in Figure 8 clearly indicate the removal of the cross-terms. The parameters of this GEP kernel are $\theta_1 = 2$, $\tau_1 = 12$, $n = 4$, and $k = 4$.

The GEP kernel designed for the third case has the capability of eliminating cross-terms on the ambiguity plane without affecting the auto-terms if the ultrasonic echoes are not overlapped in time and frequency. The ED kernel will eliminate the cross-terms at the cost of affecting the auto terms. This is depicted in Figures 9 and 10 respectively.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

The GEP kernel is applied to a simulated signal containing two echoes of different frequencies overlapped in time embedded in white Gaussian noise. The GEP kernel is designed in such a way that it is broader along θ-axis and narrower along τ-axis. The parameters of GEP are chosen as $\theta_1 = 0.4$, $\tau_1 = 1.25$, $n = 4$, and $k = 1$. These parameters are determined keeping in consideration the sampling frequency of the signal. The results indicate that not only the cross-terms of WVD are reduced but the t-f plane SNR is also enhanced. The ED kernel was also applied which reduced the cross-terms at the cost of deterioration of auto-terms. This is because the ED kernel emphasizes only the origin of the ambiguity plane.

Figure 11a shows the experimental data which contain a defect echo embedded in grain echoes. The frequency of the defect echo is very close to that of the grain echoes. Figure 11b depicts the contour plot of the WVD of this complex experimental data. The defect echo cannot be discerned from grain echoes in WVD. The features of this signal are obscured by the cross-terms of the WVD. We designed the GEP kernel with some apriori knowledge of the cross-terms of the AF of this signal. The GEP parameters are chosen to make the kernel broader along the θ-axis, emphasizing the origin of the ambiguity plane, and deemphasizing the energy on τ-axis other than around the origin. The contour plot of WVD determined from the AF of the experimental data after applying the GEP kernel to the ambiguity plane is shown in Figure 11c. This figure is close to true t-f representation of the experimental data. It is important to note that careful evaluation of this result indicate that three types of echoes with distinct t-f signatures have been detected among many random interfering echoes.

V. CONCLUSIONS

We have presented a method of designing a GEP kernel function which can reduce the cross-terms of the WVD of ultrasonic signals with close to ideal representation of the signal. It is shown that the GEP kernel reduces the cross-terms of a t-f distribution of signals better than the ED kernel. In cases of ultrasonic echoes overlapped in time but having different frequencies or echoes arriving at different times but their frequencies overlap, the GEP kernel outclasses the ED kernel in eliminating the cross-terms of a true t-f distribution of a signal. An effective GEP kernel can be designed with the true understanding of WVD and AF of a signal. Three different sets of parameters can be chosen for three different cases discussed earlier by detecting the peaks of cross terms on θ-axis and τ-axis of the ambiguity plane. The results depict that the GEP kernel reduces and in some cases eliminates the cross-terms of the bilinear t-f distribution of a signal as well as enhances the signal to noise ratio of the t-f plane. The application of the GEP kernel is nothing but 2-D t-f filtering of the WVD of the signal. All the classical concepts of filtering are also applicable for reducing cross-terms pertaining to WVD of signals.

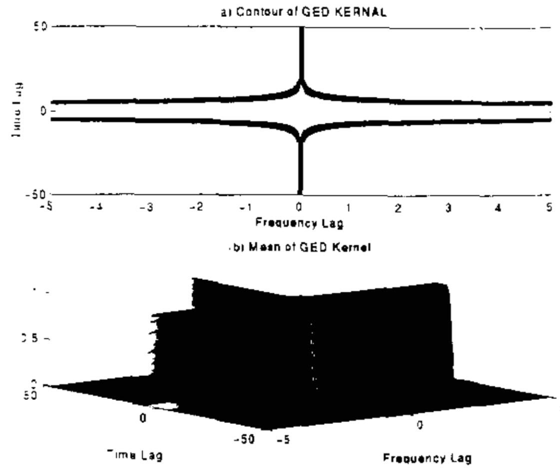

Figure 3 (a) Contour Plot of Generalized Product (GEP) Kernel for reducing cross-terms on θ-axis (b) Time-Frequency Surface Plot of GEP Kernel.

VI. REFERENCES

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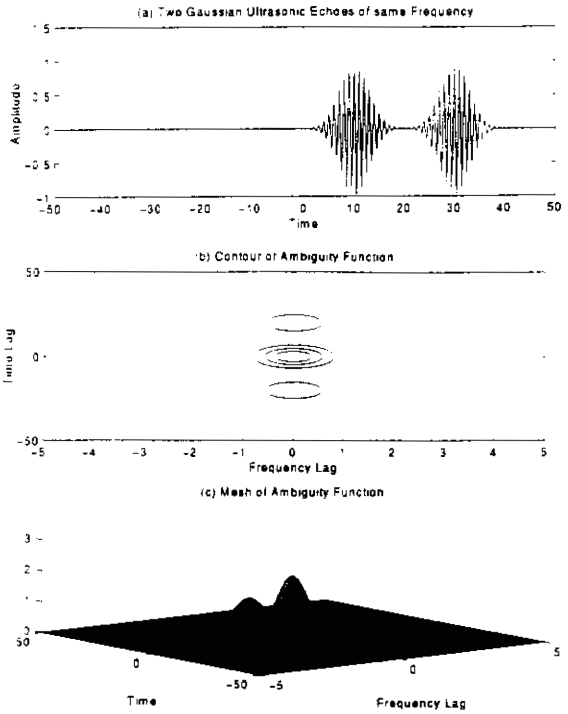


