Digital Object Identifier: 10.1109/ULTSYM.2008.0045

S-TRANSFORM APPLIED TO ULTRASONIC NONDESTRUCTIVE TESTING

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ABSTRACT- **The joint time-frequency (t-f) display of non-stationary backscattered ultrasonic echoes reveals important information to characterize the target echoes in the non-destructive testing of materials. Generalized t-f distribution (e.g., Wigner-Ville distribution, Chow-Williams exponential distribution, etc.) applied to ultrasonic backscattered echoes generates artifacts known as cross-terms. These cross-terms mask the t-f information of the target echoes and make the evaluation of the material difficult. An alternative is Gabor Transform (GT), which displays the joint t-f information without generating cross-terms. However, the fixed window of GT limits its t-f resolution. The signal components with period longer than the window width may be misinterpreted and the time resolution of high frequency signal components is limited due to the finite window width. In this paper we apply a novel t-f method, S-Transform (ST), which uses the t-f representation of GT with a frequency dependent window. We have explored the performance of S-transform as applied to ultrasonic non-destructive testing of materials. ST detects multiple target echoes in both time and frequency without the a-priori knowledge of the measurement system's characteristics. ST, combines the time-frequency representation of the GT and multi-resolution feature of Wavelet Transform (WT), is a unique t-f representation. It adapts the Fourier Transform (FT) in analyzing the localized signal by applying a frequency-dependent time-scaling window. These characteristics make the ST a useful tool to characterize the ultrasonic backscattered target echoes embedded in the microstructure noise for flaw detection in the materials. Simulation results depict correct t-f information of multiple Gaussian Echoes under low SNR environment. In addition, experimental results demonstrate better and reliable detection and characterization of backscattered target echoes in the presence of microstructure noise.**

I. INTRODUCTION

In ultrasonic nondestructive testing the frequency dependent scattering, attenuation, and dispersion make the backscattered signal information a non-stationary signal. Therefore, joint time-frequency (t-f) analysis of the ultrasonic target echoes embedded in microstructure noise reveal the important information, e.g., time of arrival and frequency, in the nondestructive testing of materials. Cohen's class [1] of quadratic/bilinear generalized time-frequency representations (GTFRs) can be applied to the ultrasonic signal to display its energy on a joint t-f plane. But GTFRs inherently generate artifacts known as cross-terms obscuring the important information regarding target echoes. An alternate method to display t-f energy of a non-stationary signal without cross-terms is the short-time Fourier transform (STFT). But STFT assumes short-time stationarity of the signal that makes it difficult to discern multiple echoes embedded in the grain noise. The Heisenberg's Uncertainty Principle imposes a lower bound on the t-f resolution of the STFT of a signal. The optimal resolution of the STFT can be obtained if the Gaussian modulated pulse is used as a window function [2] that leads to Gabor transform (GT). However, the t-f resolution of GT is limited due to its fixed width window. The signal components with period longer than the window width may be misinterpreted and the time resolution of high frequency signal components is limited due to the finite window width. Another effective method to display t-f energy of ultrasonic signals is the Wavelet transform (WT). But WT works best if the signal contains low frequency echoes of longer duration embedded in high frequency grain echoes of shorter duration [3]. Both GT and WT require a-priori knowledge of the signal and measurement system to perform optimally. S-Transform (ST) proposed by R. G. Stockwell, L. Mansinha, and R. P. Lowe [4] on the other hand takes advantage of both GT's (using Gaussian widow) optimum resolution and WT's frequency-dependent resolution.

In this study, the ST is applied to ultrasonic signals for flaw detection in the non-destructive testing of materials. The S-transform, applied to simulated data as well as experimental data, performs better than the GTFRs in discerning the target (flaw) echoes embedded in the microstructure (grain) noise. Unlike GT, the S transform does not smooth out the microstructure grain features.

II. GABOR TRANSFORM, WAVELET TRANSFORM, AND S-TRANSFROM

The GT of a signal $s(t)\ \varepsilon\ L^2(R)$ is given by:

$$GTs(\tau, f) = \int_{-\infty}^{\infty} s(t)\, g^*(t;\tau, f)\, dt \qquad (1)$$

Where, the Gaussian elementary signals are defined as follows:

$$g(t:\tau, f) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\left[\frac{(t-\tau)^2}{2\sigma^2}\right]} e^{j2\pi f t} \qquad (2)$$

