MODEL BASED TIME-FREQUENCY ESTIMATION OF ULTRASONIC ECHOES FOR NDE APPLICATIONS

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ABSTRACT - Modeling the ultrasonic signals in terms of the Gaussian echoes assures a solution for signal parameters and a technique for time-frequency representation. In this study, the parameter estimation is addressed using iterative Maximum A Posteriori (MAP) estimation algorithms. To reduce the computational complexity, we have developed a divide-and-conquer estimation procedure. Upon estimation, we have explored the merits of this model-based technique for time-frequency analysis of ultrasonic echoes. In particular, we are presenting ultrasonic experimental data where the microstructure scattering echoes dominate the flaw echoes (SNR is about 0 dB). This type of data is used to demonstrate the effectiveness of model based time-frequency representation over conventional techniques such as Wigner-Ville distribution and short-time Fourier transform.

I. INTRODUCTION

The Time-Frequency (TF) characteristics of ultrasonic echoes present valuable information pertaining to the characterization of materials and detection of flaws. However, due to the noise and overlapping echo patterns it is quiet challenging to trace ultrasonic echo characteristics. The conventional TF representations (e.g., Wigner-Ville Distribution (WVD), and Short-Time Fourier Transform (STFT)) when applied to ultrasonic signals introduce cross terms, offer poor resolution and they are sensitive to noise level. Representing the ultrasonic signal in terms of model echoes assures an analytical expression with the potential to obtain a TF solution.

In this study, the ultrasonic signals are modeled as a superposition of many Gaussian echoes with unknown parameters. A Gaussian echo wavelet offers a desirable analytic solution for TF representation. The Maximum A Posteriori (MAP) estimation principle, based on the prior knowledge of the spectral characteristic of the ultrasonic transducer has been used in order to estimate the unknown parameters. In the estimation, we first developed a coupled MAP [1] and Gauss Newton (MAPGN) [2] algorithm to estimate the parameters of a single echo in noise. Then, we coupled this algorithm into a SAGE [3] algorithm (MAP-SAGE) to estimate the parameters of a number of overlapping echoes in noise. However, using the MAP-SAGE algorithm for an ultrasonic signal composed of many echoes demands significant computation. To reduce the computational complexity, we have developed a divide-and-conquer procedure by choosing a variable length window and adaptively sliding it through the data trace.

II. MAP ESTIMATION ALGORITHMS

Consider that the ultrasonic measured signal consists of a single echo:

$$x = s(\theta) + v \qquad (1)$$

where $x \in \Re^N$ is a vector of observations, $v \in \Re^N$ is a white Gaussian noise (WGN) sequence with variance σ_v^2, and $s(\theta) : \theta \in \Re^5 \longrightarrow s(\theta) \in \Re^N$ is a Gaussian echo vector obtained by sampling the continuous time model;

$$s(\theta;t) = \beta e^{-\alpha(t-\tau)^2} \cos\{2\pi f_c (t-\tau) + \phi\} \qquad (2)$$

where the parameters of the model, bandwidth factor, time-of-arrival (TOA), center frequency, phase and amplitude are stored in a parameter vector, $\theta = [\alpha \quad \tau \quad f_c \quad \phi \quad \beta]$. We assume θ is a random variable vector with the following statistics:

$$E[\theta] = \bar{\theta} = \mu_\theta$$
$$E[(\theta - \bar{\theta})(\theta - \bar{\theta})^T] = C_{\theta\theta} \qquad (3)$$

Given above prior statistics and the observation model (see Equation 1), we find the Maximum A Posteriori (MAP) estimate of the parameter vector θ.

