A HIGH FIDELITY TIME-FREQUENCY REPRESENTATION FOR ULTRASONIC SIGNAL ANALYSIS

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Abstract - The Time-Frequency (TF) characteristics of ultrasonic echoes provide valuable information leading to the characterization of materials and localization of defects. The conventional TF analysis methods such as Wigner-Ville Distribution (WVD) and Short-Time Fourier Transform (STFT) perform inadequately when applied to ultrasonic signals because they introduce cross terms, offer poor resolution and are sensitive to noise level. In this study, we present a TF representation for ultrasonic echoes based on a Matching Pursuit (MP) method. MP decomposes a signal into a linear expansion of Gabor functions and maintains energy conservation. MP offers a TF distribution of the signal by enabling the addition of the TF distribution of each composing Gabor function. This TF representation is free of cross terms, resilient to noise, and adaptive to signal characteristics. The performance of this TF method has been tested using experimental ultrasonic data, and then compared to conventional techniques such as WVD and STFT. In particular, we present TF results for ultrasonic flaw detection where the microstructure scattering echoes dominate the flaw echoes (SNR is about 0 dB).

I. INTRODUCTION

The conventional TF analysis methods such as WVD and STFT perform inadequately when applied to ultrasonic signals because they introduce cross terms, offer poor resolution and are sensitive to noise level. In this study, we present a TF representation for ultrasonic echoes based on a Matching Pursuit (MP) decomposition method. MP iteratively decomposes a signal into a linear expansion of Gabor functions while it maintains energy conservation, i.e., the sum of the individual energies of composing functions is approximately equal to the original signal energy. Because of this property, MP offers a TF distribution of the signal by enabling the addition of the TF distributions of composing Gabor functions. A Gabor function offers a desirable analytic TF distribution through Wigner distribution and represents a concentrated blob in TF plane. TF distribution of ultrasonic data is then obtained by adding the TF distributions of decomposing Gabor functions. This TF representation, by definition, is free of cross terms and adaptive to signal characteristics. We will present the MP algorithm in Section 2, and TF distribution via MP decomposition (MPTF) in Section 3. The application of MPTF distribution to ultrasonic signals will be presented in Section 4.

II. MATCHING PURSUIT DECOMPOSITION

In the original implementation of MP, a function is chosen from the dictionary of normalized Gabor functions to best match the signal residues [1]. The best match criterion is based on the projection coefficient obtained by projecting the signal residue onto a dictionary function. The signal residue is the remaining signal after the best matching function is subtracted. MP first matches a function to the original data. Then, this best matching function is subtracted from the signal to get the signal residue. At each iteration, a Gabor function is matched to the current signal residue. When the energy of signal residue is a fraction of the energy of the original signal the decomposition is said to be complete. The final decomposition is a linear expansion of chosen dictionary functions. MP decomposition maintains energy conservation, i.e., the summation of the individual energies of composing functions is approximately equal to the original signal energy. Because of this property, MP enables superimposing the TF distribution of each Gabor function hence achieving a TF representation free of cross terms. In our implementation of MP, we optimize the parameters of a Gabor function to best match the signal residues. This saves the usage of a large number of dictionary functions and offers a more efficient implementation. Furthermore, for best matching criterion we used the mean-squared error instead of the correlation coefficient used in the original implementation. This criterion provides a better decomposition for a signal corrupted with noise, hence is more appropriate for ultrasonic applications. The following definitions are in order before the full description of the algorithm:

y: *measured signal (discrete) of length L*

g: *normalized Gabor function given by*

$$g(\theta;t) = \beta e^{-\alpha(t-\tau)^2} \cos\{2\pi f_c (t-\tau) + \phi\} \quad (1)$$

where the parameters of the function, bandwidth factor, arrival time, center frequency, phase and normalizing-amplitude are stored in a parameter vector in the order as $\theta = [\alpha \quad \tau \quad f_c \quad \phi \quad \beta]$. The time variable *t* contains discrete samples obtained by sampling the time support of a signal with a sampling frequency of f_s.

