A Comparative Study of Echo Estimation Techniques for Ultrasonic NDE Applications

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Abstract — **In this study two different echo estimation techniques with a chirplet model are evaluated: chirplet signal decomposition based on the chirplet transform (CTSD), and the matching pursuit signal decomposition framework that employs Maximum Likelihood Estimation (MPSD). Both techniques are used to decompose backscattered signals into a linear expansion of chirplet echoes and estimate the chirplet parameters. The chirplet parameter estimation is unbiased with minimum variance, i.e., it attains analytically derived Cramer-Rao lower bounds. When applied to simulated and experimental ultrasonic signals, both algorithms perform robustly, yield accurate echo estimations and result in considerable SNR enhancements. Moreover, the MPSD algorithm outperforms the CTSD in moderate noise levels whereas the CTSD performs better than MPSD in severe noise levels. Numerical and analytical results indicate that the two presented signal decomposition algorithms are effective tools for ultrasonic signal analysis accounting for narrow-band, broadband, and symmetric, skewed, dispersive or nondispersive echoes. The present study provides guidelines which can be useful when dealing with signal analysis, pattern recognition, target sizing and material characterization.**

Keywords- chirplet; signal decomposition; parameter estimation

I. INTRODUCTION

In ultrasonic nondestructive evaluation (NDE), the backscattered signal contains information pertaining to the size, shape and orientation of reflectors, the absorption and scattering properties of the propagation path. The echoes from target reflectors are often overlapping and masked by the microstructure scattering noise. Therefore, the parameters of the reflectors can not be recovered using conventional techniques. A model-based estimation framework offers an alternative approach by representing the ultrasonic signal in terms of user designed model echoes. Several different models and decomposition methods have been investigated by our group and others [1-7].

The chirplet signal model offers six degrees of freedom (i.e., bandwidth, chirp rate, time-of-arrival, center frequency, phase and amplitude) in echo representation and embodies the dispersion effect, frequency dependent absorption and scattering in complex and inhomogeneous microstructures. In this paper we present and compare two different signal decomposition algorithms that rely on the chirplet model. One is chirplet signal decomposition based on the chirplet transform (CTSD), and the other is the matching pursuit signal decomposition (MPSD) that employs Maximum Likelihood Estimation.

II. CTSD ALGORITHM

In pulse-echo ultrasonic testing the backscattered echo from a single reflector can be modeled as [4]

$$f_\Theta(t) = \beta \exp\left(-\alpha_1(t-\tau)^2 + i2\pi f_c(t-\tau) + i\phi + i\alpha_2(t-\tau)^2\right) \quad (1)$$

where $\Theta = [\alpha_1, \alpha_2, \beta, f_c, \phi, \tau]$ denotes the parameter vector, α1 is the bandwidth factor, α2 is the chirp-rate, β is the amplitude, fc is the center frequency, ϕ is the phase, and τ is the time-of-arrival of the ultrasonic echo. The chirplet transform of $f_\Theta(t)$ with respect to a chirplet kernel $\Psi_{\hat\Theta}$ is defined as

$$CT\left(\hat\Theta\right) = \int_{-\infty}^{+\infty} f_\Theta(t)\Psi_{\hat\Theta}^*(t)dt \quad (2)$$

where $\hat\Theta = \left[\gamma_1, \gamma_2, \eta, \frac{\omega_0}{2\pi a}, \theta, b\right]$ denotes the vector of estimated parameters and the chirplet kernel is defined as

$$\Psi_{\hat\Theta}^*(t) = \left(\frac{2\gamma_1}{\pi}\right)^{1/4} \exp\left[-\gamma_1(t-b)^2 - i\omega_0\left(\frac{t-b}{a}\right) - i\theta - i\gamma_2(t-b)^2\right] \quad (3)$$

In our previous work [4-5], the CTSD algorithm is elaborately discussed. It can be shown that the estimation of the peak position of the time-frequency representation $\left|CT(\hat\Theta)\right|$ (i.e. the estimation of center frequency, f_c, and time-of-arrival, τ) is not a function of the bandwidth factor, α_1, chirp rate, α_2, and phase,ϕ, of the echo. Based on the estimation of center frequency, f_c , and time-of-arrival, τ , the estimation of the bandwidth factor, α_1 , chirp rate, α_2, and phase, ϕ, of the echo is a one-dimensional estimation problem. Hence, the chirplet transform can lead to a successive estimation of the parameter vector Θ of the ultrasonic echo $f_\Theta(t)$ [4-5]. The purpose of the CTSD algorithm is to decompose the ultrasonic signal $s(t)$ into a linear combination of chirp echoes $f_{\Theta_j}(t)$ and efficiently estimate the parameter vector Θ_j of each individual echo, i.e,

$$s(t) = \sum_{j=0}^{N-1} f_{\Theta_j}(t) \quad (4)$$

The decomposition is performed as follows: First, based on the chirplet transform time-frequency representation of ultrasonic signal, the most dominant chirp echo is windowed and its parameter vector is estimated successively. Then the estimated echo is subtracted from the original signal. Next the second echo is estimated from the remaining signal. This process is repeated until the reconstruction error, E_r, is below an acceptable value E_{min}. The value of E_{min} is determined based on the requirements of the reconstruction quality of the signal.

