Compression of Ultrasonic Data Using Transform Thresholding and Parameter Estimation Techniques

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Abstract - In this paper, transform thresholding and parameter estimation techniques are studied to compress ultrasonic signals. Transform thresholding techniques are applied to select the frequency coefficients of the discrete wavelet transform (DWT), the discrete cosine transform (DCT), and the discrete Fourier transform (DFT). Furthermore, a novel algorithm using the continuous wavelet transform (CWT) is also presented as a parameter estimation approach to compress ultrasonic data. The data compression performance of these algorithms is examined using both simulated and experimental ultrasonic signals. The results show that the parameter estimation has superior compression performance when compared to the conventional transform thresholding techniques.

I. Introduction

The analysis, storage, and transmission of ultrasonic data can benefit from compression and noise suppression algorithms. The data compression performance of the DWT, the DFT, and the DCT are examined when applied to simulated and experimental ultrasonic data. An adaptive threshold is applied to remove the smaller coefficients of the frequency domains to achieve data compression (i.e., lossy data compression). Then, the signals are reconstructed from the remaining coefficients. The mean square errors of these techniques are analyzed for compression ratios of up to 90%. For higher compression ratios, the CWT is also investigated in this study. The CWT performs the correlation of a mother wavelet with the ultrasonic signal. A modified version of the Morlet wavelet is used as the mother wavelet to estimate the echo parameters (bandwidth, phase, arrival time, and center frequency). Upon the determination of these parameters, high data compression and denoising can be obtained. The CWT is applied to both simulated and experimental data, and the results are presented in this paper.

II. Thresholding Techniques

Subband transform (DWT) and transform coding techniques (DCT and DFT) provide a representation of the input signal into separate frequency bands. Data compression is therefore achieved by selecting portions of the frequency domain where the signal is expected. Energy in the frequency domain outside the bandwidth of the signal is usually due to noise. This method is most effective for bandwidth limited signals with uncorrelated noise.

A. *DWT, DFT, and DCT*

The filter bank representation of the DWT [1] is shown in Figure 1. The lowpass $h_0(n)$ and the highpass $h_1(n)$ filters are analysis filters, while $g_0(n)$ and $g_1(n)$ are lowpass and highpass synthesis filters.

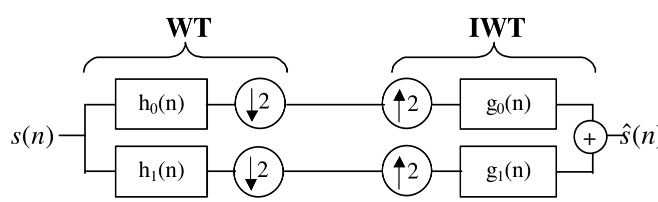

Figure 1 – Discrete Wavelet Transform

The analysis filters $h_0(n)$ and $h_1(n)$ can be selected as quadrature mirror filters (QMF) [2]. In such configuration $g_0(n)$ and $g_1(n)$ are the time reversed versions of $h_1(n)$ and $h_0(n)$ respectively. In order to recover the input signal from the DWT coefficients, $h_0(n)$ and $h_1(n)$ have to satisfy the following condition:

$$\sum h_i(n)h_i(n-2m) = \delta(m), \qquad i = 0,1 \qquad (1)$$

The DFT of a signal $f(n)$ (or f_N in a vector representation) of length N is defined in Equation (2).

The frequency domain signal F_N is a linear combination of the time domain signal f_N with the kernel W_N.

$$F_N = W_N . f_N \quad (2)$$

In the above equation W_N is an N×N symmetric matrix defined as:

$$W(k,n) = \exp\left(\frac{-j2\pi kn}{N}\right),$$
$$k = 0,1,...,N-1, n = 0,1,...,N-1$$

The inverse DFT (IDFT) is shown in Equation (3).

$$f_N = \frac{1}{N} W_N^{-1} . F_N \quad (3)$$

The DCT and its inverse (IDCT) are defined in Equations (4) and (5) respectively.

$$F_N = C_N . f_N \quad (4)$$

$$f_N = C_N^{-1} . F_N \quad (5)$$

$$C(k,n) = \sqrt{\frac{2}{N}} \cos\left(\frac{\pi}{N}(k+0.5)(n+0.5)\right)$$
$$k = 0,1,...,N-1, n = 0,1,...,N-1$$

Data compression of a given signal *f(n)* is successful when the redundant and noise components of *f(n)* are reduced or removed in the frequency domain (Figure 2). The signal $\hat{f}(n)$ is the compressed representation of *f(n)*. The following section describes how thresholding for data compression and noise suppression can be applied to the frequency coefficients of the original ultrasonic signal.

