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AN EFFICIENT SPARSE SIGNAL DECOMPOSITION TECHNIQUE FOR ULTRASONIC SIGNAL ANALYSIS USING ENVELOPE AND INSTANTANEOUS PHASE

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Abstract— **Sparse signal decomposition techniques have been widely used in recent years due to their efficiency in ultrasonic signal analysis. These techniques iteratively decompose ultrasonic signal in terms of model echoes (e.g., Gaussian echo, chirplet echo, etc.) that characterize local signal structures. The decomposed echoes (or the parameters) are then used for subsequent analysis, for example, for feature extraction and system identification. The first critical step in these decomposition techniques is the partitioning (i.e., windowing) of the ultrasonic data for identification of dominant signal features. This step has a great implication on the subsequent step that involves parameter estimation based on the assumed echo model or finding the best matched echo from a predefined dictionary of echoes. Therefore, a robust windowing technique that successively partitions ultrasonic data into dominant echo components is highly desirable. In this study, we obtain envelope and instantaneous phase via analytic signal representation to guide ultrasonic data partitioning. This type of partitioning is also meant to serve the initial guessing operation prior to the parameter estimation. Envelope and instantaneous phase provide important clues for local changes in the ultrasonic signal. The local maxima of the smooth envelope along with the changes in the instantaneous phase provide meaningful boundaries for echo structures. These boundaries are expected to provide an accurate data partition for the subsequent echo estimation. We present results that demonstrate the proposed echo windowing technique embedded with a model-based echo estimation is significantly faster than the Time-Frequency (TF) based echo localization techniques and provide meaningful echo localization and parameter estimation results. In particular, we test the algorithm in ultrasonic flaw detection using backscattered echoes from a steel sample that contains flaw echoes buried in the clutter (SNR is about 0 dB), and for the analysis of closely spaced reverberation echoes measured from a multi-layer test specimen**.

I. INTRODUCTION

Sparse signal decomposition techniques based on the Matching Pursuit (MP) algorithm have been used extensively for ultrasonic signal analysis due to their efficiency and ease of use in adaptive signal decomposition and feature extraction [1-5]. These techniques aim to decompose an ultrasonic signal in terms of a small number of waveforms obtained via parameter estimation or chosen from a redundant dictionary of functions. The decomposed functions are then used for subsequent analysis, for example, for feature extraction and system identification.

The sparse signal representation of ultrasonic echoes has been first introduced in [1] utilizing a model-based Matching Pursuit (MP) algorithm. Inspired from the model-based ultrasonic signal analysis [6], this model-based sparse signal decomposition technique is based on optimizing the parameters of a generic (i.e., model) echo to match local signal structures rather than using a predefined dictionary of functions as in the original MP technique [7]. Later, the TF representation (TFR) of the ultrasonic data is embedded into this technique to partition echoes and guide parameter estimation. This method has been used in ultrasonic data compression technique [3] and in the pulse detection of ASCAN data [8]. More recently the echo model (Gaussian echo) used in these techniques has been generalized to Gaussian chirplet model and utilized in the chirplet signal decomposition of dispersive ultrasonic echoes [4], and modeling reverberation echoes from multi-layered structures [9].

The first critical step in these decomposition techniques is the partitioning (i.e., windowing) of the ultrasonic data for identification of dominant signal features. This step has a great implication on the subsequent step that involves parameter estimation based on the assumed echo model or finding the best matched echo from a predefined dictionary of echoes. Generally, a TFR of the ultrasonic signal is utilized to partition the data in the time and frequency domains, and apply a parameter estimation algorithm subject to this data [2, 4, 9]. Even though the TFRs have been proved to be very useful in identifying echo components, it is computationally expensive. It requires the computation of TFR of the data, as well as update of the TFR after every echo-matching operation. Furthermore, the TFR and parameter estimation are bounded tightly. One generally develops a TFR based upon the assumed echo model (e.g., Gaussian or Chirplet).

In this study, we utilize envelope and instantaneous phase obtained via analytic signal representation to guide ultrasonic data partitioning. The computational complexity of this method is significantly lower than the TF based methods. It requires the envelope and phase computation via Hilbert transform prior to parameter estimation. The envelope is updated only, after a MP signal matching operation, by subtracting the envelope of the matched signal, which is readily available from the model echo. The partitioning based on the envelope and phase information provides a good initial guess for the time-of-arrival and center frequency estimation. Furthermore, this partitioning is not tightly bounded to the echo model used in parameter estimation; several commonly used models (e.g., Gaussian, chirplet) can be utilized.

