10.1109/ULTSYM.2010.0511

Chirplet Parameter Estimator Based on Ellipse Fitting in Time-Frequency Distributions for Ultrasonic NDE Applications

Alireza Kasaeifard, Jafar Saniie and Erdal Oruklu

Department of Electrical and Computer Engineering
Illinois Institute of Technology
Chicago, Illinois 60616

***Abstract*—Estimating the parameters of chirp signals is essential in several important applications such as radar, sonar and ultrasonic imaging. Chirp pulses characterize dispersive media, and they are also very effective for pulse compression and improved echo detection. Furthermore, in Doppler ultrasound velocimetry, frequency-change associated with moving object can be characterized by chirp rate estimation. Time-Frequency Distributions (TFDs) of signals with chirplet components result in elliptical contours. In this study, a chirplet parameter estimator by means of ellipse fitting in time-frequency plane is developed. This estimator performs robustly in presence of the noise and offer much lower computation complexity compared to conventional methods.**

I. INTRODUCTION

Chirplets are Gaussian-envelope functions with second order polynomial phase that are encountered in many applications such as radar [1], sonar, nondestructive evaluations [2] and medical applications [3-4]. Estimated chirp-signal parameters can be used for feature extraction, studying physical properties of the objects and system identification. Chirplet parameter estimation can be achieved via Matching Pursuit (MP) methods to choose parameters from a dictionary of functions in which the parameters are discretized [5-6]. In addition, MP method can be modified for chirped Gabor functions by applying fractional Fourier transform (FRFT) resulting in a rotation of Wigner distribution [7]. Maximum Likelihood Estimation (MLE) can be used as an alternative method for parameter estimation. [8-9]. Moreover, chirp echo parameters can be estimated based on the Time-Frequency Representation (TFR) of the signal in a successive manner [10]. In many studies, the focus has been on signal decomposition. On the other hand, lowering the computational complexity for single-component chirp parameter estimation can be found in the literature as well [1-11]. Mann and Haykin [12] have used frequency shearing as an illustration for chirp component rotation in TF plane. Bultan [7] employed FRFT operator for representing chirp atoms rotation. Modification of a mother chirplet as a result of chirping in time has been depicted pictorially in [13].

Although quadratic TFRs like Wigner-Ville distribution offer appropriate spectral and temporal resolution, they suffer from cross-terms. On the other hand, cross-terms can be avoided by using linear TFRs like Short Time Fourier Transform (STFT) at the cost of degrading TF resolution. Needless to say that maximum likelihood estimator requires excessive computations.

In case of using chirplet excitation in NDE and flow measurement applications, the reflected echo will also be a transformed chirplet governed by the dispersive property of materials or the velocity of scatterers within the propagation path. The Wigner-Ville Distribution (WVD) of a chirplet signal will be in the form of concentric ellipses in the Time-Frequency (TF) plane. In this study, we utilize Time Frequency Distribution (TFD) and ellipse fitting approach to estimate chirplet signal parameters. Furthermore, to reduce the effect of noise in measurement and to improve estimation accuracy, we use 2D Gaussian smoothing filter prior to ellipse fitting. The parameters of individual chirplet echoes are obtained by inspecting the TFDs of signal in two major steps. In the first step, two main parameters, i.e. center frequency and time of arrival are estimated by means of short time Fourier transform. In the second step, after applying Gaussian filtering, ellipses are fitted to the TF atoms in the Wigner-Ville distribution TF plane in order to estimate chirp rate, bandwidth factor and amplitude.

II. CHIRP SIGNAL REPRESENTATION IN TIME DOMAIN AND TIME-FREQUENCY PLANE

The model for a single chirp echo is:

$$f_\Theta(t) = \beta \exp[-\alpha_1 (t-\tau)^2 + j2\pi f_c (t-\tau) + j\alpha_2 (t-\tau)^2] \qquad (1)$$

where $\Theta = [\alpha_1, \alpha_2, \beta, f_c, \tau]$ is the index of the chirp echo, τ is the time of arrival, f_c is the center frequency, α_1 is the bandwidth factor, α_2 is the chirp-rate and β is the amplitude.

