PARAMETER ESTIMATION OF MULTIPLE INTERFERING ECHOES USING THE SAGE ALGORITHM

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ABSTRACT

Parametric modeling of a measured signal in terms of frequency dispersion, velocity, amplitude fading and echo skewness can be used as a quantitative technique for nondestructive evaluation. A simple model with a fixed number of parameters often performs inadequately when the ultrasonic wavelet is propagated through inhomogeneous materials and/or reflected by complex objects. In this study, we model the detected ultrasonic signal as a superposition of many Gaussian echoes corrupted with measurement noise. To estimate the parameters of multiple Gaussian echoes, a Space Alternating Generalized Expectation Maximization (SAGE) algorithm has been developed. In performance evaluation of the SAGE algorithm, Monte-Carlo simulation has been used. The estimated parameters have been found to be unbiased and their variances achieve analytical Cramer-Rao Lower Bounds (CRLB) for Signal-to-Noise Ratio (SNR) as low as 3 dB. The CRLB also constitutes the resolution bounds on the estimated parameters. Furthermore, the SAGE algorithm has been applied to experimental ultrasonic data consisting of multiple interfering echoes. It has been observed that the model fits accurately to the measured signal (estimation SNR is as high as 24 dB) and the estimated parameters display a high resolution and accurate characterization of the measurement.

I. INTRODUCTION

The frequency spectrum of the ultrasonic backscattered echoes, although it varies within the transducer bandwidth, is influenced by frequency characteristics of the propagation path (governed by the frequency dependent absorption and scattering) and also represents information pertaining to the impedance, geometric shape, size and orientation of the reflectors [1-2]. Therefore, a signal model that exploits these properties of reflection and scattering is desirable for the nondestructive evaluation of materials. In this study, we have developed a parametric signal model for ultrasonic backscattered echoes. Then, the parameters of the model have been linked to the physical properties of the reflectors and propagation path. Furthermore, the maximum likelihood criterion is used to obtain an objective function and to estimate parameters. To prove the optimality of estimated parameters, a Monte-Carlo simulation is implemented. Having confidence in estimated parameters, the estimation of superimposed echoes is considered using a Space Alternating Generalized Expectation Maximization (SAGE) algorithm.

Ultrasonic Signal Modeling

Assuming a Gaussian shape power spectrum for the ultrasonic transducer, the impulse response of the transducer can be represented as

$$g(t) = e^{-\alpha_T t^2} \cos(2\pi f_{c_T} t) \qquad (1)$$

where f_{c_T} is the transducer center frequency and α_T is the bandwidth factor. Using this ideal model of transducer for ultrasound measurement, the backscattered echo from a flat surface reflector result in the signal model:

$$f(t) = \beta\, e^{-\alpha(t-\tau)^2} \cos(2\pi f_c (t-\tau) + \phi) \qquad (2)$$

Because of its Gaussian shaped envelope, this model will be referred to as the Gaussian echo wavelet. The parameters of the model have intuitive meanings for an ideal surface reflection. The time of arrival τ links to the location of the reflector as the distance from the transducer over the speed of ultrasound in the propagation path. The amplitude β accounts for the attenuation of the original signal and the size of reflector relative to the beam field. The parameters, f_c and α are the modified center frequency and bandwidth factor resulting from the propagation path. The phase of signal ϕ is sensitive to the orientation of the reflector [3].

To demonstrate the feasibility of the model, an experimental setup, shown in Figure 1, has been used. A small piece of plexiglass sample simulating the reflector is placed at the distance d from the transducer in the water tank. Water provides a homogeneous (ideal) propagation path for the sound. Figure 2 shows the discrete values of the measured signal (i.e., dots) backscattered from the plexiglass. The first echo in the figure is backscattered from the front surface and the second echo from the back surface of the plexiglass. These two echoes are modeled according to Equation 2, and the estimated signal based on this model is shown in the same figure in a solid line.

