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A GENERIC PARAMETRIC MODEL FOR ULTRASONIC SIGNAL ANALYSIS

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***Abstract*- Parametric echo modeling and estimation approach has been widely used in recent years for ultrasonic echo analysis and assessment of NDE test parameters. Parametric models such as Gaussian echo, Gaussian Chirplet have been utilized in the context of model-based echo estimation and sparse signal decomposition. These models are mathematically tractable, convenient for numerical calculations, and have explicit Time-Frequency representations. As a result they have been used extensively in acoustic signal processing. However, most often these models are not as flexible to represent complex shape ultrasonic echoes. A generic echo model that can be optimized to represent complex shape echoes is highly desirable, for example, for modeling frequency dependent attenuation and dispersion effects in a propagation path, deconvolution of reflector echoes in presence of pulse variance. In this study, we propose a generic echo model to characterize complex shape ultrasonic echoes. This echo model is inspired from the analytic signal representation in which an ultrasonic echo is contemplated in terms of an envelope and sinusoidal components. The echo envelope is modeled as a sum of a number of fixed-width Gaussian Functions (GFs) whereas the echo sinusoidal is modeled as a linear chirp signal. The number and bandwidth of GFs are set based upon the spectral characteristics of the transducer impulse response. This Gaussian mixture representation accounts for skewed shape envelopes while the linear chirp accounts for small frequency drifts in the sinusoidal. An optimization algorithm is developed to estimate the generic model parameters. The performance of parameter estimation is verified using pulse-echo wavelets acquired from several different transducers and measurement conditions. Finally, this echo model is incorporated into a superimposed echoes estimation algorithm. Estimation results clearly demonstrate the advantage of this model compared to the existing alternatives. The potential applications of this type of model are numerous: ultrasonic deconvolution in presence of pulse variance, resolution of complex overlapping pulses, and NDE parameter estimation**.

I. INTRODUCTION

Parametric modeling and estimation of ultrasonic backscattered echoes have been introduced in [1] and become a frequently used approach in NDE signal processing. Compared to classical transform based ultrasonic signal processing methods, the parametric echo representation approach offers significant advantages such as high-resolution estimation of test parameters (e.g., time-of-arrival, center frequency, and energy), the ability to resolve closely spaced overlapping echoes, and robustness with noise. Parametric modeling concept has been widely accepted in ultrasonic NDE studies, for example, thickness sizing of thin layers [2], sizing of cracks in thin sections [3], the assessment of the bone structural parameters [4], and structural health monitoring of materials [5]. Most of these studies utilized the Gaussian echo model (i.e., real Gabor function) to represent discrete backscattered echoes. Gaussian echo model is essentially a Gaussian modulated sinusoid with five parameters (time-of-arrival, center frequency, bandwidth, phase, and amplitude) and provides a good approximation to a backscattered echo from a point or surface reflector. This approximation is not always adequate especially for cases in which accurate estimation of the pulse shape is critical as in ultrasonic deconvolution. For example, in thickness sizing of thin layers, the received echoes are often overlapping and resolution of these echoes relies on accurate estimation of the pulse-echo wavelet. To represent complex shape echoes, the composite Gaussian echoes model has been introduced [2]. This model involves estimating the transducer pulse in terms of a number of overlapping Gaussian echoes. Therefore, the composite model parameters do not have a direct interpretation for echo signal parameters.

Despite the shortcomings of Gaussian echo model in representing discrete echoes, it has been extensively used in sparse signal decomposition algorithms [2, 5, 7]. In fact, long before it is used as a parametric model in ultrasonic signal processing it had been used as an elementary signal to generate Gabor dictionary functions in Mallat's generic sparse signal decomposition method, i.e., the well-known Matching Pursuit (MP) algorithm [9]. The Gabor dictionary has been frequently used for ultrasonic signal decomposition because Gabor functions exhibit high correlation with ultrasound echoes. Later, the model-based MP method [6] is introduced to allow more flexibility in sparse decomposition by optimizing the parameters of the Gaussian echo model. In short, this model has proved to be useful in sparse decomposition of ultrasonic echoes. Later, the Gaussian echo model has been generalized to the Gaussian Chirplet with the addition of a chirp parameter in the sinusoid to allow a linear drift in frequency. This type of model has been used in [8] for a generic decomposition of an acoustic signal in terms of Gaussian Chirplets. Although this model is better than the Gaussian echo model in representing dispersive echoes, its envelope is still a Gaussian shape and symmetric with respect to its peak location. The Gaussian envelope models are insufficient to represent echoes with a non-Gaussian shape, i.e., echoes with non-symmetric and irregular envelopes.