As an alternative to the CTSD algorithm, a different signal decomposition and parameter estimation strategy has been studied for echo estimation. The MPSD algorithm is described in the next section.

III. MPSD ALGORITHM

The matching pursuit algorithm has been initially introduced by Mallat and Zhang [8]. It aims to provide a signal analysis framework for a nonstationary signal. And it is an energy conserving signal decomposition. The matching pursuit algorithm first matches a function to the original signal. Then, this best matching function is subtracted from the signal to obtain the signal residue. At each iteration, a new function is matched to the current signal residue. When the energy of signal residue is a fraction of the energy of the original signal the decomposition is completed. The final decomposition is a linear expansion of all chosen matching functions.

In the original matching pursuit algorithm, the best match criterion uses correlation criteria (the inner product between signal residue and a pre-defined dictionary function) to determine the best matching function. This matching criterion obtains decompositions adaptive to global signal characteristics but often misses local signal characteristics.

The real challenge of matching pursuit signal decomposition is that different matching criteria can produce different decomposition results [8-10]. In our MPSD algorithm, by incorporating the Maximum Likelihood Estimation (MLE) method, we adaptively optimize the parameters of the chirplet function to best match the signal residues. This approach avoids the exhaustive search of a large number of dictionary functions and leads to a more efficient implementation.

In the optimization stage of the MPSD algorithm, the signal residue is represented by a chirplet function and a remaining signal (i.e., next residue),

$$R^n s = g(t;\Theta) + R^{n+1}s \tag{5}$$

Here, $R^n s$ is the current residue of signal $s(t)$, $R^{n+1}s$ is the next signal residue and $g(t;\Theta)$ is a chirplet echo

$$g(t;\Theta) = \beta e^{-\alpha_1 (t-\tau)^2} \cos[2\pi f_c (t-\tau)^2 + \alpha_2 (t-\tau)^2 + \phi] \tag{6}$$

where $\Theta = [\alpha_1, \alpha_2, \beta, f_c, \phi, \tau]$ denotes the parameter vector of $g(t;\Theta)$. If we assume $R^{n+1}s$ has white Gaussian noise characteristics, the MLE of the parameter vector Θ can be obtained by minimizing:

$$\Theta_{MLE} = \arg_{\Theta} \min \left\| R^n s - g_{\Theta}(t) \right\|^2 \tag{7}$$

The above minimization problem can be solved by utilizing a simple Gauss-Newton algorithm (see [1]).

In summary, the MPSD algorithm can be outlined in the following steps:

1. Set iteration index $n = 0$ and first signal residue $R^n s = s(t)$.
2. Find the best parameter vector of the chirplet function such that $\Theta_n = \arg_{\Theta} \min \left\| R^n s - g(t;\Theta) \right\|^2$, and set $g_n = g(t;\Theta_n)$.
3. Compute the next residue $R^{n+1}s = R^n s - g_n$.
4. Check convergence:

   If $\dfrac{\left\| R^{n+1}s \right\|^2}{\left\| s(t) \right\|^2} \le Threshold$, STOP;

   OTHERWISE set $n \rightarrow n+1$ and go to Step 2.

IV. CRAMER-RAO LOWER BOUND

The Cramer-Rao Lower Bounds (CRLB) provides analytical bounds on the variance of unbiased estimators. In order to compare and evaluate the performance of estimation in the presence of noise, we consider a single chirp echo in white Gaussian noise with varying noise levels and observe the bias and variation in the parameter estimation. Specifically, we use the following observed chirp model

$$r(t) = g(t;\Theta) + n(t) \tag{8}$$

where $g(t;\Theta)$ represents the chirp echo(see Equation 6) and $n(t)$ represents the zero-mean white Gaussian noise with variance σ^2. The CRLB for the parameter vector Θ can be analytically computed using [11]:

$$Var\left(\hat{\Theta}\right) \ge \left[I^{-1}(\Theta)\right] \tag{9}$$

where $I(\Theta)$ is the Fisher Information Matrix (FIM). The above observed signal model $r(t)$ is assumed to be normally distributed as $N(g(t;\Theta), \sigma^2 I)$, hence the FIM can be written as [11]

$$I(\Theta) = \frac{H^T(\Theta) H(\Theta)}{\sigma^2} \tag{10}$$

where $H(\Theta)$ represents the gradients of the chirp echo model $g(t;\Theta)$. It can be derived that the variance bounds of the estimated chirplet parameters under noise are

$$Var(\hat{\alpha}_1) \ge \frac{8\alpha_1^2}{f_s \zeta} \qquad Var(\hat{\alpha}_2) \ge \frac{8\alpha_1^2}{f_s \zeta} \qquad Var(\hat{\tau}) \ge \frac{1}{\alpha_1 f_s \zeta}$$

$$Var\left(\hat{f}_c\right) \geq \frac{\alpha_1^2 + \alpha_2^2}{\pi^2 \alpha_1 f_s \zeta} \qquad Var\left(\hat{\phi}\right) \geq \frac{\frac{3}{2} + \frac{(2\pi f_c)^2}{\alpha_1}}{f_s \zeta} \qquad Var\left(\hat{\beta}\right) \geq \frac{3\beta^2}{2 f_s \zeta} \tag{11}$$

where f_s denotes sampling rate and ζ denotes signal-to-noise ratio (SNR).

V. COMPARISON AND ANALYSIS

To evaluate the performance of estimation, a Monte-Carlo simulation is performed to observe the means and variances of the estimated parameters of a single noisy echo given in Equation 8. The chirp echo is simulated according to Equation 6 with the parameter vector listed in the Actual Parameter row of Table 1. The sampling frequency is 100 MHz. The noise level is adjusted to simulate echoes with SNR levels of 20, 10, 5 dB. For each SNR level, both algorithms (i.e., CTSD and MPSD) are performed 250 times on the same simulated chirp echo with different realizations of noise. The average value and the variance of parameter estimators are listed in the Table along with the analytically computed CRLB's using Equation 11. One can observe that the parameter estimations are unbiased, i.e., the mean value of the estimated parameters achieves the actual parameter values used in simulation and the variance of estimators attains the CRLB bounds for SNR as low as 5 dB. Therefore, the CTSD and MPSD are minimum variance unbiased (MVU) estimators for a single chirp echo. Hence they provide the optimal parameter estimation results.

To quantify the SNR improvement, a chirp echo with varying noise level is simulated. After estimation is performed, the output SNR (i.e., estimation SNR) is computed as the energy ratio of the original signal and residual error (i.e., the difference between the original and the estimated signal). Fig.1a shows the output SNR as a function of the input SNR using CTSD algorithm. Each point in this plot represents a realization of the signal with a different noise level. The parameters of the single echo have not been changed. The input SNR has been varied from 5 dB (severely poor SNR) to 25 dB (high SNR). Similarly, Fig. 1b shows the output SNR as a function of the input SNR using MPSD algorithm. It has been observed that the average SNR enhancement for the single echo in WGN is well above 20 dB. It is important to point out that one should expect a smaller SNR enhancement when the signal contains overlapping chirp echoes or is corrupted with correlated noise.

In Fig. 1a and Fig. 1b, it can be seen that in moderate noise levels (i.e., input SNR varying from 10 dB to 25 dB), the estimation efficiency of MPSD algorithm is similar, even better than that of CTSD algorithm. However, in the severe noise levels (i.e., input SNR is below 5 dB), the MPSD algorithm is not as efficient as the CTSD algorithm. This can be explained by the different implementation strategies of CTSD algorithm and MPSD algorithm. First, the CTSD algorithm performs parameter estimation in time-frequency domain whereas the MPSD algorithm performs only in time domain. Hence, the noise is better suppressed in CTSD algorithm than it is in MPSD algorithm. Secondly, the MPSD algorithm is based on iterative optimization and may become more dependent on the initial guess in severe noise levels.

To demonstrate the robustness of these two algorithms, they are applied to ultrasonic experimental signals containing reverberant echoes measured from multiple layers specimen. Fig. 2 shows the experimental signal acquired from multiple thin layers using a 10 MHz transducer and sampling rate of 100 MHz. It can be seen that the analysis of reverberant echoes is particularly challenging due to the high-degree of overlap and presence of many echoes. The reconstructed signals by CTSD algorithm (39 estimated chirplets) and MPSD algorithm (35 estimated chirplets) are shown in Fig. 2a and Fig. 2b.