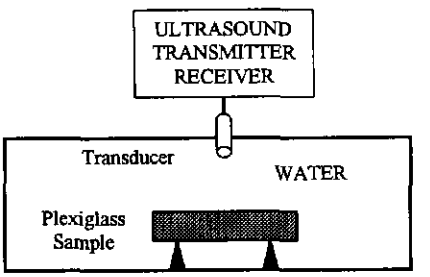

Figure 1. Experimental setup

It is important to point out that the model fits accurately to the measured signal.

In general, the medium under the ultrasonic test has many reflectors of different sizes randomly positioned within the *propagation path*. *Assuming a Gaussian echo model for each* reflector, the model for M point reflectors can be expressed as

$$y(t) = \sum_{m=1}^{M} \beta_m e^{-\alpha_m (t-\tau_m)^2} \cos(2\pi f_{c_m} + \phi_m) \; + \; v(t) \qquad (3)$$

where $v(t)$ represents the measurement noise having the *property of a white Gaussian noise (WGN) process*. The parameters of each echo can be grouped in a parameter vector as $\theta_m = [\alpha_m \;\; \tau_m \;\; f_{cm} \;\; \phi_m \;\; \beta_m]$. Each parameter vector completely defines the shape of the corresponding echo. Using the parameter vectors, the model in Equation 3 can be written in a more compact form:

$$y(t) = \sum_{m=1}^{M} f(\theta_m; t) \; + \; v(t) \qquad (4)$$

where $f(.)$ represents the Gaussian echo wavelet.

The model presented in Equation 4 has many parameters that can be adjusted to represent a complex signal. The estimation of these parameters is desirable for quantitative evaluation of ultrasonic echoes. Next section presents the Maximum Likelihood estimation of these parameters as well as the analytical relationships to evaluate the accuracy and resolution of the estimation in the presence of noise.

II. MAXIMUM LIKELIHOOD ESTIMATION OF BACKSCATTERED ECHOES

The model of ultrasonic backscattered echoes in the presence of WGN consists of many parameters that can be linked to the physical properties of the reflector and frequency characteristics of the propagation path. The estimation of these parameters is desirable for the quantitative evaluation of ultrasonic echoes. To achieve this, three major problems associated with parameter *estimation algorithms must be overcome*. *First, the* transformation from parameter space to signal space, although appropriately defined by a Gaussian echo wavelet, is nonlinear. Hence, the reverse process, the transformation from signal space to parameter space is also nonlinear and has no explicit solution. Second, the noise embedded in the measured signal obscures the estimation of the true value of the parameters. Therefore, degradation in the estimation of the parameters necessitates the quantitative evaluation of error. Third, the number of echoes may not be known a priori due to many overlapping echoes. Hence, an iterative method based on minimum acceptable error needs to be devised for estimating the number of echoes.

The first two problems will be addressed in a maximum likelihood framework considering the parameter estimation of a single echo. Then, the degradation in the estimation of parameters will be quantified using analytical Cramer-Rao bounds. *The performance of estimation method will be studied* by Monte-Carlo simulation. Finally, the parameter estimation of M-superimposed echoes will be addressed using the SAGE algorithm.

Parameter Estimation of a Single Echo

The discrete form of a single ultrasonic echo given by Equation 1 can be represented as

$$x = s(\theta) + \upsilon \qquad (5)$$

where $x \in \Re^N$ is an observation vector, $\upsilon \in \Re^N$ is a WGN vector with the variance σ^2, and $s(\theta): \theta \in \Re^5 \xrightarrow{f} s(\theta) \in \Re^N$ is a discrete Gaussian echo wavelet defined as,

$$s(\theta; t(n)) = \beta e^{-\alpha(t(n)-\tau)^2} \cos\{2\pi f_c (t(n) - \tau) + \phi\}, \; n = 0, 1, 2, \ldots, N-1 \qquad (6)$$

