Optimal Wavelet Estimation for Data Compression and Noise Suppression of Ultrasonic NDE Signals

Guilherme Cardoso and Jafar Saniie
Department of Electrical and Computer Engineering
Illinois Institute of Technology
Chicago, Illinois 60616

Abstract - Ultrasonic NDE applications often require a significant amount of data collection, storage, and in some instances data transmission and analysis. The analysis, storage, and transmission of ultrasonic data can benefit from compression and noise suppression algorithms. In this paper, effort has been focused on the determination of similarities between the wavelet kernel and the ultrasonic echoes to maximize data compression ratios. The design of wavelet kernels via optimization methods improves data compression and denoising. The influence of thresholding into data compression and denoising are investigated. Simulated data, as well as ultrasonic experimental signals are used to verify the results of this study. High data compression ratios and echo detection in very low signal-to-noise ratio is achieved with this approach.

I. Introduction

The wavelet transform (WT) has many characteristics that make it a suitable tool for ultrasonic signal analysis. Among these features are good time and scale (frequency) localization, high signal similarity (one can tailor the wavelet structure to the signal being analyzed), constant relative bandwidth (this allows an estimation of frequency components with high resolution), and orthogonality (critical for matching a unique basis element to a portion of the signal). The filter bank structure of the WT correlates the ultrasonic signal with the wavelet kernel in different channels. Lossy data compression is achieved by applying thresholding techniques over the energy coefficients from each channel. The trade off between the data compression ratio and the accuracy to the original signal is analyzed. WT has also been shown to be an excellent technique for ultrasonic signal denoising once the wavelet kernel tracks the ultrasonic echoes and not the noise. Since this algorithm allows perfect reconstruction (PR), the number of energy coefficients used in the reconstruction process can adjust the noise level of the reconstructed signal. This adjustment is achieved with thresholding in the wavelet domain. A set of different wavelet kernels including Coiflets, Daubechies, and Symmlets [1] is used in the analysis and synthesis of ultrasonic signals. A selection of wavelet kernels from this set is examined for data compression of ultrasonic signals.

II. Discrete Wavelet Transform

The filter bank representation of the Discrete Wavelet Transform (DWT) [3] is shown in Figure 1. The low pass $h_0(n)$ and the high pass $h_1(n)$ filters are analysis filters, while $g_0(n)$ and $g_1(n)$ are low and high pass synthesis filters. The input signal $s(n)$ is down-sampled by two (↓2) before the Thresholding Block (TB), and up-sampled by two (↑2) before the synthesis filters. Data compression and denoising takes place inside the TB. The Inverse Wavelet Transform (IWT) reconstructs the signal from the thresholded wavelet coefficients.

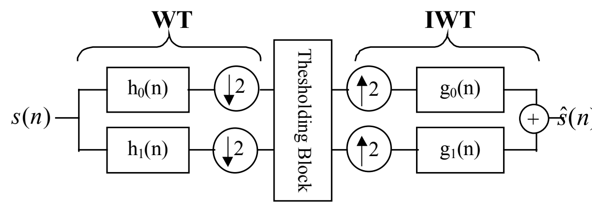

Figure 1 – Discrete Wavelet Transform

The DWT can be described in the Z domain as:

$$\hat{S}(z)=\frac{1}{2}\begin{bmatrix}G_0(z) & G_1(z)\end{bmatrix}\begin{bmatrix}H_0(z) & H_0(-z)\\ H_1(z) & H_1(-z)\end{bmatrix}\begin{bmatrix}S(z)\\ S(-z)\end{bmatrix} \quad \textbf{(1)}$$

$$\hat{S}(z)=\frac{S(z)}{2}\left[G_0(z).H_0(z)+G_1(z).H_1(z)\right]+\frac{S(-z)}{2}\left[G_0(z).H_0(-z)+G_1(z).H_1(-z)\right] \quad \textbf{(2)}$$

Where $G_0(z)$, $G_1(z)$, $H_0(z)$, $H_1(z)$, $S(z)$, and $\hat{S}(z)$ are the Z transforms of $g_0(n)$, $g_1(n)$, $h_0(n)$, $h_1(n)$, $s(n)$, and $\hat{s}(n)$ respectively. In order to have $\hat{S}(z)=c.S(z).z^{-n_0}$ (such that $\hat{S}(z)$ is a delayed and amplitude scaled version of $S(z)$) the following conditions for PR have to be satisfied [2]:

$$G_0(z).H_0(-z) - G_1(z).H_1(-z) = 0 - \begin{cases} G_0(z) = H_1(-z) \\ G_1(z) = -H_0(-z) \end{cases} \quad \textbf{(3)}$$

