Adaptive Thresholding Technique for Denoising Ultrasonic Signals

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***Abstract*— In many ultrasonic imaging applications, the signal acquired is embedded in noise, and situations with very small signal-to-noise ratio (SNR) are not uncommon. Thus, before any data analysis can be applied to the signal some level of noise removal is necessary. In this paper, we analyze the denoising performance of the discrete wavelet transform (DWT), discrete cosine transform (DCT), and Walsh-Hadamard transform (WHT) using an adaptive thresholding function (ATF) that, when applied to DWT, DCT and WHT coefficients, improves the signal-to-noise (SNR) of ultrasonic signals embedded in noise. In particular, the ATF technique is successful in denoising low SNR ultrasonic signals. Furthermore, the ATF approach outperforms the classical techniques when the ultrasonic signal has a low SNR (below 5dB). For signals with uniform noise added to DWT coefficients, the ATF technique achieves SNR improvements around 9dB over the classical thresholding techniques; these improvements are above 10dB for Gaussian noise.**

I. INTRODUCTION

Ultrasonic imaging signals are composed of ultrasonic echoes that are produced by the reflection of a transmitted pulse in scatterers inside the material under test. Often, the material under test has multiple structural scatterers. Hence, the ultrasonic signal is composed by echoes not only from the flaw in the material (in the case there is flaw in the material), but also by randomly distributed scatters. In addition to these multiple scatters, the received signal is also corrupted by thermal noise in the electronic components in the data acquisition system. Therefore, the goal of the denoising techniques presented in this paper is improving the contribution of the echoes from flaws while worsening the noise contribution from multiple scatters and electronic systems. The classical thresholding techniques analyzed (hard, soft, and Garrote thresholding) use a fixed threshold to separate noise from signal. On contrary, the adaptive thresholding function (ATF) technique removes the noise from the signal by generating a thresholding function that is obtained from the statistical noise parameters. Thus, each transform coefficient is compared to its own "threshold".

In this study, the denoising properties of the DCT, the WHT, and the DWT for ultrasonic imaging are examined when processed by the ATF technique. In particular, the relation between classical and ATF thresholding techniques and the SNR level is evaluated.

II. CLASSICAL THRESHOLDING TECHNIQUES

In this section, we present three classical thresholding techniques: hard thresholding (HT), soft thresholding (ST) [1, 2], and Garrote thresholding (GT) [3, 4]. In the HT approach all transform coefficients smaller than a given threshold, T, are set to zero. All coefficients greater than T are kept at the same original value. In the ST approach, on the contrary, instead of introducing an abrupt change in the values of the transform coefficients, ST "smoothes" the transition from zero to non-zero coefficients. GT introduces a relation to the thresholding coefficients that emphasizes the larger transform coefficients compared to the smaller transform coefficients. Hence, GT offers a compromise between HT and ST. The HT, ST, and GT can be described by

$$\zeta^{Hard}(x) = \begin{cases} 0, & |x| \le T \\ x, & |x| > T \end{cases} \tag{1}$$

$$\zeta^{Soft}(x) = \begin{cases} 0, & |x| \le T \\ x - T, & x > T \\ x + T, & x < -T \end{cases} \tag{2}$$

$$\zeta^{Garrote}(x) = \begin{cases} 0, & |x| \le T \\ x - \dfrac{T^2}{x}, & |x| > T \end{cases} \tag{3}$$

where ζ represents the transform coefficients after thresholding, T is the threshold value, and x represents the original transform coefficients. Figure 1 shows the transform coefficients before and after thresholding.

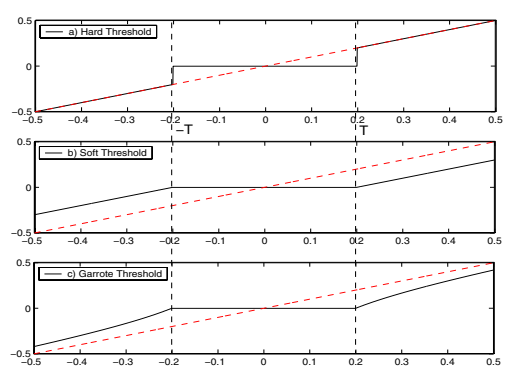

Figure 1. Thresholding techniques, before (dashed lines) and after thresholding (solid lines). a) Hard, b) soft, and c) Garrote threshold