MAPGN Algorithm: The MAP estimator of a single echo can be implemented using a Gauss Newton (GN) algorithm [2] (hence called MAPGN) in the following computational steps:

1. Start with an initial guess $\theta = \theta^{(0)}$ and prior statistics, μ_θ and $C_{\theta\theta}$.
2. Compute the model $s(\theta^{(k)})$ and the gradients

$$H(\theta) = \left[\frac{\partial s}{\partial \alpha} \quad \frac{\partial s}{\partial \tau} \quad \frac{\partial s}{\partial f_c} \quad \frac{\partial s}{\partial \phi} \quad \frac{\partial s}{\partial \beta} \right]_{\theta=\theta^{(k)}}$$

3. Iterate the parameter vector by computing the followings:

$$\tilde{x}^{(k)} = x - s(\theta^{(k)}) + H(\theta^{(k)})\theta^{(k)}$$
$$\theta^{(k+1)} = \mu_\theta + [(C_{\theta\theta}^{-1} + \frac{1}{\sigma_v^2} H^T(\theta^{(k)})H(\theta^{(k)}))^{-1} H^T(\theta^{(k)}) \frac{1}{\sigma_v^2}(\tilde{x}^{(k)} - H(\theta^{(k)})\mu_\theta]$$

4. Check convergence:

If $\left\| \theta^{(k+1)} - \theta^{(k)} \right\| < Tolerance$, STOP.

5. Set $k \to k+1$ and go to Step 2.

MAP-SAGE Algorithm: We consider the discrete version of the M-superimposed Gaussian echo model

$$y = \sum_{m=1}^{M} s(\theta_m) + v \qquad (4)$$

where $s(.)$ denotes the Gaussian echo model and v denotes a WGN sequence with variance σ_v^2. The parameter estimation problem is to estimate the parameter vectors $\theta_1, \theta_2, \ldots, \theta_M$ given the noisy observation of echoes, some of which may be overlapping. Each parameter vector is assumed to be a random vector with prior statistics given in Equation 3. In the MAP parameter estimation of superimposed echoes, one can incorporate the MAPGN algorithm in a SAGE algorithm [3] to address the multidimensional parameter estimation problem. This MAP-SAGE algorithm can be implemented in the following computational steps:

1. Make initial guesses for the parameter vectors and form $\Theta^{(0)} = [\theta_1^{(0)}, \theta_2^{(0)}, \ldots, \theta_M^{(0)}]$.
   Use prior statistics μ_θ and $C_{\theta\theta}$ for the parameter vectors. Set $k = 0$ (iteration number) and $m = 1$ (echo number).
2. Expectation Step: Compute the expected signal for the m-th echo
   $$\hat{x}_m^{(k)} = s(\theta_m^{(k)}) + \frac{1}{M}\{y - \sum_{l=1}^{M} s(\theta_l^{(k)})\}$$
3. Maximization Step: Iterate the m-th parameter vector using the MAPGN algorithm:
   $$\theta_m^{(k+1)} = \text{MAPGN}\,(\theta_m^{(k)}, \hat{x}_m^{(k)})$$
   and set $\theta_m^{(k)} = \theta_m^{(k+1)}$
4. Set $m \to m+1$ and go to Step 2 unless $m > M$
5. Check convergence:
   If $\left\| \Theta^{(k+1)} - \Theta^{(k)} \right\| \le tolerance$, then STOP.
6. Set $m = 1$, $k \to k+1$ and go to Step 2.

III. TF REPRESENTATION OF MULTIPLE ECHOES

The Wigner-Ville Distribution (WVD) of a signal, $x(t)$, is defined as

$$WVD(t,\omega) = \frac{1}{2\pi} \int x(t + \frac{\tau}{2}) x^*(t - \frac{\tau}{2})\, e^{-j\omega\tau} d\tau \qquad (5)$$

The WVD, a widely used transformation for TF representation, is particularly advantageous in analyzing dispersive signals whose frequency exhibits variations in time [4]. However, the transformation of complex signals introduces new frequency components (cross-terms) that are not in the original signal. The cross-terms may interfere and obscure the desirable auto-terms. Much effort has been spend in the literature on filtering out the cross-terms [5] but the filtering process effects the

original TF representation as well. As an alternate to filtering process, Gaussian modeling offers a TF representation by displaying the auto-terms of the WVD. The TF representation of a Gaussian echo (Equation 2) can be explicitly obtained by substituting the signal in the WVD (see Equation 5):

$$TF_{s(\theta;t)}(t,\omega) = \frac{\beta^2}{2\pi\alpha} e^{-2\alpha(t-\tau)^2} e^{-(2\pi)^2(f-f_c)^2/2\alpha} \tag{6}$$