$$G_0(z).H_0(z) + G_1(z).H_1(z) = c.z^{-n_0} \rightarrow H_0(z) = H_1(-z)$$

Many solutions for $H_0(z)$ and $H_1(z)$ satisfy the above conditions. One way to solve for $H_0(z)$ and $H_1(z)$ is to use Quadrature Mirror Filters (QMF) [4]. PR in the QMF is obtained if the filters are orthogonal to the even-shifted version of themselves:

$$\sum h_i(n)h_i(n-2m) = \delta(m), \qquad i = 0,1 \qquad \textbf{(4)}$$

The above equation sets the condition for PR. Although this is an important requirement in the solution of a wavelet kernel, there are other issues that affect the performance of the kernel in representing data. In this study we investigate wavelet kernels that not only result in near PR, but also that improve signal data compression. The next section presents optimization techniques in estimating an optimal wavelet kernel, referred to as the "Echo" wavelet. The goal is to achieve a compromise between PR and data compression.

III. OPTIMAL DESIGN OF THE ECHO WAVELET

In this study a nonlinear optimization algorithm is used since both objective and constraint functions are nonlinear. In order to estimate the echo wavelet kernel, an objective function is defined as the squared difference between the original signal $s(n)$ and its compressed version $\hat{s}(n)$ (the reconstructed signal with a given compression ratio).

$$Er = \sum_{n=0}^{N-1} \left(\hat{s}(n) - s(n)\right)^2 \qquad \textbf{(5)}$$

The goal of the optimization is to generate a wavelet kernel as similar as possible to the ultrasonic backscattered signal. Figure 2 shows a diagram of the objective function.

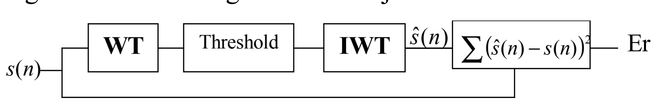

Figure 2 – Objective Function

The solution for the Echo wavelet kernel minimizes the objective function (5) subject to the constraint described in (4) [8]. Conventional wavelet kernels are used as the starting point for the optimization routine. The wavelets used are the four taps Daubechies (Daub4), six taps Coiflet (Coif6) and eight taps Symmlet (Symm8). The resultant wavelet kernels from these conventional wavelets are called Echo4, Echo6, Echo8, and are presented in Table 1. The Echo wavelets shown are the optimal wavelet kernels for the signal *s(n)* with a compression of 78%.

Table 1 – Conventional and Echo wavelet kernels

	Daub4	**Echo4**
$h_0(0)$	0.48296292	0.57276109
$h_0(1)$	0.83651631	0.78891378
$h_0(2)$	0.22414387	0.19175563
$h_0(3)$	-0.12940952	-0.13921694
	Coif6	**Echo6**
$h_0(0)$	0.03858078	0.05387916
$h_0(1)$	-0.12696913	-0.17110494
$h_0(2)$	-0.07716156	-0.04582076
$h_0(3)$	0.60749164	0.62389061
$h_0(4)$	0.74568756	0.72505672
$h_0(5)$	0.22658427	0.22831278
	Symm8	**Echo8**
$h_0(0)$	-0.10714890	-0.07576571
$h_0(1)$	-0.04191097	-0.02963553
$h_0(2)$	0.70373907	0.49761867
$h_0(3)$	1.13665824	0.80373875
$h_0(4)$	0.42123453	0.29785780
$h_0(5)$	-0.14031762	-0.09921954
$h_0(6)$	-0.01782470	-0.01260397
$h_0(7)$	0.04557034	0.03222310

IV. DATA COMPRESSION AND NOISE SUPPRESSION

Data compression of a given signal *s(n)* is successful when the redundant and noise components of *s(n)* are reduced or removed. The signal $\hat{s}(n)$ is the compressed representation of *s(n)*. The following section describes how thresholding for data compression and noise suppression can be applied to the wavelet coefficients of the original ultrasonic signal.