In the case where the signal is corrupted by additive white Gaussian noise (AWGN) the observed signal, x, can be modeled as

$$x = s + n \tag{4}$$

where s is the original signal and n is the AWGN. To derive the best threshold T to denoise the signal x, the approach used by Donoho and Johnstone [1, 2] is based on the minimization of the mean-square-error (*MSE*) between the original, s, and the reconstructed signal, $\hat{s}$,

$$MSE(\hat{s}, s) = \frac{1}{N}\sum_{i=1}^{N} E(\hat{s}_i - s_i)^2 \tag{5}$$

The minimization of MSE in the previous equation leads to the "universal" threshold, T, given by

$$T = \frac{c}{\sqrt{N}}\sqrt{2\ln(N-1)} \tag{6}$$

where N is the total number of transform coefficients and σ is the standard deviation of the AWGN. The statistical parameters of the AWGN can be estimated from the original signal using the unbiased estimation [Kay93a]

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu)^2} \tag{7}$$

where the mean, μ, is estimated by

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{8}$$

III. Adaptive Thresholding Function

In this section, we present the procedure for estimating the adaptive thresholding function (ATF) in order to improve SNR of ultrasonic signals embedded in noise. Different from the classical thresholding techniques shown in the previous section, the ATF technique relies on a thresholding function, *T(x)*, instead of a thresholding coefficient, *T*. The shape of this thresholding function is determined by the source of random noise in the signal. More specifically, *T(x)* has the shape of the probability distribution function of the noise source. Thus, instead of modifying each transform coefficient based on a single threshold coefficient, in the ATF technique a thresholding function is subtracted from the sorted transform coefficients, or

$$\zeta_s^{ATF}(x_s) = x_s - T(x_s) \tag{9}$$

where $\zeta_s^{ATF}(x_s)$ represents the transform coefficients after ATF thresholding, $T(x_s)$ is the ATF, and x_s is the sequence of sorted coefficients of x. This result is then resorted and inverse transformed. The following sections show that the ATF technique leads to a reconstructed signal in the time domain with higher SNR than using the classical thresholding techniques. The performance of the ATF and classical thresholding techniques in denoising ultrasonic signals using different transforms (DWT, DCT, and WHT) is also the subject of the following section.

IV. Description of the ATF Technique

The first step in the implantation of the ATF technique is to apply the appropriate transform to the original signal, x. The next step in the algorithm is to sort the transform coefficients to obtain x_s, as shown in the flowchart in Figure 2.

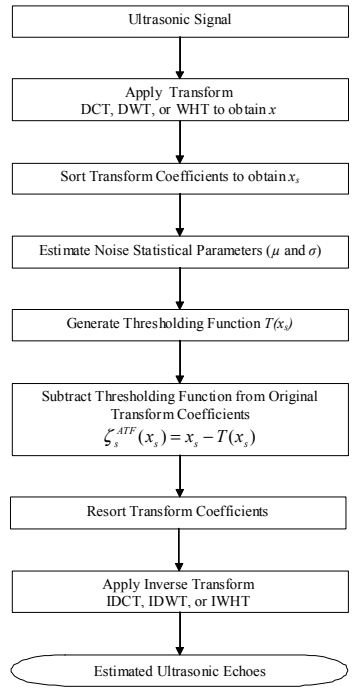

Figure 2. Flowchart of ATF algorithm

The sorting algorithm organizes the transform coefficients from smallest to greatest in amplitude while keeping the position (or order) information of each of these coefficients. In the ATF technique the transform coefficients are sorted such that the observed signal, x_s, is given by

$$x_s = s_s + n_s \tag{10}$$

where s_s and n_s are the sorted sequence of signal and noise transform coefficients, respectively. The shape of s_s is unknown, as the number, position, and distribution of the ultrasonic echoes in s is by definition unknown. On the contrary, if the type of the underlying random process that generates n_s is known, as we assume it is, the shape of n_s is the probability distribution function of n. Hence, the ATF is generated from the noise parameters so that n_s and $T(x_s)$ are similar to each other, and as a consequence