If one can decompose an ultrasonic signal in terms of Gaussian echoes, it is possible to achieve a time-frequency representation of the signal. Figure 1-a shows an example where an ultrasonic signal is composed of three interfering Gaussian echoes corrupted by WGN (the SNR is about 10 dB). The estimated signal is shown in Figure 1-b. The direct WVD of these echoes (Figure 1-c) does not reveal the respective frequency components of the signal at their arrival times. Similarly, the STFT often used for TF representation of the signal lacks resolution (see Figure 1-d). However, using a model-based estimation before TF representation, one can clearly discriminate the time and frequency information among these three echoes (Figure 1-e). In summary, these examples demonstrate the three major advantages using the model based TF representation of ultrasonic signals. First, only the auto-terms of individual echoes appear in the TF plane, the cross-terms have disappeared. Second, high-resolution time frequency representation can be obtained. Third, by prior estimation of the signal before transformation, the effects of noise in the TF representation are minimized.

IV. ULTRASONIC FLAW DETECTION USING TF REPRESENTATION

In ultrasonic testing, the flaw echoes exhibit different frequency content than those of the microstructure echoes (see Figure 2-a for a measured signal). This measured signal acquired from a steel block that contains a flaw. The flaw echo exhibits lower frequency content than the grain echoes (i.e., scattering noise). The TF representation is used as a discriminative tool in detecting the flaw echoes in the presence of grain echoes. The estimated signal is shown in Figure 2-b. The WVD results (Figure 2-c) in cross-terms due to the grain echoes and obscures the detection of flaw echo (see Figure 2-c). The STFT of this measured signal shows poor resolution (see Figure 2d). The model based TF representation (i.e., displaying the individual auto-terms of the detected echoes) is able to discriminate the frequency content of the flaw echo from that of the grain echoes (compare Figure 2-e with Figure 2-c and Figure 2-d).

V. CONCLUSIONS

In this study, we have developed a model based TF representation for ultrasonic NDE applications. In particular, an iterative MAP estimator is developed to estimate the parameters of the signal consisting of many Gaussian echoes. The spectral characteristic of the transducer is incorporated as prior statistics in the parameter vector of the model. The performance of the model based TF method has been compared to classical TF methods (WVD and STFT) using simulated and experimental ultrasonic signals. It has been observed that the model based TF method is superior in terms of resolution, immunity to noise and undesirable cross-terms.

VI. REFERENCES

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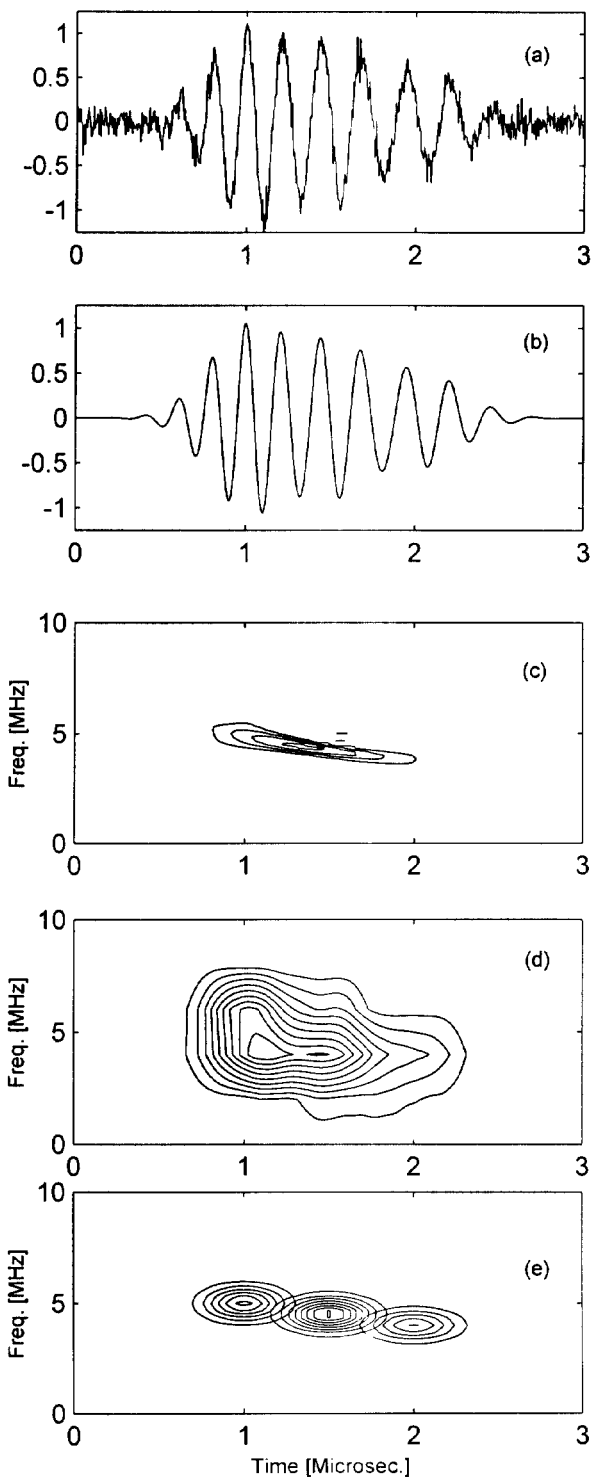