Figure 4 (a) Ambiguity Function(AF) of Two Ultrasonic Gaussian Echoes of same Frequency and separated in Time
(b) Time-Frequency Surface Plot of AF of the Signal in (a)
(c) Contour Plot of AF of the signal in (a).

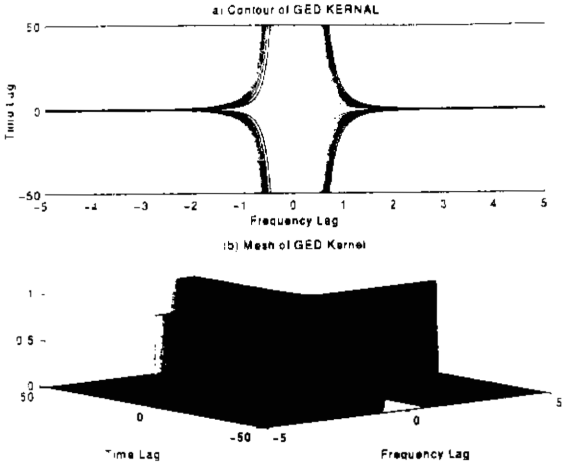


Figure 5 (a) Contour Plot of Generalized Product (GEP) Kernel for reducing cross-terms on τ-axis
(b) Time-Frequency Surface Plot of GEP Kernel.

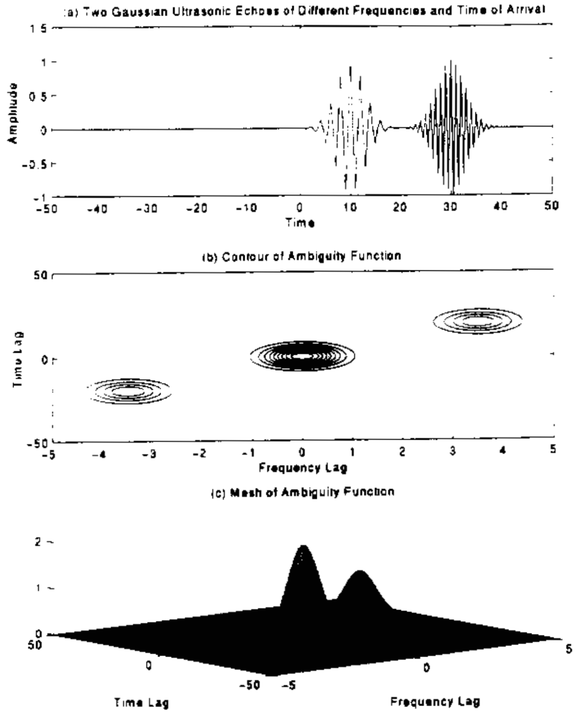


Figure 6 (a) Ambiguity Function(AF) of Two Ultrasonic Gaussian Echoes separated in both Time and Frequency
(b) Time-Frequency Surface Plot of AF of the Signal in (a)
(c) Contour Plot of AF of the signal in (a).

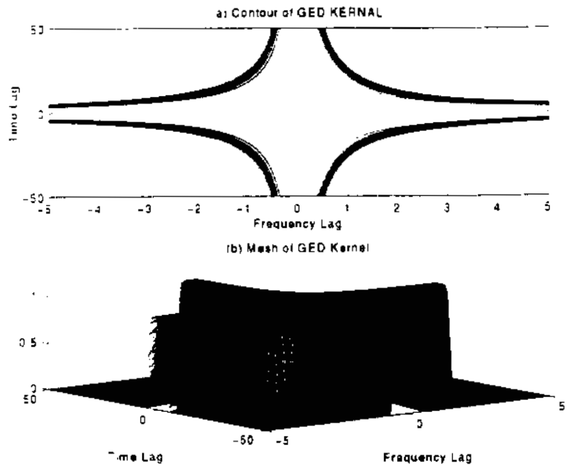


Figure 7 (a) Contour Plot of Generalized Product (GEP) Kernel for reducing cross-terms away from both τ-axis and θ-axis
(b) Time-Frequency Surface Plot of GEP Kernel.