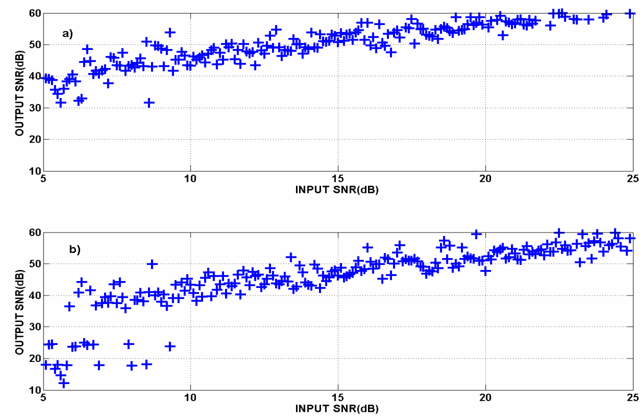

Figure 1. a) Input SNR vs. output SNR for a single noisy chirp echo using CTSD algorithm b) Input SNR vs. output SNR for a single noisy chirp echo using MPSD algorithm

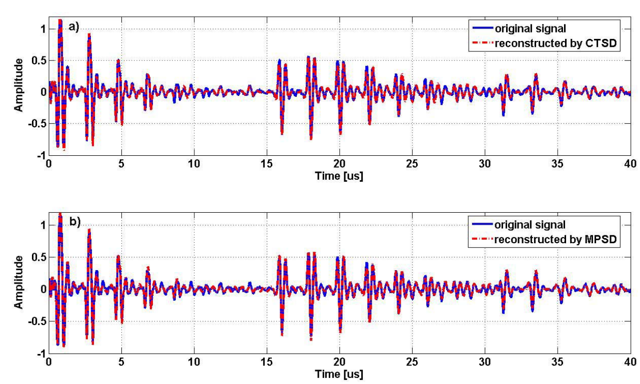

Figure 2. a) Experimental signal superimposed with the reconstructed signals by CTSD b) Experimental signal superimposed with the reconstructed signals by MPSD

VI. CONCLUSION

In the comparative study of chirplet model-based echo estimation techniques, two different signal decomposition and parameter estimation algorithms (i.e., CTSD and MPSD) are analyzed. It has been shown through analytical CRLB derivation, computer simulations and experimental verification that both algorithms are robust and efficient in ultrasonic signal analysis. These algorithms can also be utilized in the analysis of signals often encountered in sonar target detection and seismic signal analysis.

TABLE COMPARISON OF THE CRLB'S WITH THE VARIANCES OF CTSD AND MPSD FOR DIFFERENT SNR

	α_1 [MHz]^2	α_2 [MHz]^2	τ [μs]	f_c [MHz]	ϕ [rad]	β
Actual Parameter	**25**	**15**	**1**	**5**	**1**	**1**
20.00 dB SNR						
MEAN_CTSD	25.0266	15.0080	0.9999	4.9996	0.9959	1.0007
MEAN_MPSD	25.0683	14.9570	1.0000	4.9989	0.9991	1.0004
VAR_CTSD	4.4831e-1	5.6883e-1	4.5664e-6	3.4852e-4	4.5799e-3	1.5671e-4
VAR_MPSD	5.0075e-1	5.5020e-1	4.3575e-6	3.2463e-4	4.5524e-3	1.4666e-4
CRLB	5.0000e-1	5.0000e-1	4.0000e-6	3.4449e-4	4.0978e-3	1.5000e-4
10.00 dB SNR						
MEAN_CTSD	25.0242	14.8368	0.9997	4.9967	0.9911	1.0011
MEAN_MPSD	24.8818	14.9922	0.9998	5.0057	0.9914	0.9991
VAR_CTSD	4.0620	5.4588	3.5395e-5	3.4286e-3	3.4439e-2	1.4117e-3
VAR_MPSD	5.3476	4.8644	3.7189e-5	2.9160e-3	3.9118e-2	1.4352e-3
CRLB	5.0000	5.0000	4.0000E-5	3.4000e-3	4.1000e-2	1.5000e-3
5.00 dB SNR						
MEAN_CTSD	24.7932	15.2223	0.9997	4.9933	0.9905	1.0080
MEAN_MPSD	24.2034	13.7460	1.0699	5.4025	1.0268	0.9984
VAR_CTSD	14.4450	16.9490	1.3875e-4	1.1230e-2	1.4020e-1	3.8953e-3
VAR_MPSD	19.2300	95.1000	1.2318	5.0545e-1	2.3073e-1	4.1424e-3
CRLB	15.8114	15.8114	1.0000e-4	1.0900e-2	1.2960e-1	4.7000e-3

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