This paper is organized as follows. The next section reviews the envelope and instantaneous phase estimation via Hilbert Transform (HT) and introduces the echo partitioning technique. Section III presents the MP signal decomposition method based on the aforementioned partitioning method. Section IV discusses the application of this technique in ultrasonic NDE applications. This section also presents decomposition results for the backscattered echoes measured from steel samples and reverberation echoes measured from a multi-layer specimen.

II. Ultrasonic Echo Partitioning via Envelope and Instantaneous Phase

Ultrasonic echoes are characterized as bandpass signals due to the bandpass property of the ultrasonic transducer. The analytic (complex) signal representation is useful for representing a bandpass signal. It allows simple determination of the envelope and instantaneous phase of a real signal. The analytic signal for a real signal $s(t)$ is defined as

$$\varphi(t) = s(t) + j\hat{s}(t) \tag{1}$$

where $\hat{s}(t)$ is the HT of the signal. The HT is a standard operation and available in many scientific computing libraries. One can represent an ultrasonic echo by,

$$s(t) = \beta(t)\cos\{2\pi f_c t + \phi(t)\} \tag{2}$$

where $\beta(t)$ is the envelope, f_c is the center frequency and $\phi(t)$ is the instant phase of the signal. It is known that envelope and phase of the echo changes much slower than the sinusoidal term. Such signals are called narrow-band signals. Using narrow-band signal property, the analytic signal representation for the ultrasonic echo can be written as

$$\varphi(t) = \beta(t)\exp(\, j(2\pi f_c t + \phi(t))\,) \tag{3}$$

The magnitude of this complex signal is called the envelope and identical to the magnitude of the real signal, i.e.,

$$\beta(t) = |\varphi(t)| = \sqrt{s^2(t) + \hat{s}^2(t)} \tag{4}$$

Similarly, the phase of the real signal can be obtained from the phase of the analytic signal, i.e.,

$$\phi(t) = \angle\varphi(t) = \tan^{-1}(\hat{s}(t)/s(t)) \tag{5}$$

The envelope detection based on HT is sensitive to noise even for high SNR levels. In order to obtain a smooth envelope we apply a frequency domain low-pass filtering to the ultrasonic signal. The low-pass filter is designed based on the spectral characteristics of the measuring transducer. The cut-off frequency is set to the upper frequency limit of the transducer magnitude response. Low-pass filtering will provide a smooth envelope that is more useful for echo partitioning. The next step is to determine the local minima and maxima points of the smooth envelope. The objective is to find the peaks and valleys of the smooth envelope. The peaks and valleys, a subset of the local minima and maxima, are used to partition the echoes. The local maxima mark the approximate location in time (peak points) of discrete echoes, whereas the two local minima neighboring a local maxima mark the end points. The local minima and maxima are identified based upon the first and second derivatives of the smooth envelope. First derivative is used to find all local minima/maxima, and second derivative is used to separate local minima and maxima. The derivative operation is sensitive to noise, hence a smooth derivative operator is utilized to obtain first and second order derivatives. Some of the local minima and maxima are due to the small variations in the envelope and not necessarily mark the correct peaks and boundaries of echoes. The phase information is used to eliminate these spurious peaks and valleys. Phase-based constraints are enforced on the consecutive maxima/minima points based upon the time-domain characteristics of the transducer pulse. The phase difference between two consecutive local minima is required to be greater than a preset threshold value. Furthermore, the phase difference between two consecutive local maxima is required to be greater than a preset threshold value, and the phase difference between a consecutive minima followed by a maxima (and vice versa) is required to be greater than a preset threshold value. The first and second thresholds are chosen to be 2π so that there is at least a full-cosine cycle between two consecutive local minima or maxima. The third threshold is chosen to be a π such that there is at least half a cosine cycle between a maxima and minima, and vice versa. These thresholds can be adjusted based upon the time-domain characteristics of the transducer pulse used for measurement.