$R^n y$: n-th residue of the signal obtained by subtracting the best matching Gabor function

c_n : Projection coefficient obtained by projecting the n-th signal residue onto Gabor function, i.e., $c_n = < R^n y, g(\theta_n;t) > = (R^n y)^T . g(\theta_n;t)$

E_y : energy of signal *y*, i.e., $E_y = y^T y$

MP Algorithm

1. Set iteration index $n = 0$ and first signal residue $R^0 y = y$
2. Find the best parameter vector of Gabor function such that

   $\theta_n = \arg_\theta \min \left\| R^n y - g(\theta;t) \right\|^2$

   Set $g_n = g(\theta_n;t)$ and compute $c_n = < R^n y, g_n >$
3. Compute the next residue $R^{n+1} y = R^n y - c_n g_n$
4. Check convergence: if $\frac{E_{R^{n+1}y}}{E_y} \le Threshold$, STOP.

   Otherwise, set $n \to n+1$ and go to Step 2.

Step 2 of the algorithm matches a Gabor function to signal residue by optimizing the parameters of the Gabor function. This optimization problem is critical in achieving the best decomposition and requires special care. In our earlier work, we proposed a fast Gauss-Newton algorithm coupled with an initial guess strategy to solve this problem [2]. Step 3 computes the next signal residue by subtracting the best match Gabor function. Step 4 checks convergence: if the residue energy is some fraction of the signal energy, the algorithm stops, otherwise a new Gabor function is matched to current signal residue.

III. Time Frequency Distribution Via MP Decomposition

The MP algorithm, after M iterations, decomposes a signal into M-Gabor functions and remaining residue:

$$y = \sum_{n=0}^{M-1} c_n g_n + R^M y \quad (2)$$

When the residue energy is a small fraction of original signal energy, the decomposition is said to be complete. It is shown that as M increases, the sum energy of composing Gabor functions approaches to the signal energy [1]. MP decomposition maintains energy conservation, i.e., at any step of the algorithm, the signal energy is equal to the sum of the energy of the composing functions and the energy of signal residue. When the decomposition is complete, the sum of the individual energies of composing functions is approximately equal to the original signal energy. Because of this property, MP offers a TF distribution of the signal by enabling the addition of the TF distribution of each composing Gabor function. The TF of a Gabor function can be obtained by taking the Wigner-Ville transformation of Equation 1 [3]:

$$TF_{g(\theta)}(t,f) = \frac{\beta^2}{2\pi\alpha} e^{-2\alpha(t-\tau)^2} e^{-(2\pi)^2 (f-f_c)^2 / 2\alpha} \quad (3)$$

This TF distribution has a desirable analytic solution: it only depends on the parameters of the Gabor function. This distribution represents a concentrated 2-D Gaussian shape centered at (τ, f_c) in the TF plane and its concentration is determined by the bandwidth factor α. The TF distribution of signal *y* is a superposition of the TF of each composing Gabor function weighted by the square of its projection coefficient (energy):

$$MPTF \quad \{y\} = \sum_{n=0}^{M-1} c_n{}^2 \; TF \; \{g(\theta_n)\} \qquad (4)$$

By definition, MPTF distribution is free of cross terms. It provides a good energy concentration. The time and frequency resolutions are determined by the sampling frequency and the number of samples. Its time and frequency resolutions are as good as those of WVD, and its frequency resolution is better than the STFT. Furthermore, this TF representation is immune to noise because of the de-noising process in the MP algorithm. The next section discusses the application of MPTF method to ultrasonic echoes and compares its performance to WVD and STFT.