If the bandwidth parameter (effective window width), σ, of GT is taken as the inverse of absolute frequency as, $\sigma = 1 / |f|$, then it leads to the definition of the ST as follows:

$$S_s(\tau, f) = \frac{|f|}{\sqrt{2\pi}} \int_{-\infty}^{\infty} s(t)\, e^{-\left[\frac{(t-\tau)^2 f^2}{2}\right]} e^{-j2\pi f t}\, dt \qquad (3)$$

ST defined in Eq. (3) succinctly depicts that it is a GT whose widow is scaled with respect to frequency inversely. The window duration is wider for lower frequency and therefore

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can provide better time and frequency resolution while its duration is narrower for high frequency and thus can provide higher time resolution. In essence the scaling of the dimension of the analyzing widow accommodates same number of cycles for each spectral component as in wavelet [5]. Thus the ST is a specific GT with a window linearly and inversely varying with frequency. Therefore, the ST, unlike GT, performs better in terms of t-f resolution without the a-priori knowledge of the signal and measurement system.

The continuous wavelet transform (CWT) of a signal $s(t) \varepsilon L^2(R)$ is given by:

$$CWTs(\tau, \mathrm{a}) = \frac{1}{\sqrt{\mathrm{a}}} \int_{-\infty}^{\infty} \mathrm{s(t)}\, \psi^*(\frac{t-\tau}{a}) dt \qquad (4)$$

Where, τ is the translation parameter, a (i.e., the dilation parameter or scaling factor) governs the frequency, and $\psi(t)$, called a basic or mother wavelet, is a square integrable function satisfying the following conditions [6]:

1. $\psi(t)$ is a continuous function and has exponential decay [i.e. $\psi(t) = \leq M e^{-C|t|}$ for some constants C and M].
2. The mother wavelet must have zero mean.

If the dilation factor, a, in the Eq. (4) is the inverse of the frequency as: $\sqrt{\mathrm{a}} = 1 / |\mathrm{f}|$, and the wavelets, $\psi(t,f)$, are defined as:

$$\psi(t, f) = \frac{|f|}{\sqrt{2\pi}} exp\left(-\frac{t^2 f^2}{2}\right) e^{-j2\pi f t} \qquad (5)$$

The CWT of s(t) can be written as:

$$CWT_s(\tau, \mathrm{f}) = \mathrm{e}^{\mathrm{j}2\pi \mathrm{f} \tau} \frac{|\mathrm{f}|}{\sqrt{2\pi}} \int_{-\infty}^{\infty} s(t) e^{-\frac{(\mathrm{t}-\tau)^2 f^2}{2}} e^{-j2\pi \mathrm{f t}} dt \qquad (6)$$

The mother wavelet in Eq. (5) does not satisfy the admissibility condition of a wavelet in CWT. Therefore, Eq. (6) is not strictly a CWT. Comparing Eq. (3) and Eq. (6), ST of s(t) can be defined as a CWT with a specific mother wavelet multiplied by the phase factor as follows:

$$S_s(\tau, \mathrm{f}) = \mathrm{e}^{-\mathrm{j}2\pi \mathrm{f} \tau}\, CWT_s(\tau, \mathrm{f}) \qquad (7)$$

ST of a signal s(t) in this sense can be considered as the 'phase correction' [4] of CWT of the signal.

Another description of the ST as an operation on Fourier transform $S(f)$ of $s(t)$ is as follows[4]:

$$S_s(\tau, \mathrm{f}) = \int_{-\infty}^{\infty} \mathrm{S}(\alpha + \mathrm{f})\, \mathrm{e}^{-\frac{2\pi^2 \alpha^2}{f^2}} e^{j2\pi \alpha \tau} d\alpha, \quad f \neq 0 \qquad (8)$$

Hence the discrete ST is computed using the following discrete version of Eq. (8), letting f → n / NT and τ → jT, T being the sampling interval.

$$S_s\left[jT, \frac{n}{NT}\right] = \sum_{\mathrm{m}=0}^{\mathrm{N-1}} \mathrm{S}\left[\frac{\mathrm{m+n}}{\mathrm{NT}}\right] \left[\mathrm{e}^{-\frac{2\pi^2 m^2}{n^2}}\right] \left[\mathrm{e}^{\frac{\mathrm{i}2\pi \mathrm{m j}}{\mathrm{N}}}\right], \quad n \neq 0 \qquad (9)$$

where, S [(m + n) / NT] is shifted localized spectrum (m is the shift lag) of the discrete time series s[kT], k = 0,1, , N-1, and discrete Fourier transform of the time series is given by

$$S\left[\frac{n}{NT}\right] = \frac{1}{\mathrm{N}} \sum_{\mathrm{m}=0}^{\mathrm{N-1}} \mathrm{s}[\mathrm{mT}] \mathrm{e}^{-\frac{\mathrm{i}2\pi \mathrm{m n}}{\mathrm{N}}}, \quad \mathrm{n} = 0, 1, \ldots, \mathrm{N}-1 \qquad (10)$$

$exp(-2\pi^2 m^2 / n^2)$ is a band-pass filter function (voice Gaussian), and $exp(i\, 2\, m\, \pi\, j / N)$ is inverse DFT Kernel.