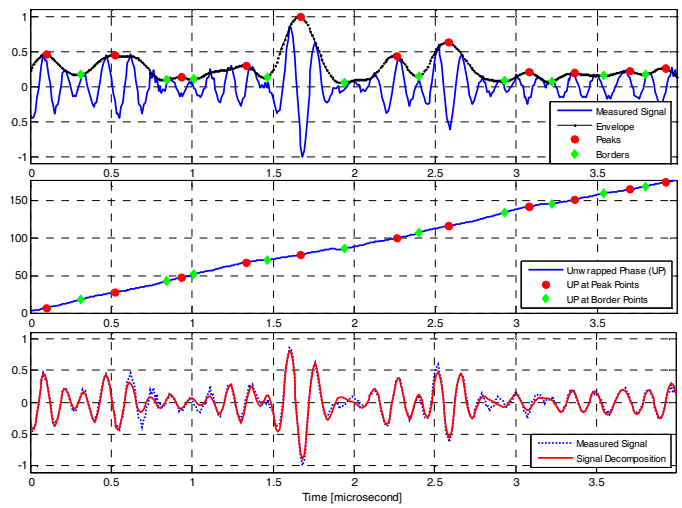

Figure 1. The illustration of ultrasonic echo partitioning and decomposition technique using backscattered echoes measured from a steel sample: a) The measured signal (solid blue line), smooth envelope (dotted dark line), and peak points (red disks) and border points (green diamonds), b) The unwrapped phase (solid blue line) and its values at the peak and border points, c) The measured signal (dotted blue line) and decomposition echoes (solid red line).

We demonstrate the aforementioned echo partitioning technique using an exemplary experimental ultrasonic data. Figure 1a displays the ultrasonic signal (solid blue line) obtained from a steel sample and the envelope of the signal (dotted dark line). The peak points are shown in red disks whereas the border points are shown in green diamonds. These points are obtained from the local minima and maxima of the envelope after enforcing phase based constraints. Figure 1b shows the unwrapped phase of the signal with the peak and border points marked. The phase differences between consecutive minima and maxima points are assessed based on the unwrapped phase. The partition for the most prominent echo is determined by finding the global peak point and the neighboring two border points. This partitioned echo is subject to model-based echo estimation. Then, the next prominent echo is windowed by identifying the second largest global peak and its neighboring borders after removal of the envelope of the first matched echo. This procedure will be repeated until all the peak points greater than a small amplitude threshold are utilized. Using this technique, Figure 1c displays the

decomposed echoes (thick red line) along with the measured signal (dotted blue line) based upon the partitioning defined by the peaks and border points above. The parameter estimation technique for the partitioned echo will be described in the next section.

III. Sparse Representation of Ultrasonic Echoes Using MP

In model-based ultrasonic signal decomposition techniques [1, 2, 4, 5], the generic procedure is to partition the most prominent echo from the ultrasonic echoes sequence, fit a model echo by parameter estimation, and subtract the estimated echo from the original signal. This generic procedure is illustrated in Figure 2.

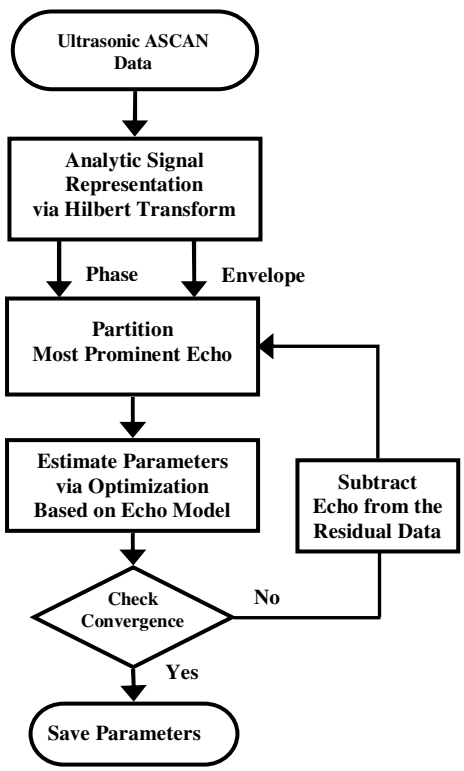

Figure 2. The flowchart of the signal decomposition algorithm

The first step of the algorithm, partitioning the most prominent echo from the ultrasonic data is described in the previous section. The second step of the algorithm, estimating the parameters of the model echo, can be accomplished using a variety of model-based echo estimation techniques [1, 2, 4]. In this study we adopt the maximum a posteriori (MAP) [10] parameter estimation technique presented in the reference [1] using the Gaussian echo model (i.e., real Gabor function):