IV. TF DISTRIBUTION OF ULTRASONIC ECHOES VIA MP DECOMPOSITION

The TF distribution of ultrasonic echoes suffers from three major problems: noise in the measurement, overlapping echo patterns, and dramatic changes in signal characteristics. The MP algorithm can decompose a signal into Gabor functions while de-noising the signal. Through the optimization process, the parameters of the Gabor functions are adaptively changed to match the signal characteristics. The original MP algorithm does not care about the bandwidths of the decomposed Gabor functions as long as they match signal residues. However, the bandwidths of measured echoes should be within the vicinity of the transducer (impulse response) bandwidth. For a more meaningful decomposition, one can put a constraint on the bandwidths of the reconstructed functions. This can be done in the optimization process by placing a constraint on the bandwidth parameter of the Gabor function. Another way of doing this is to use the maximum a posteriori (MAP) estimation principle. One can assume prior statistics on the desired parameters and incorporate this in the estimation. The work given in the reference [4] provides a detailed analysis and a MAP-Gauss Newton (MAPGN) algorithm to optimize the parameters of Gabor function with respect to given data. Thus, the Gauss Newton algorithm proposed to solve Step 2 of the MP algorithm can be replaced with the MAPGN algorithm. This algorithm ensures that the decomposed Gabor functions have bandwidths close to transducer bandwidth and offers a more meaningful decomposition for ultrasonic signals.

To demonstrate the MP algorithm applied to ultrasonic echoes, we use ultrasonic data acquired from a steel block that contains a flaw (see Figure 1-a). We apply the MP algorithm above with MAPGN algorithm utilized in Step 2. We impose a prior knowledge for the bandwidth factor parameter. The expected value of this parameter is computed by fitting a Gabor function to the impulse response of the transducer. The bandwidth parameters of the decomposed functions are expected to change around this value. Using this constrained bandwidth estimation strategy, the MP decomposition of the signal is estimated as displayed in Figure 1-b. This signal is composed of 12 Gabor functions whose parameters are listed in Table 1 in a timely order.

Func. Num.	BW Factor	Arrival Time	Center Freq.	Project. Coeff.	Energy
1	14.88	3.26	4.31	0.95	0.90
2	15.30	3.79	5.09	2.83	8.01
3	15.20	4.36	5.42	3.07	9.42
4	14.96	4.93	6.62	0.55	0.30
5	14.93	5.10	4.07	0.51	0.26
6	14.37	5.20	5.44	3.11	9.67
7	15.18	5.68	6.02	1.20	1.44
8	15.15	6.19	5.52	1.63	2.66
9	14.14	6.76	3.93	4.23	17.89
10	14.92	6.87	5.85	0.56	0.31
11	14.70	7.31	4.35	1.59	2.53
12	15.07	7.84	4.60	0.78	0.61

Table 1: The parameters of Gabor functions for MP decomposition of experimental data

This table provides valuable information about the TF energy distribution of ultrasonic data. The energy of signal at the time and frequency given by arrival time and center frequency parameters is the square of projection coefficient (the energy column in the table). The energy concentration is controlled by the bandwidth factor parameter which changes slightly. This is due to the prior knowledge incorporated in the estimation algorithm. The decomposed signal is a linear addition of these 12 Gabor functions weighted by their projection coefficients (Figure 1-b). The remaining signal residue (error) after the decomposition is shown in Figure 1-c. The remains of decomposition contain noise but no significant echo components. The magnitude spectrums of the experimental and decomposed signals are shown in Figure 1-d in dotted and solid lines respectively. There is a good agreement between the signal and decomposition both in time and frequency domains.