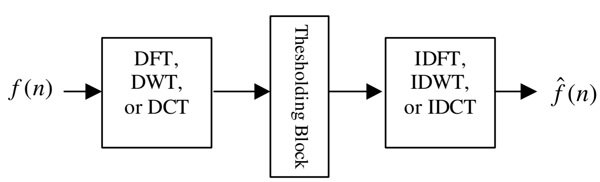

Figure 2: Thresholding of frequency coefficients

B. *Adaptive Threshold*

For the case of a signal corrupted by additive white Gaussian noise (AWGN) with variance σ^2, reference [3] has shown that the optimal threshold is given by:

$$\tau = \frac{\sigma}{\sqrt{N}} \sqrt{2.\ln(N-1)} \quad (6)$$

This "universal" threshold τ is applied to ultrasonic data corrupted by WGN. In the threshold approach all transform coefficients smaller than τ are set to zero. All coefficients greater than τ are kept the same:

$$\hat{W}_f(n) = \begin{cases} 0, & |W_f(n)| < \tau \\ W_f(n), & |W_f(n)| \geq \tau \end{cases}, n = 0...N-1 \quad (7)$$

This approach is referred to as a hard thresholding technique [4]. In the above equation $W_f(n)$ represents the frequency coefficients of the ultrasonic signal *f(n)*, while $\hat{W}_f(n)$ is the set of frequency coefficients used in the IDWT, IDCT, and IDFT. Figure 3 shows the input/output mapping for the hard thresholding case.

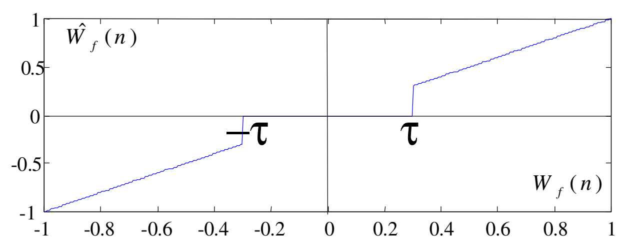

Figure 3: Hard thresholding rule.

III. Parameter Estimation Technique

The parameter estimation method is based on the CWT to estimate the parameters of echoes in an ultrasonic signal. The CWT performs a correlation of the mother wavelet with the input signal. The CWT is defined in Equation (8).

$$WT(a,b) = \frac{1}{\sqrt{a}} \int f(t).\psi^*\left(\frac{t-b}{a}\right) dt \quad (8)$$

In general the ultrasonic signal can be modeled by the following function:

$$f(t) = \beta.\exp\left(-\alpha(t-\tau)^2\right).\cos\left(w_c.(t-\tau)+\phi\right) \quad (9)$$

Where β is the amplitude, α is the bandwidth factor, τ is the arrival time, w_c is the frequency, and ϕ is the phase of the ultrasonic signal.

In this study a modified wavelet kernel is introduced to closely match the shape of the ultrasonic echoes.

$$\psi(t) = \exp\left(\gamma t^2\right)\cos\left[t+\theta\right] \quad (10)$$

This kernel includes two additional parameters: θ (phase) and γ (bandwidth). These parameters are optimized to maximize the correlation between the ultrasonic echo and the wavelet kernel. This algorithm allows better correlation, and consequently improves parameter estimation.

The first step of the algorithm is to estimate the time of arrival and center frequency of the ultrasonic echoes using the CWT. Based on this estimation, a windowing scheme isolates the dominant echo.

The next step is the estimation of the echo bandwidth and phase. These two parameters are obtained by correlating the mother wavelet with the ultrasonic echo and representing the results in the phase and bandwidth domain. The peaks of the correlation reveal the optimal values of the phase and bandwidth. In a situation where the ultrasonic signal contains multiple interfering echoes (see Equation (11)) this approach becomes sub optimal.

$$f_N(t) = \sum_{j=1}^{N} \beta_j \exp\left(-\alpha_j (t-\tau_j)^2\right).\cos\left(w_{c_j}.(t-\tau_j)+\phi_j\right) \quad (11)$$

Where β_j, α_j, τ_j, w_{cj}, and ϕ_j are the parameters of the j[th] echo in $f_N(t)$. To search for an optimal result this method is iterated until the reconstruction error (i.e., mean square error between the estimated signal and original signal) is below an acceptable value (E_{min}), as shown in Figure 4.