$$g(\theta;t_k) = \beta e^{-\alpha(t_k-\tau)^2} \cos\{2\pi f_c(t_k-\tau)+\phi\} \qquad (6)$$

where $t_k = kT$ is the discrete time samples and T is the sampling interval, and $k = 0,1,2,\ldots,K-1$ is the time index. The parameters of the Gabor function, bandwidth factor, arrival time, center frequency, phase, and amplitude, are stored in this order in a parameter vector, $\theta=[\alpha \;\; \tau \;\; f_c \;\; \phi \;\; \beta]$. Assuming the partitioned echo is corrupted with white Gaussian noise, the MAP estimator for the parameter vector of this echo can be obtained by solving the following optimization problem

$$\hat{\theta} = \arg_\theta \min \left\| y_p - g(\theta) \right\|^2$$

$$\text{while } E[\theta] = \mu_\theta \text{ and } E[\theta\theta^T] = C_\theta \qquad (7)$$

where y_p represents the partitioned echo, and μ_θ and C_θ represent the a priori mean and covariance assumed for the parameter vector. The a priori means for bandwidth factor and center frequency parameters are obtained by fitting a Gabor function to the transducer pulse measured from a surface reflector. The a priori mean for phase is set to 0. The a priori means for arrival time and amplitude are set to the peak location and the peak value respectively, based upon the envelope of the partitioned echo. Note that these two a priori means are determined adaptively based upon the partitioned echo. The covariance matrix C_θ is chosen to be diagonal with each element indicating the prior variance for the respective parameter. The variance for each parameter is chosen in accordance with the expected variation in each parameter. These variances control the degree of change in the parameter value in the optimization process. Reference [1] provides more details for the MAP parameter estimation and outlines a fast Gauss-Newton (GN) algorithm to solve the optimization problem stated in Equation 7.

Following the flowchart presented in Figure 2, the estimated echo is subtracted from the original signal and its envelope is subtracted from the original envelope. Next step checks for convergence: if the residue energy is some fraction of the original signal energy, the algorithm stops, otherwise a new model echo is matched to the new partitioned echo. This energy ratio is by definition an inverse signal-to-noise ratio (SNR); hence the convergence threshold can be set according to the expected noise level (i.e., SNR) in the signal.

IV. Sparse Decomposition Results and Discussions

To demonstrate the above signal decomposition algorithm, we use ultrasonic data acquired from a steel block that contains a flaw (see Figure 3a) using a broadband transducer with a center frequency of 5 MHz. The sampling rate is 200 MHz. First, we determine the prior statistics on the bandwidth factor (α) and center frequency (f_c) parameters. The prior means for these parameters are set to 30 $(MHz)^2$ and 6.2 (MHz) respectively by fitting a Gabor function to the impulse response of the transducer. The bandwidth and center frequency parameters of decomposed functions are expected to vary around these values. The a priori variances for these parameters are set to $100\,(MHz)^2$ and $10\,MHz$ respectively to control variation from the prior means. Using these a priories, the decomposition signal is displayed in Figure 3a in solid red line along with the measured signal in dotted blue line. Furthermore, the TFR of the decomposed Gaussian echoes using the TF estimation technique described in [1] is shown in

Figure 3b. The TFR obtained via this type of decomposition can be useful for feature extraction. In particular, flaw detection in large-grained materials is challenging because the grain scattering echoes dominate the flaw echoes. In the Rayleigh scattering region, it has been shown that grain scattering results in an upward shift in the expected frequency of the ultrasonic signal [11]. On the other hand, flaw echoes exhibit a downward shift in their expected frequency due to the overall effect of attenuation because flaws are generally larger in size than the grain hence behave like geometrical reflectors. In summary, the flaw echoes exhibit high energy profiles at low frequency levels while grain echoes exhibit low energy profiles at high frequency levels [11]. This downward frequency shift of the flaw echo can be utilized as a discrimination tool. The high energy density circles at low frequency levels in the TF contour plot (see Figure 3b) represent the flaw echo. One can determine the exact location, frequency and energy of this echo by examining the parameters of the associated Gabor function.