For zero frequency voice (for n=0) , ST is defined as a constant as follows:

$$S_s[jT, 0] = \frac{1}{\mathrm{N}} \sum_{\mathrm{m}=0}^{\mathrm{N-1}} \mathrm{s}[\mathrm{mT}] \qquad (11)$$

Where, j, m, and $n = 0, 1, \ldots, N-1$. Thus the constant time average of the signal (DC) is mapped to zero frequency voice.

ST can be implemented using Eq. 9 by shifting the spectrum to a new frequency, applying the voice Gaussian, and taking inverse DFT to create the voice [a function of time with fixed frequency, $S_s(\tau, f_0)$].

III. PROPERTIES OF THE S-TRANSFORM

The following properties of the ST make it distinct from other t-f representations and are useful for improving the SNR for estimating correctly the time of arrival and frequency of ultrasonic echoes.

1. Direct Relationship with Fourier Transform

The normalizing factor $|f| / \sqrt{2\pi}$ in Eq. (3) ensures that the time-average of ST of the signal s(t) is the FT of the signal , S(f) as follows:

$$\int_{-\infty}^{\infty} S_s(\tau, \mathrm{f})\, \mathrm{d}\tau = \int_{-\infty}^{\infty} s(t) e^{-j2\pi \mathrm{f t}} dt = \mathrm{S(f)} \qquad (12)$$

It is clear from Eq. (13) that signal s(t) can be recovered from $S_s(\tau, f)$, thus rendering ST to be invertible,

$$s(t) = \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} \mathrm{S_s}(\tau, \mathrm{f})\, \mathrm{d}\tau \right\} e^{j2\pi \mathrm{f t}} df \qquad (13)$$

This characteristic of ST is generalization of the FT to nonstationary time series, is not manifested by the other t-f representations such as CWT. Therefore, the frequency information of the ultrasonic echoes can be directly obtained by time averaging the ST of the signal.

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2. Progressive Resolution

ST has progressive resolution of the t-f plane. Longer periods have better frequency resolution and high frequencies have good time resolution. Although the temporal and frequency resolutions are directly governed by Heisenberg's *Uncertainty Principle* but the ST provides better t-f resolution and improves the SNR (signal-to-noise ratio) on the joint time-frequency plane.

3. Linearity

The bilinear t-f representations (TFRs) of signals with additive noise are marred by the cross-products equal to twice the product of TFR of the signal and TFR of noise. The STFT and GT eliminate these cross-products but some of the features embedded in noise are smoothed out and the t-f resolution of the signal is degraded. The ST, unlike the bilinear class of TFRs, is a linear operation. It does not generate cross-products nor does it smooth out any other features embedded in noise but provide better t-f resolution of the signal and improves the SNR.

IV. RESULTS AND DISCUSSIONS

Comparison of GT and ST of Two Gaussian Echoes with Additive Noise

Two Gaussian echoes of different frequencies (5 MHz and 10 MHz) and times of arrival (0.45µs and 0.9 µs approximately) with additive white Gaussian noise (AWGN) are simulated and shown in Figure 1a. Figure 1b is the spectrum of these two echoes. We have applied the ST and GT to the simulated data and the ST and GT of the two noisy echoes are shown in Figure 1c and Figure 1d, respectively. It is clear from Figure 1c and Figure 1d that the ST displays very good t-f resolution as compared to that displayed by GT of the signal. Unlike GT, the ST does not smooth out the noise but displays the true t-f energy of the signal without a-priori knowledge of the measurement systems' characteristics. Although the t-f resolution of GT of the signal may be optimized by having the a-priori knowledge of the signal as well as the measurement systems' characteristics [8] and cross-terms are deleted, but due to the smoothing effect the true t-f energy is not displayed. The ST on the other hand does not generate the cross-terms and display the true t-f energy with good t-f resolution of the echoes. The estimates of time of arrival and frequency of the two echoes obtained from the ST of the signal are more reliable than those obtained by the GT of the signal.