$$\begin{aligned} \zeta_s^{ATF}(x_s) &= s_s + n_s - T(x_s) \\ \zeta_s^{ATF}(x_s) &\cong s_s \end{aligned} \tag{11}$$

or the thresholded transform coefficients are approximately equal to the transform coefficients of the original signal s_s. Two examples of the outcome of sorting the realizations of random processes are shown in Figure 3. Figure 3a shows 2048 samples of a Gaussian process with zero mean and unit variance (μ = 0 and σ = 1, i.e., N(0,1)). Figure 3b shows the sorted sequence of the samples in Figure 3a, where the coefficient amplitude is plotted in the horizontal axis and the

coefficient number is plotted in the vertical axis. This figure shows that the sorted coefficients in Figure 3a produce the probability distribution function (PDF) of the Gaussian random process, which has the shape of the error function. Similarly, Figure 3c shows 2048 samples of a uniform process with $\mu = 0$ and $\sigma^2 = 0.1$. Figure 3d shows the sorted sequence of the samples in Figure 3c, where the coefficient amplitude is plotted in the horizontal axis and the coefficient number is plotted in the vertical axis. Similarly, this figure shows that sorting the DWT coefficients in Figure 3d lead to the PDF of the realization in Figure 3c.

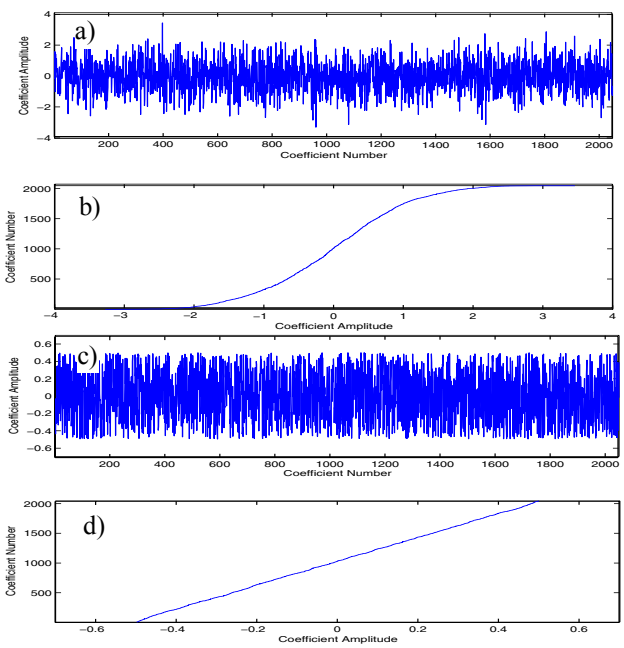

Figure 3. 2048 samples of a) Gaussian process realization and c) uniform process realization, b) sorted samples of the random noise realization in a) and d) sorted samples of the random noise realization in c).

The noise statistical parameters can be estimated from the sorted transform coefficients, x_s, or the original transform coefficients, x. The noise estimated parameters (mean, μ, and standard deviation, σ) are used to generate the ATF, $T(x_s)$, given by

$$T(x_s)_z = \frac{1}{\sqrt{2\pi}} \int_0^z \exp\left[-\frac{(k-\mu)^2}{2\sigma^2}\right] dk \qquad (12)$$

in the case that the noise source is Gaussian (a common source of noise in experimental ultrasonic signals), and z represents the individual samples of $T(x_s)$ and it is defined in the range from the minimum coefficient of x (i.e., $x_s(1)$) to the maximum coefficient of x (i.e., $x_s(N)$) in $\frac{x_s(N) - x_s(1)}{N}$ steps.

In the case that the noise source is uniform, the ATF is given by

$$T(x_s)_z = \frac{z - x_s(1)}{x_s(N) - x_s(1)} = \frac{1}{2} + \frac{z - \mu}{2\sqrt{3}\sigma} \qquad (13)$$

where $\mu = \frac{x_s(1) + x_s(N)}{2}$ and $\sigma = \frac{x_s(N) - x_s(1)}{2\sqrt{3}}$.

The next step is to subtract $T(x_s)$ from the original signal, x_s, to obtain the thresholded transform coefficients ζ_s^{ATF}. The thresholded coefficients are then resorted using the original position of the transform coefficients, and the last step of the algorithm is to apply the inverse transform to the resorted transform coefficients.