Elliptical shape can be traced in the contour levels in the TF plane even in the presence of noise. Since we employ an ellipse fitting method to estimate chirp rate, bandwidth factor and amplitude, STFT no longer guarantees an appropriate TF resolution. To solve this problem, WVD is used.

In order to have a normalized signal that has unit energy, the term $\sqrt[4]{2\alpha_1/\pi}$ is used as amplitude. Considering a Gaussian chirplet:

$$s(t)=\sqrt[4]{2\alpha_1/\pi}\exp[-\alpha_1(t-\tau)^2+j2\pi f_c(t-\tau)+j\alpha_2(t-\tau)^2] \quad (2)$$

The Wigner-Ville distribution of the chirplet is [18-19]:

$$WVD_s(t,f)=2\exp-\{2\alpha_1(t-\tau)^2+[2\pi(f-f_c)-2\alpha_2(t-\tau)]^2/2\alpha_2\} \quad (3)$$

The WVD of that chirplet signal is in the form of concentric ellipses that their slope, center, major and minor axis correspond to the parameters Θ (see Figure 1).

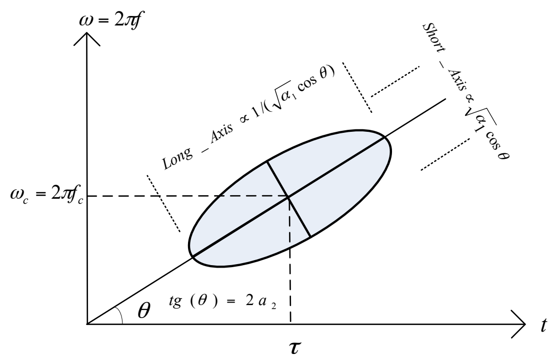

Figure 1. Elliptical shape in WVD of a chirplet signal

The goal is obtaining chirp rate, bandwidth factor and amplitude based on the elements of the ellipses that can be seen in the TF plane (see Figure 1). Slope of major axis of the ellipse represents the chirp rate i.e. $\tan(\theta)=2\alpha_2$. For the ease of obtaining bandwidth factor and amplitude, consider that the elliptical contour corresponds to the level of e^{-2}, f_c=0 and $\tau=0$, where the center of the ellipse coincides with the TF coordinate origin (see Figure 2).

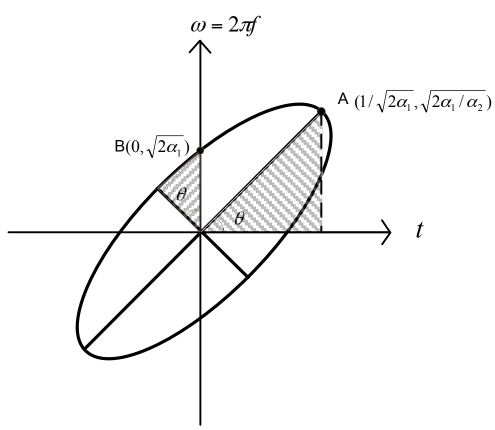

Figure 2. WVD of a chirplet signal concentric with TF coordinate origin

Based on the WVD distribution in equation 3, the position of two points on the ellipse (i.e. A and B in Figure 2) are A $(1/\sqrt{2\alpha_1},\sqrt{2\alpha_1/\alpha_2})$ and B $(0,\sqrt{2\alpha_1})$.

By obtaining hypotenuse in two right triangles marked in gray shade in Figure 2:

$$\text{Long\_Axis/2}=1/(\sqrt{2\alpha_1}Cos(\theta)) \quad (4)$$

$$\text{Short\_Axis/2}=\sqrt{2\alpha_1}Cos(\theta)) \quad (5)$$

Therefore:

$$\alpha_1=Short\_Axis/(Long\_Axis\cdot\cos^2(\theta)) \quad (6)$$

It should be noted that the value for α_1 is independent of the elliptic contour's level.