Experimental Results: A Flaw Echo Embedded in Microstructure Grain Noise

The experimental data in Figure 2 shows a flaw (target echo) embedded into microstructure grain noise. Time-Frequency analysis tools are best suited to discern the flaw echo from the microstructure noise and to estimate its time of arrival and frequency.

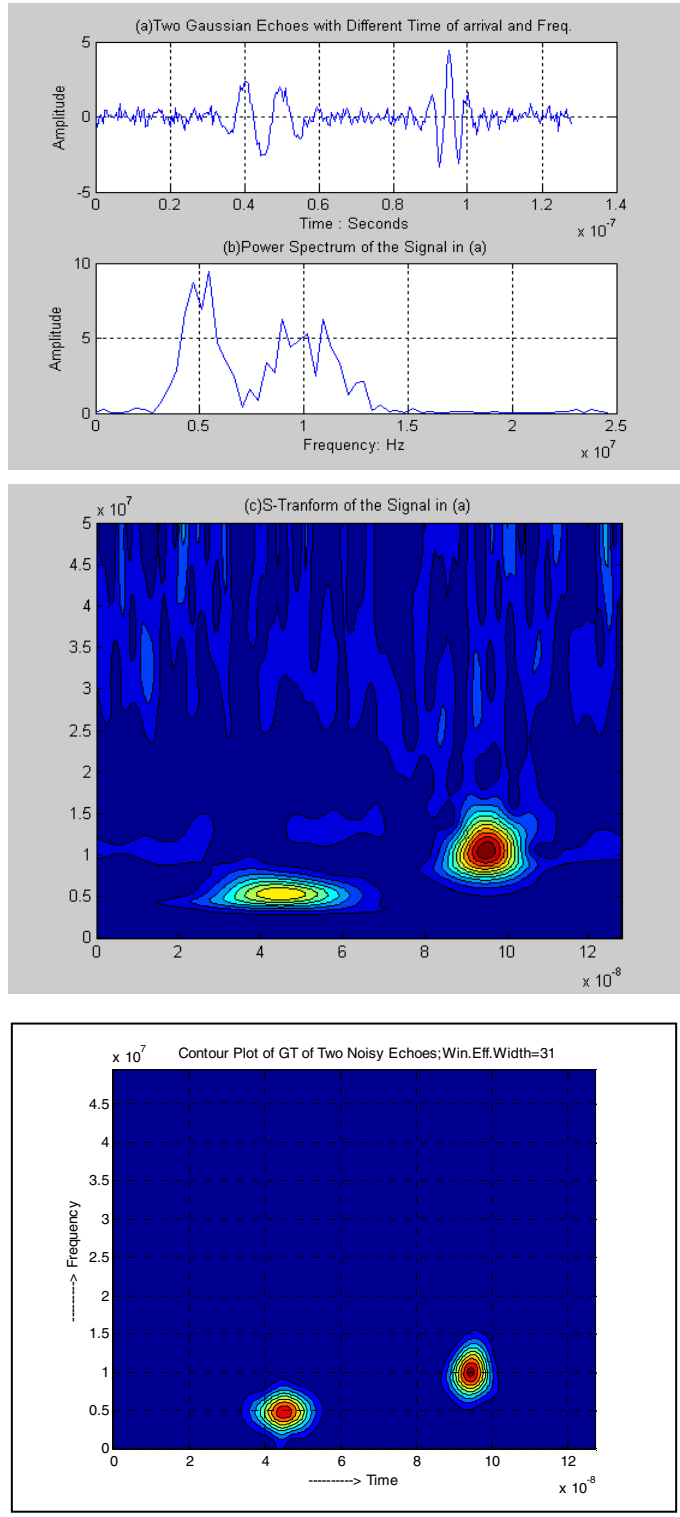

Figure 1. (a) Two Gaussian echoes of different frequencies and time of arrival with AWGN, (b) Spectrum of the signal in (a), (c) ST of the signal in (a), and (d) GT of the signal in (a).

The experimental data is shown in Figure 2 has a flaw echo embedded in the microstructure noise. It is very difficult to discern this flaw echo from the microstructure noise. The

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ST of the experimental data in Figure 3 clearly detects the flaw echo by estimating its arrival time and frequency simultaneously. The ST of the experimental signal displays the true time-frequency energy of the signal without generating the artifacts (cross-terms) and depicting clearly the microstructure noise with an improvement in the signal-to-noise ratio. The GT of the same experimental data is shown in Figure 4 with GT Effective window width of 63 samples. The GT of the experimental data with effective window width used show better time resolution but the degraded frequency resolution due to larger window width. In order to correctly estimate the frequency and the arrival time of the flaw echo using GT, the a-priori knowledge of the signal and measurement system characteristics are required. It is clear from the GT of the signal in Figure 4 that the cross-terms are deleted at the cost of microstructure noise being smoothed out. Comparing Figures 3 and 4 the overall performance of the ST in detecting the flaw (target) echo and characterizing the material is better than that of the GT.