V. Performance of ATF and Classical Techniques

In this section, the performance of the ATF technique is compared to the performance of the classical techniques, i.e., hard thresholding (HT), soft thresholding (ST), and Garrote thresholding (GT) in denoising ultrasonic signals with multiple echoes using the DCT, DWT, and WHT. In these simulations the pulse-echo ultrasonic testing the backscattered echo from a single reflector can be modeled as

$$f_\Theta(t) = \beta \exp\left[-\alpha(t-\tau)^2\right] \cos(2\pi f_c(t-\tau) + \phi) \qquad (14)$$

where $\in = [a, \beta, f_c, \phi, \tau]$ denotes the parameter vector and ultrasonic signals consisting of multiple interfering echoes can be modeled as

$$\begin{aligned} s_\Theta(t) &= \sum_{j=0}^{M-1} f_{\Theta_j}(t) = \\ &= \sum_{j=0}^{M-1} \beta_j \exp\left[-\alpha_j (t-\tau_j)^2\right] \cos\left(2\pi f_{c_j}(t-\tau_j) + \phi_j\right) \end{aligned} \qquad (15)$$

where M is the total number of echoes in the signal. A Monte Carlo (MC) analysis was done to evaluate the performance of the thresholding techniques when the applied to multiple interfering echoes in the DCT, DWT, and WHT domains. The SNR of the original (input) signal was varied from around -5dB to 10dB. The MC analysis generated 25 noise realizations for each input SNR, thus in Figures 4-6, each point in the plots shows the mean of the improvement obtained and the error bars show one standard deviation around this mean. Figures 4-6 show the performance of the ATF, HT, ST, and GT techniques when applied to multiple ultrasonic echoes embedded in uniform Gaussian random noise. To better appreciate the performance of the ATF and the classical thresholding techniques, Figure 7 shows one realization of the multiple interfering echo signal embedded in uniform noise with SNR = -2.5dB.The multiple interfering echoes used have the parameters shown in Table I.

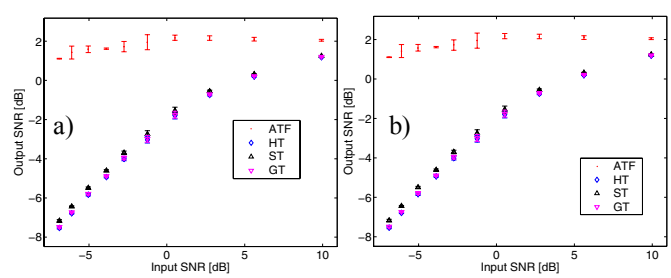

Figure 4. SNR enhancement of adaptive thresholding function (ATF), hard thresholding (HT), soft thresholding (ST), and Garrote thresholding (GT) techniques applied to 10 ultrasonic echoes with DCT in a) uniform and b) Gaussian noise.

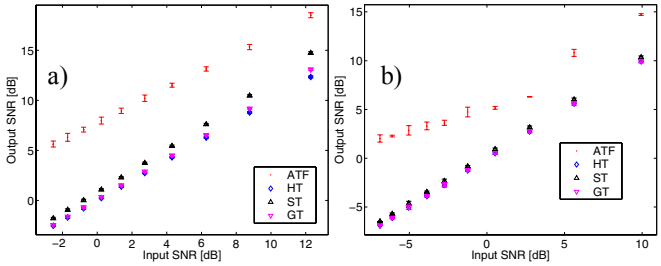

Figure 5. SNR enhancement of adaptive thresholding function (ATF), hard thresholding (HT), soft thresholding (ST), and Garrote thresholding (GT) techniques applied to 10 ultrasonic echoes with DWT in a) uniform and b) Gaussian noise.

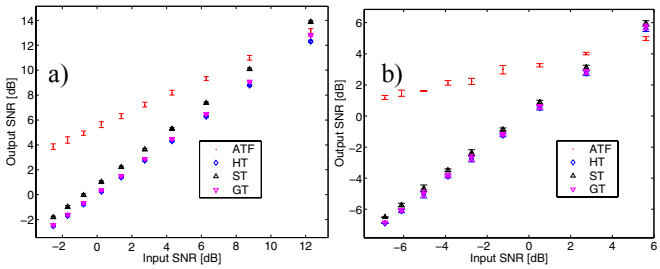

Figure 6. SNR enhancement of adaptive thresholding function (ATF), hard thresholding (HT), soft thresholding (ST), and Garrote thresholding (GT) techniques applied to 10 ultrasonic echoes with WHT in a) uniform and b) Gaussian noise.