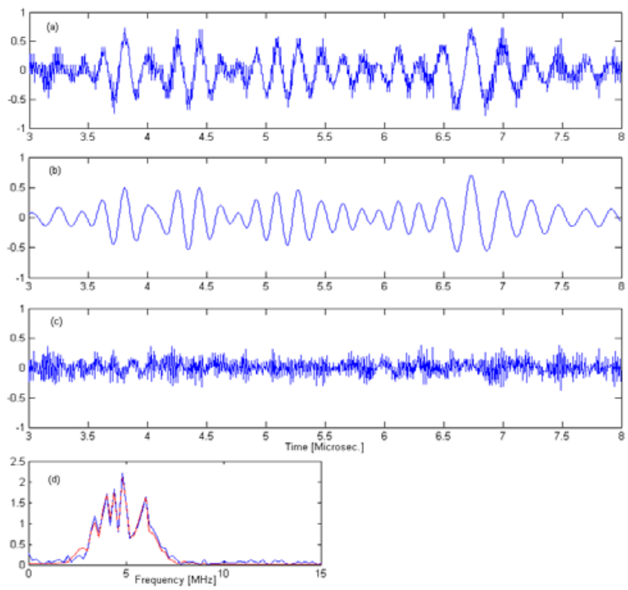

Figure 1 – MP Decomposition (b) of Ultrasonic Experimental Echoes (a). Decomposition error (c), magnitude spectrums of the experimental data and decomposed signal (d).

Although Table 1 presents information about the TF distribution of the signal, one can map the Gabor functions into the TF plane and achieve a MPTF distribution as described by Equation 4. The contour plot of this TF distribution is shown in Figure 2-d. For comparison, we plot the TF distributions obtained by WVD (Figure 2-b) and STFT (Figure 2-c). The WVD is cluttered with cross terms and noise. STFT captures the general trend in the signal, but smears fine details. MPTF displays strong energy components with much better resolution. Only the most significant 6 of 12 Gabor functions listed in Table 1 are visible in the MPTF. These TF concentrations are also visible in STFT with less resolution. This TF representation can be used for ultrasonic flaw detection. The high energy density circles with lower frequency content in the MPTF plot represents the flaw echo in the experimental data. This flaw echo is also visible in STFT with somehow more ambiguity. One can determine the exact location, frequency and energy concentration of this echo by examining the parameters of the associated Gabor function. For computational complexity of MPTF, it takes 12 MP iterations for this decomposition, and each MP takes about 20 Gauss-Newton iterations. Once the parameters of Gabor functions are estimated, they are used to compute the TF distribution given by Equation 3.

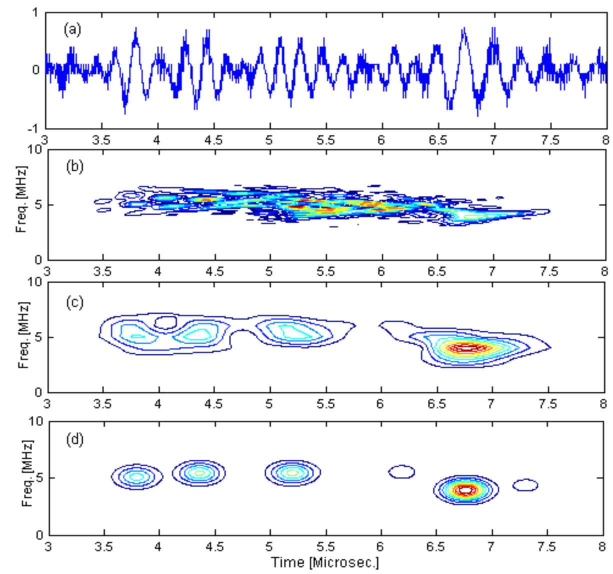

Figure 2 – TF Distributions of Ultrasonic Experimental Data (a) Using WVD (b), STFT (c), and MPTF (d).

V. CONCLUSIONS

We developed a matching pursuit algorithm that decomposes an ultrasonic signal into Gabor functions. The TF distribution of the ultrasonic signal is then obtained by adding the TF distributions of Gabor functions. The performance of this MPTF has been compared to classical TF methods (WVD and STFT) using experimental echoes. It has been observed that the MPTF method is superior in terms of resolution, immunity to noise, and suppressing the cross-terms.

VI. REFERENCES

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