For obtaining amplitude, consider

$$s(t)=Amp.\sqrt[4]{2\alpha_1/\pi}\exp[-\alpha_1(t-\tau)^2+j2\pi f_c(t-\tau)+j\alpha_2(t-\tau^2)] \quad (7)$$

For the elliptical contour that corresponds to the level□:

$$\square=2Amp^2\exp[-2\alpha_1(Long\_Axis\cdot Cos(\theta))^2] \quad (8)$$

Therefore:

$$Amp=\sqrt{\square/2\exp[-2\alpha_1(Long\_Axis\cdot Cos(\theta))^2]} \quad (9)$$

III. THE PARAMETER ESTIMATION PROCEDURE

In the following, the steps of proposed method for estimating the chirplet parameters are described. First, the STFT of the signal is evaluated. Since STFT is not prone to cross-terms, for a sparse signal, maxima points in the spectrogram of the signal indicate the number of chirplet components. Also two main parameters i.e. τ and f_c for each component are obtained from maxima locations in the TF plane of the signal. In spite of the fact that choosing window size in STFT determines how one can select between an acceptable frequency resolution and time resolution, obtaining the maxima locations in STFT plane does not rely on window size.

WVD of the signal is evaluated as the second step. Existence of cross-terms adds some ambiguity for obtaining the proper contour lines. One can overcome this ambiguity by considering just the contour lines encircling the maxima points found in the previous step. Considering concentric contour line corresponds to each maxima, the farther from the centers, more prone to the noise. For the SNRs 1 dB and more, the contour levels down to 80% of their peak value or more offer satisfactory estimation results.

Noise has the effect of deformation on the elliptic contour lines in TF plane. A two dimensional Gaussian smoothing approach is used to lessen contour noise. The approach proves to be effective to reduce the noise (see Figures 3c and 3d). To avoid excessive computation, filtering is performed for the area containing deformed contour lines.

Among existing conics fitting approaches, direct least squares ellipse-fitting method satisfies our objective of chirplet parameters estimation [14]. Since the contour lines corresponding to the chirplet components in TF plane are in elliptical shape, the proposed elliptic-specific approach shows satisfactory performance. A general conic curve is generally represented by:

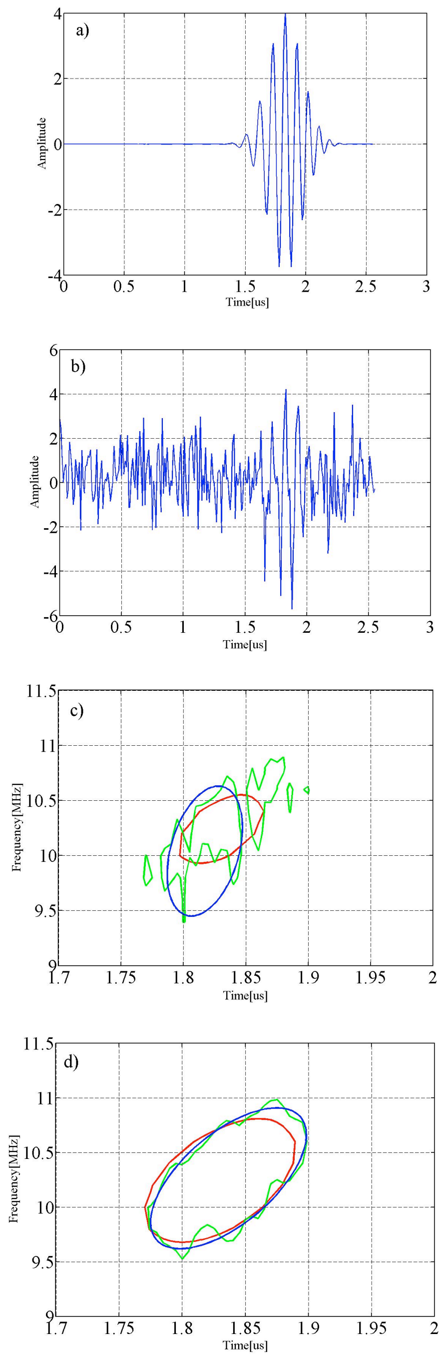

Fig. 3. (a) chirplet signal (b) Noisy echo (c) Results of ellipse fitting before Gaussian filtering (d) after Gaussian filtering. Red ellipses shows the undeformed contours not suffering from noise, green curve in (c) shows the deformed contours suffering from noise and in (d) shows the contour after smoothing. Blue ellipses are the results of ellipse fitting in two cases.