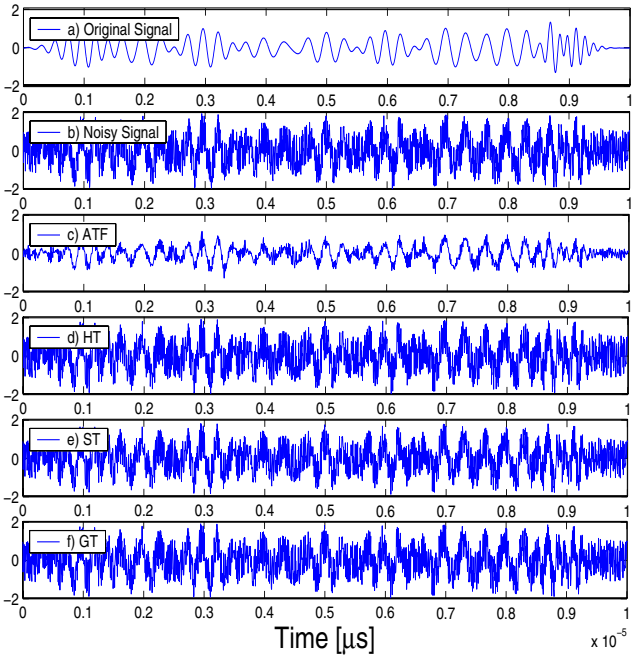

Figure 7. SNR enhancement of thresholding techniques in an ultrasonic signal with 10 interfering echoes corrupted by uniform noise with SNR = -2.5dB using DWT. a) Original signal, b) noisy signal, reconstructed signal using, c) adaptive thresholding function (ATF), d) hard thresholding (HT), e) soft thresholding (ST), and f) Garrote thresholding (GT).

TABLE I. ORIGINAL PARAMETERS OF TEN INTERFERING ECHOES.

Echo	f [MHz]	τ [µs]	α [MHz]^2	β	ϕ [rad]
a	4.5	1	4	1	1
b	4	6	4.8	0.9	0.8
c	4	3	4	1	0.9
d	7	9	8	1	1.5
e	3	2	5	0.9	0.8
f	5	8.7	8	0.8	0.2
g	3.3	5	5	0.8	0
h	3	7	5	1	0.7
i	3	8	4	1	0.5
j	3	4	4	0.5	0.7

Figures 4-6 show that the best transform to recover the signal embedded in low SNR is the DWT using the ATF. This result is a consequence of the time localization of the DWT kernel and its packing efficiency [5]. Furthermore, these results show that for small SNR (-2dB) the ATF consistently outperforms the classical thresholding techniques, but the situation reverses as the input SNR is improved (15dB). As expected, the ATF is better suited for applications with poor SNR.

VI. CONCLUSIONS

In this paper, we introduced an adaptive thresholding function technique that uses the statistical parameters of the noise embedded in the signal to generate a thresholding function based on the probability distribution function of the noise. This thresholding function is then subtracted from the sorted transform coefficients, leading to transform coefficients with a superior SNR. The results presented show that this is a very powerful technique that allows the detection of ultrasonic backscattered echoes embedded in low SNR environments. For signals with uniform noise added to DWT coefficients, the ATF technique achieves SNR improvements around 9dB over the classical thresholding techniques; these improvements are above 10dB for Gaussian noise. This paper also compares the denoising performance of the ATF technique when implemented in combination with DCT, DWT, and WHT coefficients. The results show that the DWT has the best performance among the transforms in the group analyzed. The energy packing capabilities of the DWT, along with the time localization of the wavelet kernel, allow that only a few DWT coefficients are necessary to correctly identify the location of the ultrasonic echo. Such localization is paramount for signal denoising and compression. Thus, the adaptive thresholding function technique presented in this paper offers data denoising capabilities for ultrasonic signals suitable for target detection, pattern recognition, and material characterization.

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