Figure 1. (a) Simulated signal (b) Estimated echoes (c) WVD (d) STFT (e) Model-based TF

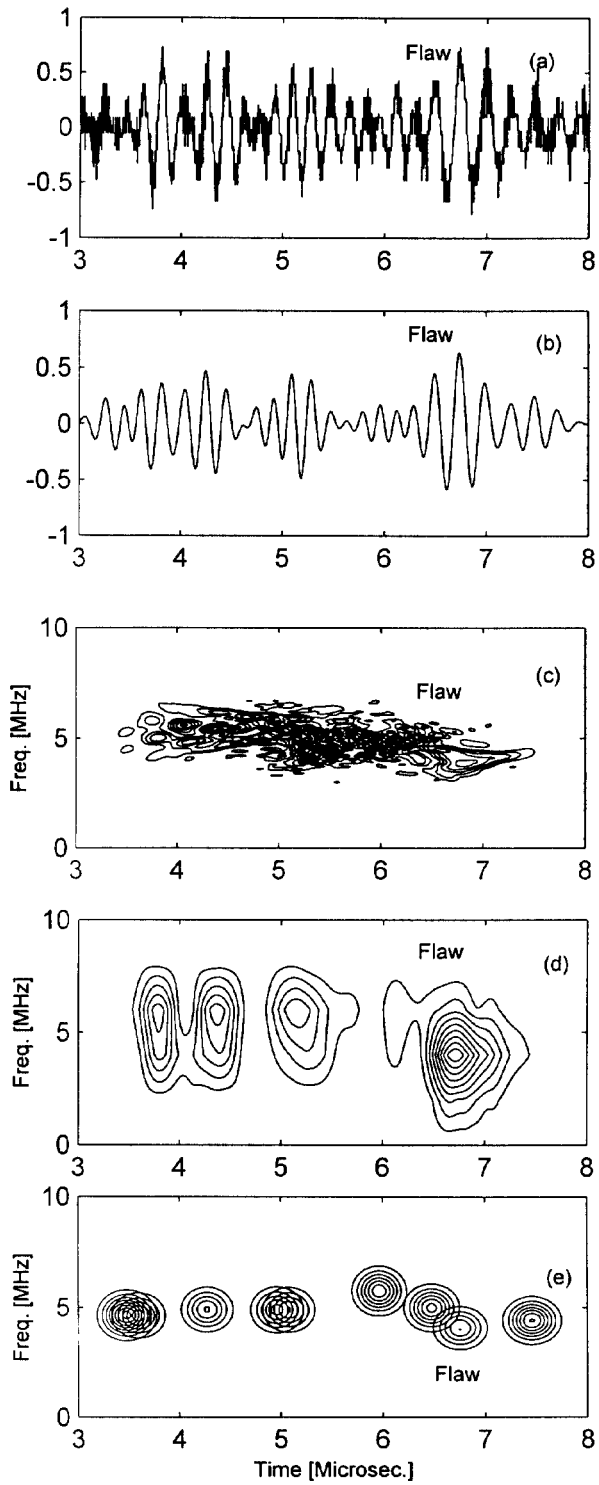

Figure 2. (a) Experimental data (b) Estimated echoes (c) WVD (d) STFT (e) Model-based TF