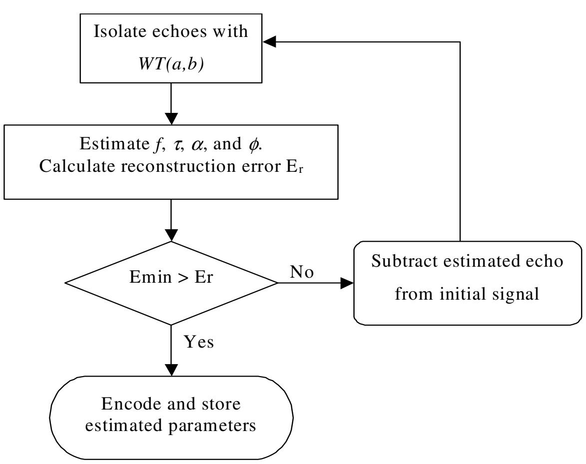

Figure 4: Parameter estimation functional diagram

If the error is not acceptable, the estimated signal is subtracted from the original signal, and whole estimation process is repeated until the error is within the acceptance level.

IV. RESULTS

The performance of the thresholding technique applied to the DWT, DFT, and DCT coefficients is shown in Figure 5. This figure shows the compression ratio performance in ultrasonic experimental data. Since the DWT kernel resembles the ultrasonic echoes, it offers better data compression compared to the DCT and DFT.

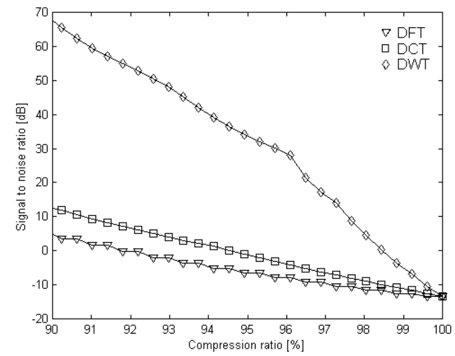

Figure 5 – Compression ratio performance with experimental data

The parameter estimation technique is applied to the signal in Figure 6a,d). The noisy version of the same signal is shown in Figure 6b,e), while the reconstructed data is shown in Figure 6c,f).

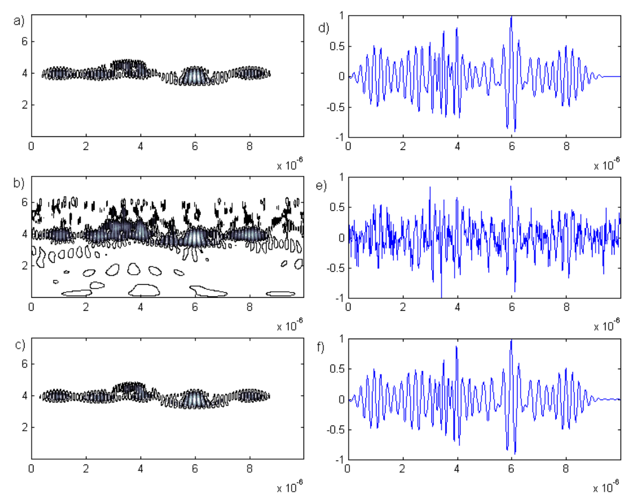

Figure 6 – CWT of: a) original signal, b) noisy signal, c) reconstructed signal. Time domain of: d) original signal, e) noisy signal, f) reconstructed signal.

The signal-to-noise ratio (SNR) of the signal is 6dB. In this example the parameter estimation technique is able to correctly isolate the 7 echoes of the ultrasonic signal. A total of 35 coefficients are necessary to reconstruct the original signal with a SNR of 36dB.

V. CONCLUSIONS

This paper presents two different methods to denoise and compress ultrasonic data. Thresholding techniques were analyzed using the DCT, DFT and DWT as a means to obtain frequency domain coefficients. The selection of the higher energy coefficient using an adaptive thresholding scheme reduces the amount of data to be stored and/or transmitted. In the experimental and simulated ultrasonic signals evaluated, the DWT outperformed the DFT and DCT. The high compression ratio obtained shows that these transform techniques are appropriate for the analysis and compression of ultrasonic signals.

The parameter estimation technique achieves even higher compression ratios. If an ultrasonic signal contains multiple echoes, individual echoes are estimated separately. The CWT representation of the signal is used to design a window that isolates each single echo. As the number of echoes increases the interference between them is higher, and consequently the performance of the estimation algorithm deteriorates. The same argument is applicable for noise interference. As SNR decreases, the algorithm will iterate to estimate the noise rather than the echoes.

Thresholding techniques provide a high compression ratio, and are computationally simple to implement (e.g., various commercial chips are available to compute the DWT, DCT, and DFT). The parameter estimation demands a higher computational complexity, but with a higher compression ratio. The two techniques analyzed provide an elegant trade-off between implementation complexity and compression ratio.

VI. REFERENCES

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