A. *Thresholding*

The thresholding rule is applied to an ultrasonic signal after the WT (Figure 1) to reduce the number of non-zero wavelet coefficients used in the IWT. For the case of a signal corrupted by White Gaussian Noise (WGN) with variance σ^2, references [5] and [6] have shown that the optimal threshold is given by τ:

$$\tau = \frac{c}{\sqrt{N}}\sqrt{2.\ln(N-1)} \qquad \textbf{(6)}$$

This "universal" threshold τ is applied to ultrasonic data corrupted by WGN. The implementation of the thresholding rule includes hard and soft thresholding (HT and ST, respectively). In the HT approach all wavelet coefficients smaller than τ are set to zero. All coefficients greater than τ are kept the same:

$$\hat{W}_s(n) = \begin{cases} 0, W_s(n) < \tau \\ W_s(n), W_s(n) \geq \tau \end{cases}, n = 0 \ldots N-1 \qquad \textbf{(7)}$$

In the above equation $W_s(n)$ represents the wavelet coefficients of the ultrasonic signal *s(n)*, while $\hat{W}_s(n)$ is the set of wavelet coefficients used in the IWT. Figure 3a shows the input/output mapping for the hard thresholding case.

Instead of introducing an abrupt change in the value of the wavelet coefficients, ST "smoothes" the transition from zero to non-zero coefficients. For more details refer to [6]. The decision rule in this case is described in the following equation and plotted in Figure 3b.

$$\hat{W}_s(n) = sign(W_s(n))(|W_s(n)| - \tau), n = 0 \ldots N-1 \qquad \textbf{(8)}$$

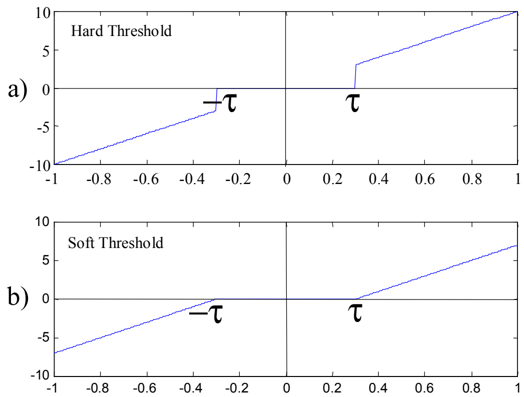

Figure 3 – Thresholding techniques. a) Hard threshold, b) Soft threshold.

V. Performance Evaluation of Wavelet Kernels

In order to evaluate the performance of different wavelet kernels to noise supression, WGN was added to experimental ultrasonic data. The reconstructed signal is given by $\hat{s}(n) = IWT\{Thresh[WT(s(n) + g(n))]\}$, where g(n) ~ N(0,1). The signal *s(n)* is shown in Figure 4.

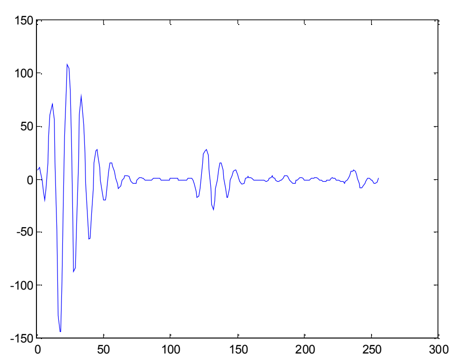

Figure 4 – Experimental A-Scan data from a thin metal sample

A Monte Carlo Simulation (MCS) with 1,000 realizations was performed over this experimental data. A Gaussian fit of the set of realizations provides a comparative measure of the wavelet kernels' performance. The fitting for the Daub4 and Echo4 is presented in Figure 5, where the vertical axis represents the number of events and the horizontal axis the reconstruction error. Table 2 summarizes the reconstruction error results for all wavelet kernels. These results show that the optimal wavelet kernels have a superior performance when compared to the conventional wavelet kernels.

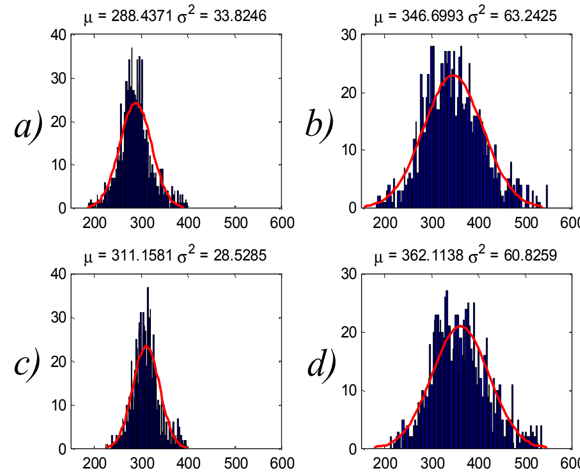

Figure 5 – Fitting results of MCS with Daub4 and Echo4. a) Echo4 with HT, b) Echo4 with ST, c) Daub4 with HT, and d) Daub4 with ST.