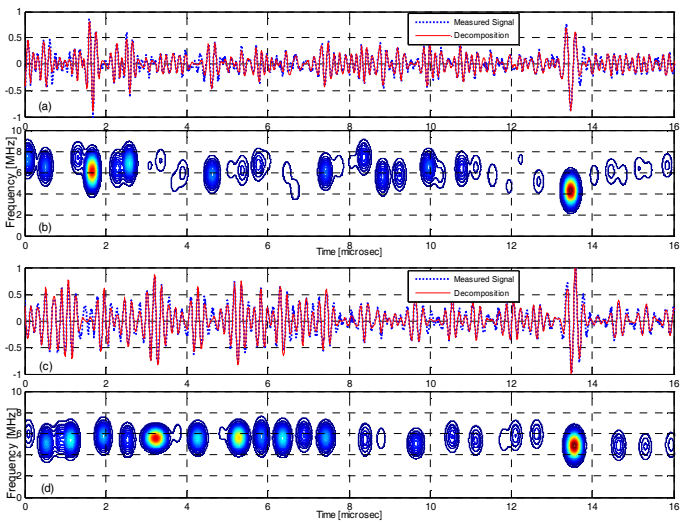

Figure 3. The backscattered echoes from the steel sample and flaw detection results: a) Measured echoes (dotted blue line) when transducer is positioned along the normal axis of the target and decomposition echoes (solid red line), b) TF contour plot of (a), c) Measured echoes (dotted blue line) and signal decomposition (solid red line) when the transducer is moved away slightly from the normal axis, d) TF contour plot of (c).

Another measurement after amplitude normalization is shown in Figure 3c. This data is captured using the same experimental setup and transducer as the experimental data in Figure 3a with the exception that the transducer is moved away slightly from the targets normal line to de-emphasize flaw echo with respect to the grain echoes. The signal decomposition (solid red line) and its TFR for this experimental data are shown in Figure 3c and 3d respectively. The flaw echo still exhibits high-energy at lower frequency levels than grain echoes hence is detectable in the presence of heavy clutter.

One desirable property of this decomposition technique is its ability to extract echoes in a hierarchical order. The order of estimation progresses from high-energy echoes to low-energy echoes. This hierarchical order is illustrated in Figure 4 where the residual energy decreases monotonically after each MP iteration (i.e., echo estimation and subtraction) for the ultrasonic data shown in Figure 3a. This property is useful in extracting the prominent energy echoes from the ultrasonic data.

The signal decomposition algorithm has also been tested for the analysis of reverberation echoes acquired from a multi-layered test specimen [9]. The test specimen is prepared by placing an aluminum plate and a steel block in water, where there is a gap (water) between the plate and the block. The echoes are captured with a 10 MHz nominal frequency transducer and sampled with 100 MHz frequency. These echoes are shown in Figure 5a along with the envelope, and peak and border points marked.

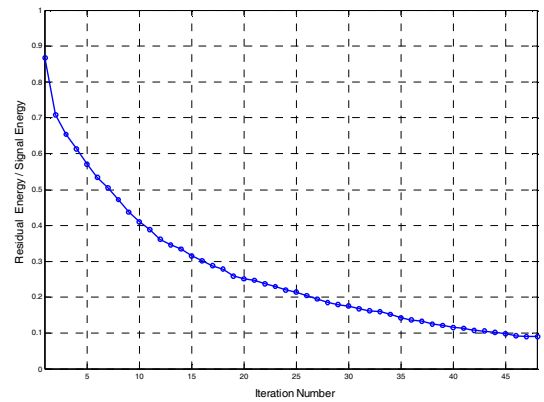

Figure 4. The normalized residual energy after each MP iteration (i.e., echo matching) for the backscattered echoes shown in Figure 3a.