The parameter vector $\theta = [\alpha \;\; \tau \;\; f_c \;\; \phi \;\; \beta]$ represents all the parameters of the echo. Our goal is to estimate θ given the observations x. The maximum likelihood estimate (MLE) of θ is defined as the value of the parameter that maximizes the *likelihood function* defined as the joint *probability density function* of the observations x

$$p(x;\theta) = \frac{1}{(2\pi)^{N/2} |C(\theta)|^{1/2}} \exp\{-\frac{1}{2}(x - \mu(\theta))^T C^{-1}(\theta)(x - \mu(\theta))\} \qquad (7)$$

where $\mu(\theta) = E\{s(\theta) + \upsilon\}$ is the mean vector and $C(\theta) = E\{(x - \mu(\theta))(x - \mu(\theta)^T\}$ is the covariance matrix. The above *Normal Distribution* is only valid when all the observations are independent and identically distributed. Under constant parameters and WGN assumption, that is $\mu(\theta) = s(\theta)$ and $C(\theta) = \sigma^2 I$, the *likelihood function* reduces to

$$J(\theta) = (x - s(\theta))^T (x - s(\theta)) \qquad (8)$$

known as the Mean Square Error (MSE). The MLE of θ can be found by minimizing this MSE with respect to θ. This efficient method of solving MLE is known as the least-squares (LS) optimization. The simplification was achieved by assuming the measurement noise is a white Gaussian noise process. Once the LS estimation is performed, the consistency of Gaussian noise modeling can be examined by the characteristics of the residual error between the estimated and the observed echo.

The MSE (i.e., the objective function) given in Equation 8 is a nonlinear function of parameter vector θ. Numerical optimization methods can be employed to minimize this objective function, but a special care has to be taken to ensure convergence to the global solution. One approach is to use the Gauss-Newton optimization method [6], an iterative algorithm, based on the following formula:

$$\theta_{k+1} = \theta_k + (H(\theta_k) H(\theta_k))^{-1} H^T(\theta_k)(x - s(\theta_k)) \qquad (9)$$

where $H(\theta)$ is the gradient of the model defined as:

$$H(\theta) = \left[\frac{ds(\theta)}{d\alpha} \;\; \frac{ds(\theta)}{d\tau} \;\; \frac{ds(\theta)}{df_c} \;\; \frac{ds(\theta)}{d\phi} \;\; \frac{ds(\theta)}{d\beta} \right] \qquad (10)$$

In improving the estimate, the gradient matrix has to be computed using current θ_k. Although this can be done numerically, the analytical derivation speeds up the computation. By introducing the following kernel functions:

$$f(\theta) = e^{-\alpha(t-\tau)^2} \cos\{2\pi f_c (t - \tau) + \phi\},$$

$$g(\theta) = e^{-\alpha(t-\tau)^2} \sin\{2\pi f_c (t - \tau) + \phi\}$$

the analytical derivatives can be obtained:

$$\frac{ds(\theta)}{d\alpha} = -\beta(t-\tau)^2 f(\theta), \qquad \frac{ds(\theta)}{df_c} = -2\pi\beta(t-\tau)g(\theta),$$
$$\frac{ds(\theta)}{d\tau} = -2\alpha(t-\tau)f(\theta) + 2\pi f_c \beta g(\theta),$$
$$\frac{ds(\theta)}{d\phi} = -\beta g(\theta), \qquad \frac{ds(\theta)}{d\beta} = f(\theta). \tag{11}$$

Finally, the iteration can be terminated when the improvement in the parameter vector is smaller than a set tolerance. Therefore, the Gauss-Newton algorithm for parameter estimation of a single echo can be summarized in the following computational steps:

1. Start with an initial guess: $\theta = \theta_0$, set $k = 0$
2. Compute the gradient $H(\theta_k)$
3. Iterate: $\theta_{k+1} = \theta_k + (H(\theta_k)H(\theta_k))^{-1} H^T(\theta_k)(x - s(\theta_k))$
4. Check convergence: $\|\theta_{k+1} - \theta_k\| < Tolerance$
5. Go to Step 2.