$$\mathrm{F}(\boldsymbol{\gamma},\mathbf{z}) = \boldsymbol{\gamma}.\mathbf{z} = ax^2 + bxy + cy^2 + dx + ey + f = 0 \tag{11}$$

Where $\boldsymbol{\gamma} = [a\ b\ c\ d\ e\ f]^T$ and $\mathbf{z} = [x^2\ xy\ y^2\ x\ y\ 1]^T$. $\mathrm{F}(\boldsymbol{\gamma},\mathbf{z})$ is considered as algebraic distance of a point (x_i, y_i) to the curve $\mathrm{F}(\boldsymbol{\gamma},\mathbf{z}) = 0$. Best fitting to the N data points is achieved by minimizing the sum of squared algebraic distance:

$$\sum_{i=1}^{N} [\mathrm{F}(\boldsymbol{\gamma},\mathbf{z})]^2 \tag{12}$$

Where $\boldsymbol{\gamma}$ is constrained to avoid the trivial solution $\boldsymbol{\gamma} = 0$. The constraint applied [14] in (12) is $4ac - b^2 = 1$ which can be expressed in the form $\boldsymbol{\gamma}^T \mathbf{C} \boldsymbol{\gamma} = 1$ where $\mathbf{C}$ is:

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

The minimization can be solved by considering eigenvalue system [15]:

$$(\mathbf{D}^T\mathbf{D} - \lambda\mathbf{C})\boldsymbol{\gamma} = 0 \tag{14}$$

Where

$$\mathbf{D} = \begin{bmatrix} x_1^2 & x_1 y_1 & y_1^2 & x_1 & y_1 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ x_n^2 & x_n y_n & y_n^2 & x_n & y_n & 1 \end{bmatrix} \tag{15}$$

After obtaining the eigenvalues λs by solving, $\det(\mathbf{D}^T\mathbf{D} - \lambda\mathbf{C}) = 0$ we can readily calculate the parameter vector $\boldsymbol{\gamma}$ using equation 14 [16].

The proposed fitting ellipse method is computationally efficient compared to the other iterative and general conic fitting methods [17]. The algorithm also shows robustness to the noise. The effect of noise can be reduced further by obtaining the mean of chirplet parameters by using ellipse fitting for multiple contour levels.

IV. Parameter Estimation Results

To demonstrate the proposed estimation approach, means and variances of the estimated parameters of a simulated chirplet echo signal given in equation 1 are examined. The sampling frequency is 100 MHz. The chirplet echo is perturbed with noise level with SNRs of 20, 10, 5 and 2.5 dB. The detection process is performed 50 times for the same chirplet echo for each level of SNR but with different noise pattern. Both average and variance of estimated parameters are presented in Table I. This table confirms that ellipse fitting method offers robust performance in terms of low variance for estimated parameters in presence of noise.

V. Conclusion

Using different TF transformations combined with MLE algorithms are common in chirplet parameter estimation and signal decomposition. These methods are computationally heavy, and consequently lowering the computational complexity for single component chirplet estimation is advantageous for ultrasonic signal analysis. In this study, we developed an efficient method by ellipse fitting to the chirplet components in the TF plane for real-time ultrasonic testing.

TABLE I

Chirplet echo parameter estimation results with SNRs of 20, 15, 10, 5 and 2.5 dB

	α_1 $[MHz]^2$	α_2 $[MHz]^2$	τ $[\mu s]$	f_c [MHz]	Amp.
Parameter	25	15	1.84	5	1
SNR= 20 dB					
Mean	25.5435	14.8772	1.8406	5.0781	0.9758
Variance	0.2040	0.2156	9.8367e-6	0.0002	4.7058e-5
SNR= 10 dB					
Mean	25.7881	14.6654	1.8400	5.0547	0.9744
Variance	2.0251	2.1047	2.0000e-4	0.0212	3.6207e-4
SNR= 5 dB					
Mean	25.5496	15.0627	1.8386	5.0547	0.9826
Variance	6.1908	7.2340	5.4290e-4	0.0711	0.0017
SNR= 2.5dB					
Mean	26.6570	14.7712	1.8438	5.0938	0.9820
Variance	10.4263	11.3877	9.0567e-4	0.1807	0.0020

REFERENCES

[1] O. A. Yeste-Ojeda, J. Grajal, G. Lopez-Risueno, "Atomic Decomposition for Radar Application," *Aerospace and Electronic Systems, IEEE Transactions on* , vol. 44, no. 1, pp. 187- 200, 2008.