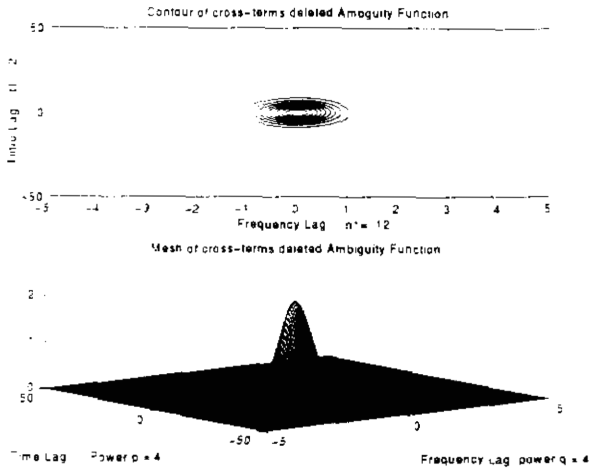


Figure 8 (a) Cross-Terms Reduced Ambiguity Function (AF) of Two Ultrasonic Gaussian Echoes separated in both Time and Frequency Using GEP kernel
(b) Time-Frequency Surface Plot of AF

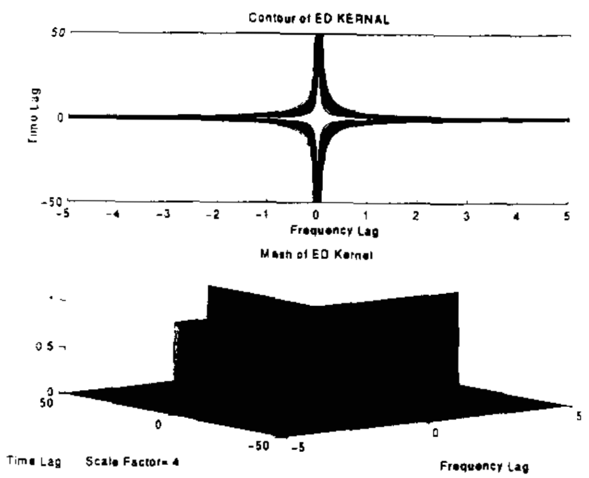
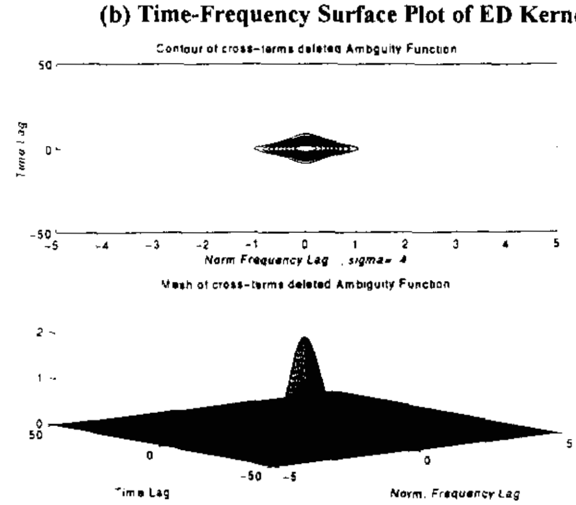


Figure 9 (a) Contour Plot of ED Kernel for reducing cross-terms away from both τ-axis and θ-axis
(b) Time-Frequency Surface Plot of ED Kernel.



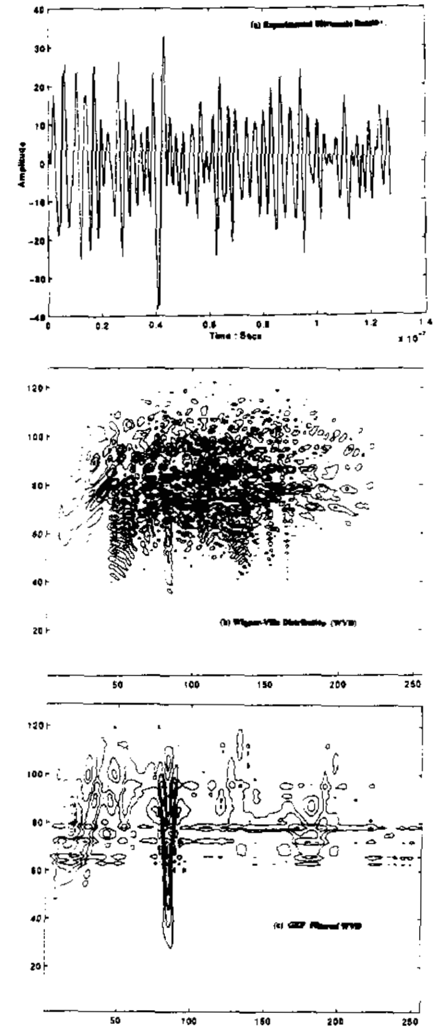


Figure 11 (a) Ultrasonic Experimental Data containing Defect Echoes Embedded in Random Grain Echoes
(b) Contour Plot of WVD of the Signal in (a)
(c) Contour Plot of Filtered WVD of the signal in (a).

Figure 10 (a) Cross-Terms Reduced Ambiguity Function (AF) of Two Ultrasonic Gaussian Echoes separated in both Time and Frequency Using ED kernel
(b) Time-Frequency Surface Plot of AF