Cramer-Rao Analytical Bounds

It is critical to evaluate the accuracy and resolution of estimation in the presence of noise. The *Cramer-Rao Lower Bounds* (CRLB) provides an analytical evaluation of errors in estimation of parameters. The CRLB for a vector parameter θ is defined by the following inequality [6]:

$$\mathrm{var}(\hat{\theta}_i) \geq \left[I^{-1}(\theta)\right]_{ii} \tag{12}$$

where $I(\theta)$ is the *Fisher Information* matrix defined as:

$$\left[I(\theta)\right]_{ij} = -E\left[\frac{\partial^2 \ln p(x;\theta)}{\partial\theta_i \partial\theta_j}\right] \tag{13}$$

and $\ln p(x;\theta)$ is the log-likelihood function given by Equation 7. For the observation model x given in Equation 5, normally distributed as $N(s(\theta),\sigma^2 I)$, the *Fisher Information* matrix can be simplified to

$$I(\theta) = \frac{1}{\sigma^2} H(\theta)^T H(\theta) \tag{14}$$

where $H(\theta)$ is the gradient previously defined in Equation 10. Having analytical expressions for the gradient, the *Fisher Information* matrix can also be derived analytically as

$$I(\theta) = \left(\frac{E_s}{\sigma^2}\right)\left(\frac{1}{f_s}\right)\begin{bmatrix} \frac{3}{16\alpha^2} & 0 & 0 & 0 & \frac{-1}{4\alpha\beta} \\ 0 & \alpha + (2\pi f_c)^2 & 0 & -2\pi f_c & 0 \\ 0 & 0 & \frac{\pi^2}{\alpha} & 0 & 0 \\ 0 & -2\pi f_c & 0 & 1 & 0 \\ \frac{-1}{4\alpha\beta} & 0 & 0 & 0 & \frac{1}{\beta^2} \end{bmatrix} \tag{15}$$

where E_s is the signal energy and f_s the sampling frequency. The inversion of the matrix in Equation 15 can also be carried out analytically. Defining SNR as, $\varsigma = E_s/E_v$, where $E_v = \sigma^2$ is the noise energy, the terms along the diagonal of the inverse *Fisher Information* matrix yield the analytical CRLB associated with parameters, i.e.,

$$\mathrm{var}(\hat{\alpha}) \geq \frac{f_s 8\alpha^2}{\varsigma}, \qquad \mathrm{var}(\hat{\tau}) \geq \frac{f_s}{\alpha\varsigma}, \qquad \mathrm{var}(\hat{f}_c) \geq \frac{f_s \alpha}{\varsigma\pi^2},$$
$$\mathrm{var}(\hat{\phi}) \geq \frac{f_s(\alpha + 4\pi^2 f_c^2)}{\alpha\varsigma}, \qquad \mathrm{var}(\hat{\beta}) \geq \frac{f_s 3\beta^2}{2\varsigma}. \tag{16}$$

Monte-Carlo Simulation Results

A Gaussian echo with the parameters (bandwidth factor $\alpha = 25$, arrival time $t_o = 1\,\mu s$, center frequency $f_c = 8\,MHz$, phase $\phi = 1\,rad$, and amplitude $\beta = 1$) summarized in a parameter vector, $\theta = [10 \;\; 1 \;\; 8 \;\; 1 \;\; 1]$, is simulated using Equation 6. A realization of a zero-mean white Gaussian noise process with the variance σ^2 is added to the echo. Then, the Gauss-Newton algorithm presented above is applied to estimate echo parameters. This process is repeated 100 times, yielding a different estimate each time, due to the random nature of the additive noise. The means of estimators are compared to the actual parameter vector used in simulation and the variances of estimators against analytical CRLB (see Table 1). Table 1 clearly indicates that the estimated means closely match the actual parameter vector and the variances attain CRLB. Therefore, the estimator implemented by the Gauss-Newton algorithm above, is a minimum variance unbiased estimator, hence optimal.