Table 2 – MCS summary of results

	HT		ST	
	μ	σ^2	μ	σ^2
Daub4	311.16	28.53	362.11	60.83
Echo4	288.44	33.82	346.70	63.24
Coif6	287.42	31.38	350.81	62.57
Echo6	287.76	31.89	351.63	63.23
Symm8	253.11	40.71	316.82	67.48
Echo8	252.34	42.37	317.39	68.23

Figure 6 shows that the optimal Echo wavelet achieves a higher compression ratio when compared to Daub4.

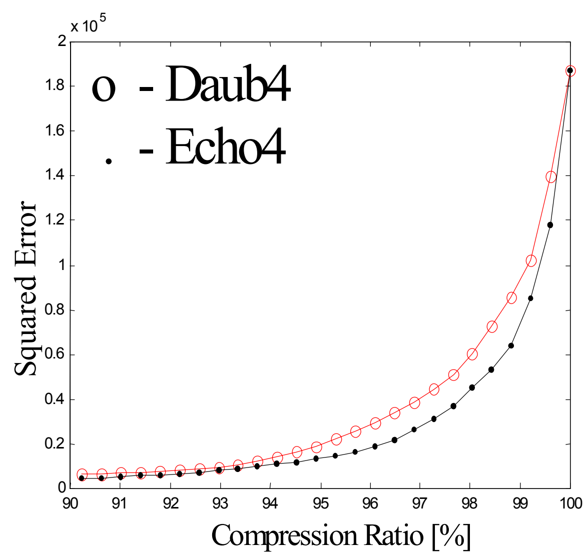

Figure 6 – Compression ratio with Daub4 and Echo4

Similar results are shown for Coif6, Echo6, Symm8, and Echo8 in Figure 7. This figure also confirms that the reconstruction error is smaller using the optimal wavelet kernels than when using other conventional wavelet kernels.

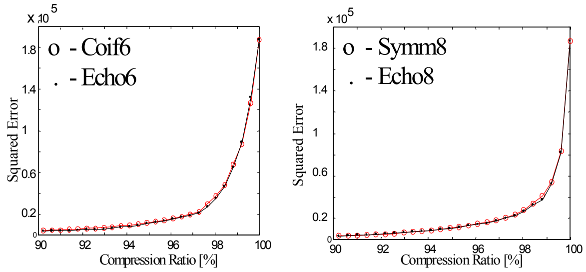

Figure 7 – Compression ratio with Coif6, Echo6 (left) Symm8, and Echo8 (right).

B-Scan images of ultrasonic signals are widely used in NDE applications. Figure 8 shows the B-Scan image from a coin with a rough surface. This image was compressed using Daub4 and Echo4, and the results are shown in Figure 8b and Figure 8c. Table 3 presents a quantitative comparison of the wavelet kernels applied to the B-Scan image. This result demonstrates the superiority of the optimal Echo wavelet kernel when compared to conventional wavelet kernels.

Table 3 – B-Scan compression ratio comparison

	Compression Ratio [%]	Reconstruction Error
Daub4	87	1
Echo4	87	0.84
Coif6	75	1
Echo6	75	0.89
Symm8	95	1
Echo8	95	0.91

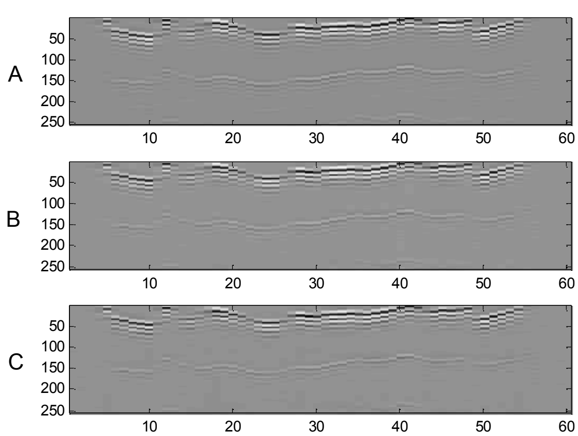

Figure 8 – a) Original B-Scan image. 87% compressed image with b) Echo4, c) Daub4.

VI. CONCLUSIONS

This paper presents a method to design an optimal Echo wavelet kernel. Compression and noise suppression of ultrasonic signals is achieved using a wavelet kernel with a higher similarity to the original signal than the conventional wavelet. The optimal Echo wavelet kernel is obtained through a nonlinear constrained optimization method that allows near perfect reconstruction. The optimization is achieved by minimizing the reconstruction error constrained to the even shift orthogonality of the optimal Echo wavelet kernel. Experiments in simulated and experimental data indicate that the optimal Echo wavelet kernels outperform conventional wavelet kernels like Daubechies, Symmlet, and Coiflet.

VII. REFERENCES

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