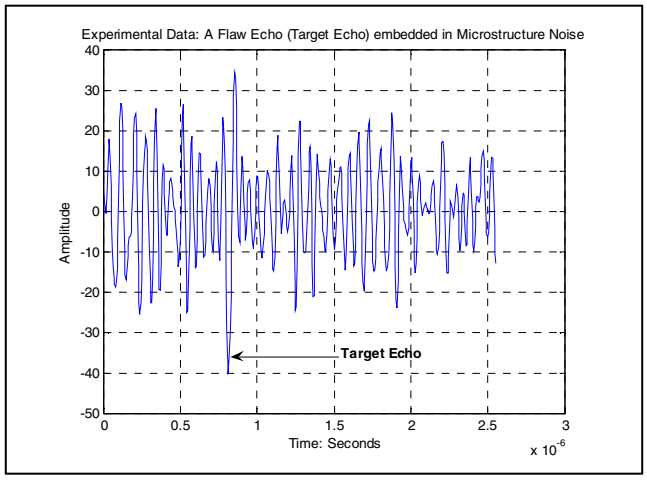

Figure 2. Experimental Data: A flaw echo embedded in microstructure scattering noise.

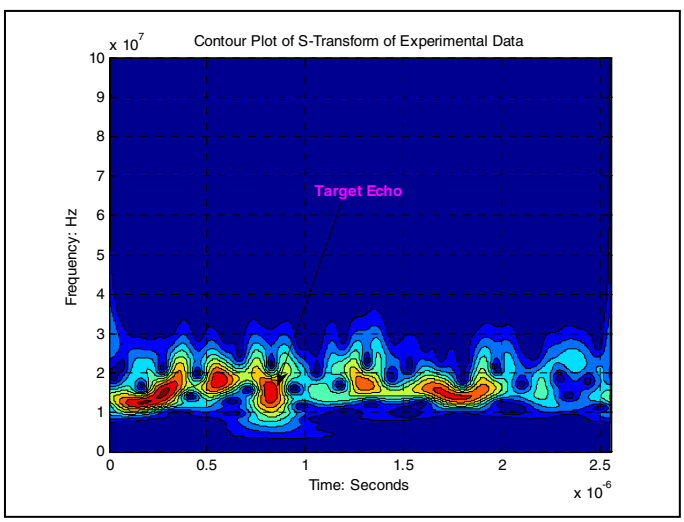

Figure3. The S transform of the experimental data in Figure 2.

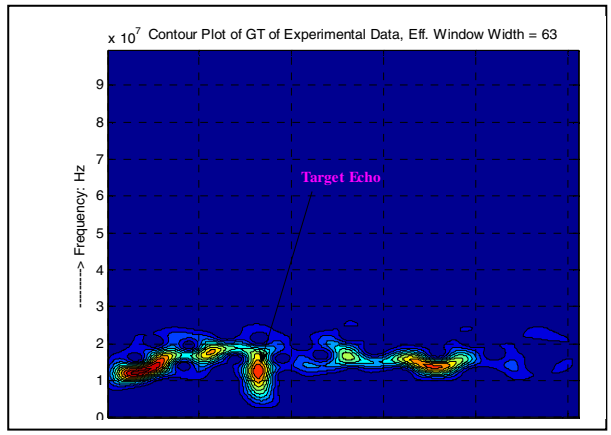

Figure 4. GT of the experimental data in Figure 2.

V. CONCLUSIONS

The advantages of the ST over GT for ultrasonic non-destructing testing are manifested by the results of the simulation and experimental data. The window of the ST varying inversely with the frequency provides good frequency localization, better time and frequency resolution at low frequencies and higher time resolution at high frequencies. These characteristics of the ST along with its linearity make it a good choice for t-f analysis of the ultrasonic signals for non-destructive testing. With the a-priori knowledge of the measurement systems' characteristics and using the t-f concentration curves one can obtain an optimal GT window parameter for better detection of multiple echoes in both time and frequency. But the microstructure scattering features are smoothed out at the cost of better estimates of frequency and time of arrival of multiple echoes without the interference of artifact (cross-terms). The ST on the other hand provides better time and frequency resolution for the detection of multiple echoes as well as keeps the microstructure scattering features.

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