In this study we introduce a new parametric echo model that has a flexible and smooth envelope. This echo model is inspired from the analytic signal representation in which an ultrasonic echo is represented as a product of envelope and sinusoidal components. In analytic signal representation, the envelope is positive and low-pass. To represent such an envelope, a mixture model composed of a number of fixed-width and equally spaced Gaussian Functions (GFs) are used. The number and bandwidth (or time-width) of GFs are set based upon the spectral characteristics (nominal center frequency and bandwidth) of the transducer impulse response. Furthermore, the weights of GFs are constrained to be larger than a small noise threshold so that the echo envelope is always positive and smooth. This Gaussian mixture representation accounts for skewed shape envelopes while the linear chirp sinusoidal accounts for small frequency drifts in the time-support of the echo. This model can also represent the existing Gaussian echo and Gaussian Chirplet models.

The rest of the paper is organized as follows. The next section introduces the new generic model and its properties, and echo parameters related to NDE testing. Section III presents an optimization algorithm to estimate the parameters of this model in the presence of white Gaussian noise. This section also presents results regarding the parameter estimation of echoes obtained from several different transducers and measurement conditions. Section IV discusses the incorporation of this model into a superimposed signal estimation algorithm to demonstrate the estimation of a number of discrete echoes, some of which maybe overlapping.

II. Generic Echo Model

We represent a discrete ultrasonic echo (e.g., a backscattering echo from a planar surface reflector, or a point reflector) in terms of an envelope and a chirp sinusoidal component:

$$g(t) = env(t-\tau)\cos\{2\pi f_c(t-\tau) + \psi(t-\tau)^2 + \phi\} \quad (1)$$

where $env(t)$ is the envelope, τ is the reference location (e.g., peak point) of the model echo, f_c is the center frequency, ψ is the linear chirp rate, and ϕ is the phase of the signal. The envelope is a finite duration low-pass signal and modulates the sinusoidal term. This envelope essentially constitutes the echo location, bandwidth, amplitude and hence its energy. The envelope of an echo represented in Equation 1 can be obtained via Hilbert Transform:

$$env(t) = \sqrt{g^2(t) + \hat{g}^2(t)} \quad (2)$$

where $\hat{g}(t)$ denotes the Hilbert Transform of the signal $g(t)$. For Gaussian echo and Gaussian Chirplet models [1, 8], this envelope term is assumed to be a Gaussian Function (GF):

$$env(t-\tau) = \beta e^{-\alpha(t-\tau)^2} \qquad t \in [0 \;\; T_s] \quad (3)$$

where β is the amplitude and α is bandwidth factor controlling the echo-bandwidth, and τ is the echo time-of-arrival marking the peak location of the GF. The Gaussian envelope model has desirable mathematical properties (n-th order integrable and differentiable) and is convenient for numerical calculations. The time-of-arrival of this envelope is explicitly defined with one parameter τ. However, this model restricts the envelope to be symmetric and monotonically rising and decaying before and after the time-of-arrival. Most often, this envelope model is insufficient to represent echo envelopes that are often sharply rising before the peak and slowly decaying after. Therefore we introduce the following envelope model composed of linear combination of fixed-width and equally-spaced GFs:

$$env(t-\tau_c) = \sum_{k=-K}^{K} w_k e^{-\lambda(t-k\Delta T-\tau_c)^2} \qquad t \in [0 \;\; T_s] \quad (4)$$