SNR [dB]		Bandwidth Factor	Arrival Time (microsec.)	Center Frequency (MHz)	Phase (rad)	Amplitude
	Actual Parameters	25.0000	1.0000	8.0000	1.0000	1.0000
	Initial Guess	20.0000	0.8000	6.4000	0.8000	0.8000
20.10	Mean	25.0207	0.9999	7.9986	0.9965	1.0015
	Variance	0.2435	1.62E-04	1.40E-04	0.0041	6.58E-06
	CRLB	0.2443	1.95E-04	1.23E-04	0.0049	7.33E-06
10.07	Mean	25.0325	0.9998	8.0027	0.9911	1.0038
	Variance	2.5764	0.0020	0.0013	0.0527	0.0007
	CRLB	2.4576	0.0019	0.0012	0.0501	0.0007
2.51	Mean	25.3903	0.9977	7.9957	0.8862	1.0095
	Variance	13.7007	0.0104	0.0072	0.2663	0.0033
	CRLB	14.0078	0.0112	0.0070	0.2659	0.0042

Table 1. Monte Carlo Simulation Results, Sampling Frequency 200 MHz, 100 trials of estimation performed.

III. PARAMETER ESTIMATION OF SUPERIMPOSED ECHOES USING SAGE ALGORITHM

The Gauss-Newton implementation of single echo estimation has been demonstrated to be efficient (i.e., given a reasonable initial guess, it produces the optimal solution for parameters). However, in addressing a more general problem, the MLE of multiple superimposed echoes in WGN (the model given by Equation 4) requires the minimization of the term

$$\left\| y - \sum_{m=1}^{M} s(\theta_m) \right\|^2 \tag{17}$$

with respect to parameter vectors $\theta_1, \theta_2, \ldots, \theta_M$. The term y is the observed data consisting of M echoes plus noise. In general, minimization of Equation 17 is not practical because of computational load and potential poor convergence. One efficient method of estimating superimposed echoes is to use complete-data space (CDS), a hypothetical data set, for each composing echo in estimating their corresponding parameter set. This method is known as the Expectation Maximization (EM) algorithm and can be applied to estimate superimposed echoes [4]. An alternative to EM is the Space Alternating Generalized EM (SAGE) [5], which offers a fast convergence property. The SAGE algorithm involves estimating a CDS for each composing echo and computing the MLE of the corresponding parameter vector using the estimated CDS. The CDS is computed using the observed data and the value of parameters at the current iteration. This SAGE algorithm can be summarized in following steps:

1. Start with initial guesses, $\theta_1^{(0)}, \theta_2^{(0)}, \ldots, \theta_M^{(0)}$ and set k=0 (iteration number) and m=1 (echo number).
2. Expectation Step: Compute
$$\hat{x}_m^{(k)} = f(\theta_m^{(k)}) + \frac{1}{M}\{y - \sum_{i=1}^{M} f(\theta_i^{(k)})\}$$
3. Maximization Step: Iterate the m-th parameter vector
$$\hat{\theta}_m^{k+1} = \arg\min \left\| \hat{x}_m^{(k)} - s(\theta_m^{(k)}) \right\|^2, \ \hat{\theta}_m^{k} = \hat{\theta}_m^{k+1}$$
4. Set m=m+1 and go to Step 2 unless m=M
5. Check convergence, if converged, STOP.
6. Set m=1, k=k+1 and go to Step 2.