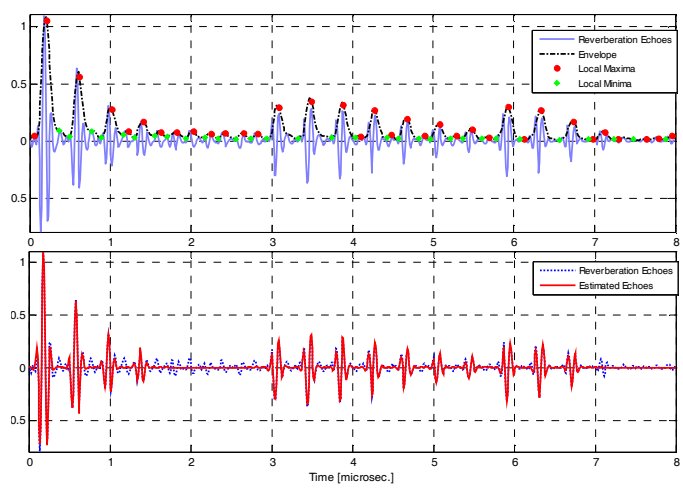

Figure 5. The illustration of ultrasonic signal partitioning and decomposition technique using reverberation echoes measured from a three-layer specimen: a) The reverberation echoes (solid blue line), smoothed envelope (dotted dark line), peak points (red disks) and border points (green diamonds), b) The decomposed echoes (red-thick line) and measured reverberation echoes (dotted-blue line).

These echoes can be classified according to the reverberation model [12] to identify which echoes are due to a specific layer. Then the parameters of the classified echoes are used to estimate the thickness and transmission coefficient of the respective layer [12]. The reconstructed echoes are shown in Figure 5b. The arrival times and amplitudes of the recovered echoes can be used to classify the layers in the specimen and measure the thicknesses of the respective layers [9]. The estimated parameters of these reverberation echoes are listed in

Table 1. The relationship between arrival-time and amplitude parameters with the thickness and reflection/transmission coefficients of the layer has been established in reference [12]. In this study we demonstrate that these two important parameters can be estimated with high accuracy and consistency using the proposed signal decomposition technique.

Table 1. The estimated parameters of Gabor functions for reverberation echoes shown in Figure 5a.

Echo Num	Bandwidth Factor $[MHz^2]$	Arrival Time $[\mu s]$	Center Freq. $[MHz]$	Amplitude
1	239.9753	0.1830	11.1472	1.1487
2	239.9848	0.5842	11.2334	0.6298
3	239.9598	0.9855	11.9584	0.3137
4	239.9536	1.3821	12.2484	0.1850
5	239.9836	3.0603	10.9911	0.3282
6	239.9947	3.4610	11.0829	0.3896
7	240.0007	3.8603	11.4130	0.3516
8	239.9949	4.2579	11.6332	0.2861
9	239.9792	4.6570	11.6921	0.2119
10	239.9862	5.0579	11.8083	0.1566
11	239.9847	5.4593	11.8743	0.1109
12	239.9944	5.9152	11.6199	0.3208
13	239.9939	6.3173	11.7019	0.2910
14	239.9923	6.7173	11.9208	0.1816

Finally, the computational complexity of the proposed decomposition technique is very low. For example, the first experimental data (Figure 3a) presented above (containing 2048 samples, ($N = 2048$) takes 48 MP iterations for decomposition, and each MP iteration takes about 20 GN iterations. On a dual-core Pentium PC processor with 1.66 GHz clock-rate, the MATLAB implementation of the algorithm takes about 0.6 seconds to process this data including the pre-processing (envelope detection and filtering) operations. The GN algorithm presented in reference [1] is rather fast. The major computational load for one GN iteration is 5 inner products in a reduced dimension M (average partitioned echo length) which is much smaller than the data length N. On average, one echo estimation requires $100xM$ inner products. This is significantly smaller than the number of inner products ($N \log_2 N = 22528$) the dictionary-based MP algorithm [7] would require for one MP iteration. Note that this computation time (0.6s) can be reduced drastically with an efficient implementation of the algorithm. Overall, this low computational complexity offers a very practical and efficient solution for real time ultrasonic testing.

V. Conclusions

The echo windowing techniques utilized for ultrasonic signal analysis are often application specific. For example, TFRs of ultrasonic data via wavelet transform and chirplet transform have been used to localize echoes in TF plane. Even though these techniques perform well in localization, the windowing technique is inherently dependent on the assumed echo model. Furthermore, obtaining a TFR of the data is computationally expensive and requires repetitive updates, i.e., calculating the TF of the estimated echo and subtracting its contribution from that of the original data. In this study we proposed an echo windowing and estimation technique that eliminates these redundant computations and offers a more efficient and practical solution for real-time ultrasonic signal analysis. The proposed algorithm is computationally efficient in orders of magnitude compared with the TF based-based MP algorithms.

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