where w_k denotes the weight (amplitude) of the GF positioned at $k\Delta T$, λ is the parameter controlling the time-width of a GF, and ΔT denotes the spacing between two neighboring GFs, and $2K+1$ is the number of GFs in the mixture. The parameters λ, ΔT, and K can be set a priori according to spectral characteristics of the measuring ultrasonic transducer. The determination of these parameters will be addressed later using an exemplary echo. The unknown parameters of the model are w_k, $k = -K,..,0,..K$, the weights of the GFs, and τ_c, the time-center of all the GFs. Note that this time center is not necessarily same as the echo arrival time. The definition of echo arrival time is not unique for an echo with an asymmetric envelope; one can use different reference points such as the echo peak location, echo start location, or centeroid location (i.e., center of gravity of the envelope). In this paper we use the peak point of the envelope as the reference point. Before we address parameter estimation, we note that echo signal parameters (time-of-arrival, bandwidth, energy) can be calculated in terms of the composite GFs model parameters and weights. The energy of the generic echo model can be calculated as the energy of the envelope term since the energy of the sinusoidal is unity. The energy of envelope will depend on GF weights, decay rate (λ), spacing (ΔT), and *(M)*, the number of components. The energy of the echo can also be numerically calculated as the sum of squared sampled envelope values. The bandwidth of the echo is determined from the bandwidth of the envelope which can be calculated using the FFT magnitude spectrum. Reference [1] outlines a procedure to estimate the energy and bandwidth of an echo from its magnitude spectrum.

III. Parameter Estimation of Model Parameters

3.1 Estimation of GFs Weights

Our objective is to estimate weights w_k and center point τ_c of the GFs mixture. Assuming the center location τ_c is known, one can estimate the weights using the Least Squares (LS) estimation criterion. The GFs mixture (Equation 4) can be written in a matrix-vector multiplication form:

$$env = E_B . w \quad (5)$$

where *env* is an N x 1 vector with real-positive numbers obtained by sampling the echo envelope, *w* is an M x 1 vector of GF weights ($M = 2K+1$), and E_B is an N x M matrix containing the sampled Gaussian basis functions. The above system can be solved by optimizing the weights vector ***w*** to minimize the LSE between the measured and GFs mixture envelope:

$$\hat{w} = \arg_w \min \|env - E_b w\|^2 \quad (6)$$

The solution of the above minimization is a LS solution:

$$\hat{w} = (E_b{}^t E_b)^{-1} E_b{}^t env \quad (7)$$

This estimation frequently leads to weights w_k that are smaller than zero, or very close to zero which may cause fitting GFs to noise. Since the echo envelope is positive, $env \ge \overline{0}$, the weights of the composing GFs can be restricted to be positive. This will ensure the GFs mixture model to be additive and enable the addition of individual GF energies to obtain the echo energy. Therefore, we propose to use the following constrained LS estimation criterion:

$$\hat{w} = \arg_W \min \left\| env - E_b w \right\|^2 \quad \text{subject to} \quad w \geq b \tag{8}$$

where b denotes a vector with prescribed thresholds. All the elements of this vector can be set to a small positive number ($\delta > 0$) to place a minimum acceptable threshold for any GF weight w_k. This parameter δ serves as a noise control threshold and prevents fitting GFs to noise. The above constrained estimation problem can be solved using a non-negative least squares (NNLS) estimation algorithm.

We illustrate the envelope estimation using an ultrasonic echo obtained from a planar surface reflector in water. The reflector is placed from 4.5 cm perpendicular to a broadband transducer with a 5MHz center frequency and 0.375 MHz nominal bandwidth. The pulse is sampled at 100 MHz. This measured echo is shown in Figure 1a. The envelope of this echo obtained from an envelope detector. To represent this envelope, we use a GFs basis shown in Figure 1b. The time-width, spacing and number of GFs are chosen based on the nominal center frequency and bandwidth (or maximum expected timewidth) of the echo. The parameter λ is adjusted to make the time-width of a GF large enough to cover one period of the sinusoid (0.4 μs). The spacing ΔT is set to 0.1 μs so that the addition of neighboring GF's gives raise to a convex and smooth curve. The number of GFs is set to 13 so that this basis comfortably covers the maximum possible echo duration (1.5 μs). Using this basis, the estimated envelope is shown in Figure 1c with a solid line along with the original envelope curve in a dotted line. The estimated GFs weights using the procedure above are also shown in Figure 1c as a bar graph. The estimated envelope is a linear combination of GFs whose weights are shown in the bar graph.