[2] J. R. Berriman, D. A. Hutchins, A. Neild, T. H. Gan, and P. Purnell, "The Application of Time-Frequency Analysis to the Air-Coupled Ultrasonic Testing of Concrete," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Vontrol*, vol. 53, no. 4, April 2006.

[3] Y. Zhang; H. Zhang; N. Zhang; "Microembolic Signal Characterization Using Adaptive Chirplet Expansion," *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control,* vol.52, no. 8, pp. 1291-1299, 2005.

[4] J. Cui, and W. Wong; "Investigation of Short-Term Changes in Visual Evoked Potentials With Windowed Adaptive Chirplet Transform," *IEEE Transactions on Biomedical Engineering,* vol. 55, no. 4, pp. 1449-1454, 2008.

[5] S. G. Mallat, and Z. Zhang "Matching pursuits with time-frequency dictionaries", *IEEE Transactions on Signal Processing,* vol. 41, no. 12, pp. 3397-3415, Dec. 1993.

[6] R. Demirli and J. Saniie, "A high-fidelity time-frequency representation for ultrasonic signal analysis", *Proc. of the IEEE Int. Ultrasonics Symposium*, pp. 1376-1379,Oct 2003.

[7] A. Bultan, "A four-parameter atomic decomposition of chirplets", *IEEE Trans. Signal Processing,* vol. 47, no 3, pp. 731 – 745, 1999.

[8] R. Demirli and J. Saniie, "Model-based estimation of ultrasonic echoes, part II: analysis and algorithms ", *IEEE Trans. on Ultrasonics Ferroelectrics and Frequency Control*, vol. 48, no. 3, pp. 787-802, May 2001.

[9] J. C. O'Neill, P. Flandrin, "Chirp hunting Time-Frequency and Time-Scale Analysis", *Proceedings of the IEEE-SP International Symposium,* pp. 425-428, 1998.

[10] Y. Lu, R. Demirli, G. Cardoso, and J. Saniie, "A successive parameter estimation algorithm for chirplet signal decomposition," *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control,* vol. 53, no. 11, pp. 2121-31, November 2006.

[11] Y. Li, H. Fu, P. Y. Kam, "Improved, Approximate, Time-Domain ML Estimators of Chirp Signal Parameters and Their Performance Analysis", *Signal Processing, IEEE Transactions on*, vol. 57, no. 4, pp. 1260-1272, April 2009.

[12] S. Mann and S. Haykin, "The chirplet transform: Physical considerations,"*IEEE Trans. Signal Processing,* vol. 41, pp. 2745–2761, Nov.1993.

[13] L. Angrisani, M. D'Arco, "A Measurement Method Based on a Modified Version of the Chirplet Transform for Instantaneous Frequency Estimation", *IEEE Transactions on Instrumentation and Measurement*, vol. 51, PP.704-711, 2002.

[14] A. W. Fitzgibbon, M. Pilu, and R.B. Fisher. Direct least squares fitting of ellipses, "*IEEE Transactions on Pattern Analysis and Machine Intelligence,* vol. 21, no. 5, May 1999.

[15] A.W. Fitzgibbon and R.B. Fisher, "A Buyer's Guide to Conic Fitting," *Proc. British Machine Vision Conf*., Birmingham, England, 1995.

[16] J. Wu, "Robust Real-Time Ellipse Detection by Direct Least-Square_Fitting" *Computer Science and Software Engineering International Conference on*, pp. 923 – 928, 2008.

[17] K. Kanatani, "Statistical Bias of Conic Fitting and Renormalization,"*IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 16, no. 3, pp. 320-326, 1994.

[18] L. Cohen, Time Frequency Analysis. Englewood Cliffs, NJ: Prentice-Hall, 1995.

[19] S. Qian, Introduction to Time-Frequency and Wavelet Transforms, Englewood Cliffs, NJ: Prentice-Hall, 2002.