In Step 2 (E-Step) of the algorithm, the expectation of the m-th echo, $\hat{x}_m^{(k)}$, is computed using the current estimate of the parameter vector $\theta_m^{(k)}$ and the observed data y. Then using this expected signal in the maximization step (M-Step), θ_m^{k+1} is computed as the ML estimate for θ_m^{k}. In other words, the M-Step corresponds to the MLE of a single echo subject to estimated data, $\hat{x}_m^{(k)}$. Note that the maximization step can be implemented using the Gauss-Newton algotihm developed earlier for single echo estimation. Once the MLE of one echo is performed, in the next E-Step the expectation of the next signal $\hat{x}_{m+1}^{(k)}$ will be computed using the estimated parameter θ_m^{k+1}.

The SAGE algorithm has been applied to detect and estimate two non-interfering echoes (Figure 2) and two interfering echoes (Figure 3). The signals are collected using the experimental setup given in Figure 1. The non-interfering echoes are the reflections from the plexiglass sample as discussed in Section II. The interfering echoes are generated by placing a thin, ultrasonically semi-transparent layer on top of a steel sample using the same test setup. The parameter estimation of non-interfering echoes (Figure 2) demonstrates the agreement of the model with the measurement where the estimation error is insignificant (i.e., the estimation SNR is 24 dB). The estimation of the parameters associated with two interfering echoes (Figure 3) demonstrates the ability of the method in discriminating closely spaced reflectors. This superb performance of modeling and estimation leads to the system identification and quantitative characterization of the material.

IV. CONCLUSION

In this study, we developed a parametric signal model for ultrasonic nondestructive testing. Then, the MLE of these parameters is addressed when the signal has been corrupted by WGN. A Gauss-Newton algorithm has been implemented for the MLE of a single echo. In the performance analysis of the estimation, the analytical CRLB has been derived in order to obtain the lower bound for the variance of the estimated parameters, which quantifies the resolution and accuracy of the estimation. Monte-Carlo simulation results demonstrate that the estimator is a minimum variance unbiased estimator (that it achieves the CRLB for a SNR as low as 3 dB), hence optimal. Then, the parameter estimation problem has been extended to a more general case, the MLE of M-superimposed echoes. In estimating superimposed echoes, a SAGE algorithm has been developed. The algorithm has been tested on simulated and experimental echoes. Results suggest the consistency of the model being used as well as the ability of the method in discriminating closely spaced reflectors. This method of modeling and estimation leads to the quantitative evaluation of ultrasonic signals linked to the physical properties of targets and the propagation path.

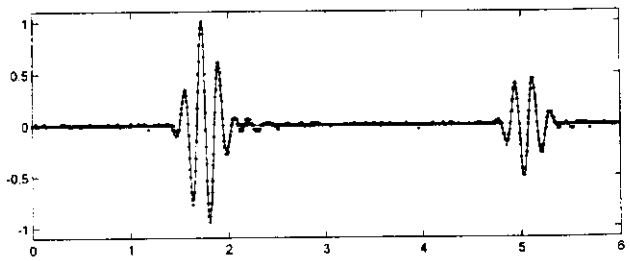

Figure 2. Two non-interfering echoes backscattered from plexiglass sample (4.8 mm thick). Dots indicate the measured echoes and solid line indicates the estimated echoes.

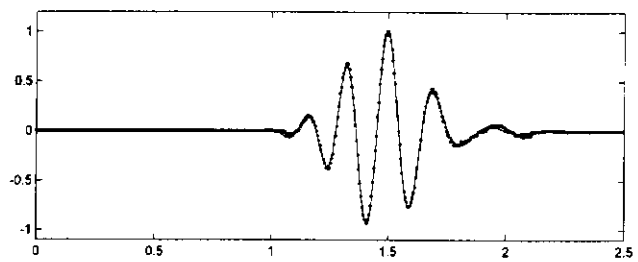

Figure 3. Two interfering echoes backscattered from a thin transparency film (0.1 mm). Dots indicate the measured echoes and solid line indicates the estimated echoes.

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