3.2 Estimation of Arrival Time, Center Frequency, Chirp Rate, and Phase

Assuming the envelope is known we now describe a procedure to estimate the time-of-arrival and sinusoidal parameters of the generic echo model presented in Equation 1. An observed echo with measurement noise can be written as

$$y = g(\theta) + \upsilon \tag{9}$$

where $g(\theta)$ is the discrete version of the generic model in (1):

$$g(\theta) = env(t_k - \tau)\cos\{2\pi f_c (t_k - \tau) + \psi (t_k - \tau)^2 + \phi\} \tag{10}$$

where $t_k = kT$ is the discrete time samples and T is the sampling interval. The parameters of the model, arrival time, center frequency, chirp rate, and phase are stored in a parameter vector $\theta = [\tau \quad f_c \quad \psi \quad \phi]$. Assuming the noise υ is white Gaussian (WGN), the Maximum Likelihood Estimator (MLE) for the parameter vector can be obtained by solving:

$$\hat{\theta} = \arg_\theta \min \left\| y - g(\theta) \right\|^2 \tag{11}$$

where y represents the discrete measured echo. This non-linear optimization problem can be solved using a Gauss-Newton (GN) algorithm. The GN algorithm presented to estimate Gaussian echo parameters in [1] can be customized to estimate the parameters of the generic model.

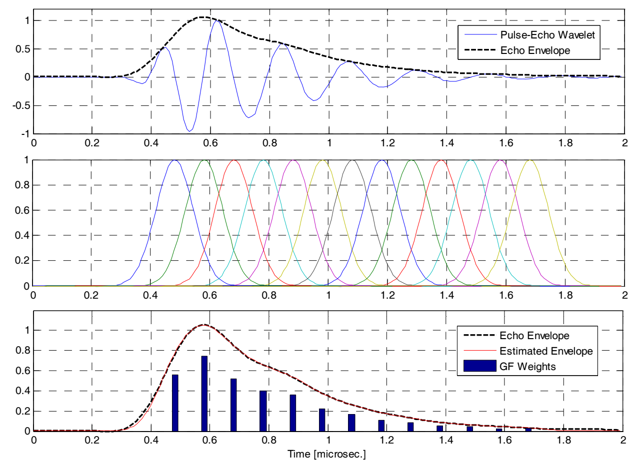

Figure 1. The illustration of ultrasonic echo envelope modeling and estimation using GFs mixture model. a. The measured echo from a broadband transducer and its envelope obtained via envelope detection, b. The basis functions of the GFs mixture, c. The estimated envelope (solid red) along with detected envelope (dotted line), and the estimated weights of the GFs in (b).

Using the estimation procedure above, the parameters of the generic echo is optimized to represent pulse-echo wavelets obtained from surface reflectors. Figure 2 shows 3 different echo measurements and estimations. Figure 2a shows a measured echo (dotted line) from a planar front surface reflector in water using a 5 MHz broadband transducer. The envelope of this echo (see Figure 1a) is of skewed shape and tapers off slowly. The generic model represents this envelope with high accuracy (see Figure 1c) and the sinusoidal chirp tracks the frequency drift noticeable in the echo (Figure 2a). Figure 2b shows another measured echo (dotted line) from a back-surface reflector using the same transducer as in Figure 1a. Figure 2c shows another front-surface echo measured using a 10 MHz broadband transducer. The generic echo model nicely fits the measured echoes. Note that the same GFs basis displayed in Figure 1b is used for all these measurements. One can use this basis for transducers whose center frequency is in the range of 5-10 MHz.

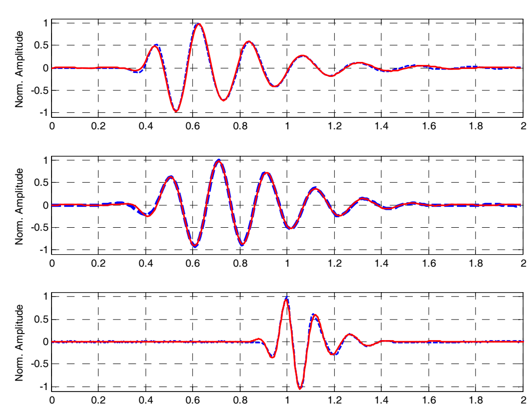

Figure 2. The illustration of generic echo modeling and estimation: a. A measured echo (dotted blue) from a flat surface reflector in water and its estimate (solid red) using a 5 MHz transducer, b. A back surface reflector echo (dotted blue) for the same transducer used in (a) and its estimate (solid red), c. Another flat surface reflector echo (dotted blue) and its estimate (solid red) using a 10 MHz transducer.

IV. Parameter Estimation of Superimposed Echoes

We introduce the superimposed generic echoes model to represent ultrasonic measurements containing a number of echoes, some of which maybe overlapping. It is assumed that the number of echoes is known a priori. The discrete version of such a model containing M-superimposed echoes in noise:

$$y = \sum_{m=1}^{M} \beta_m g(\theta_m) + \nu \qquad (12)$$

where $g(.)$ denotes the generic echo model with parameter vector θ_m and ν denotes measurement noise. In order to represent the measurement in terms of generic echoes, one needs to estimate the parameter vectors $\theta_1, \theta_2, \ldots, \theta_M$ given the noisy observation of echoes in y, some of which may be overlapping. This type of representation is well studied in terms of the Gaussian echo model [1]. The Space Alternating Expectation Maximization (SAGE) algorithm has been successfully used to estimate model parameters. The algorithm is also applicable to the generic model. In this study, the SAGE algorithm has been tested on simulated echoes using a real pulse-echo wavelet. A simulated signal is shown in Figure 3a using the transducer pulse in Figure 2a. Figure 3b shows estimated echoes. This type of algorithm is especially useful for resolving overlapping pulses of complex shape. Note that resolution of Gaussian echoes is extensively studied in [1]. Here the model is extended to cover overlapping echoes with free-form envelopes and linear chirp sinusoidals. It is important to note that in the superimposed echoes case, the generic model assumes a free-form but fixed envelope. The envelope of the model is assumed to be known priori. This restriction can be relaxed for non-overlapping echoes; however it is difficult to resolve the individual echo envelopes from the overlapping signals without assuming any parametric form. One needs to make certain assumptions for the envelope shape. This can be done, for example, by assigning prior statistics to the GFs basis functions to form a certain shape envelope and estimate GF weights. This will be a further investigation for this study.

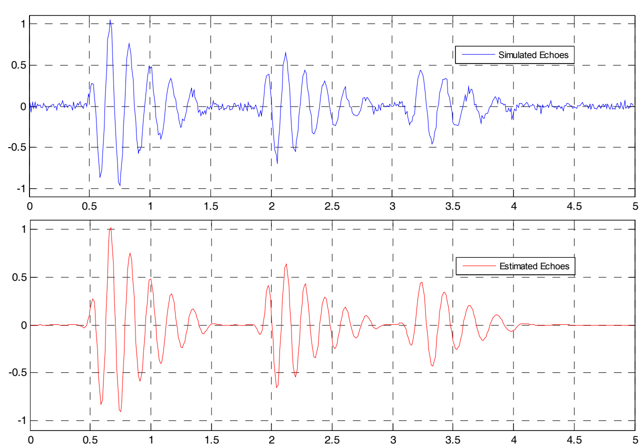

Figure 3. a. Three superimposed echoes simulated using the transducer pulse in Figure 2a (SNR is about 10 dB), b. The estimated echoes using generic echo model and SAGE algorithm.

V. Conclusions

In this study we introduced a new parametric echo model that has a generic shape envelope and linear chirp sinusoidal. This echo model is designed to represent any shape discrete echo (e.g., a backscattered signal from a planar surface reflector). Then, we developed an algorithm to estimate the envelope and sinusoidal parameters of this model. The envelope is modeled as a mixture of a GFs basis whose basis functions are designed based upon the spectral characteristics of the measuring transducer. The sinusoidal term is modeled as a liner chirp. The model is tested using echo measurements obtained from different transducers and measurement conditions. It has been shown that the proposed model can represent diverse shape echoes with high fidelity. Then the superimposed generic echoes model is introduced to deal with ultrasonic measurements involving multiple echoes. The previously used SAGE algorithm has been used for generic echoes model. The algorithm has been tested using simulated signals containing non-overlapping echoes. It has been demonstrated that it is possible to estimate superimposed echoes of arbitrary shapes. In summary this model generalizes the existing Gaussian echo and Chirplet models, offers a greater flexibility in superimposed echoes estimation, and likely to improve the performance of model-based ultrasonic echo estimation algorithms.

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