

# A Computationally Efficient Algorithm for Ultrasonic Signal Decomposition and Flaw Detection

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**Abstract**— Chirp pulses are often used as an effective technique for ultrasonic imaging in NDE or flaw detection applications. Chirp echoes characterize dispersive media while enabling pulse compression and improved echo detection. Characterizing defects in large grain materials is a complex and challenging problem in ultrasonic imaging applications. The complexity is caused by microstructure scattering which often results in interference and masking of ultrasonic flaw echoes. Therefore methods to unravel these complex signals are advantageous. In this investigation, a computationally efficient method for parameter estimation of ultrasonic signals, consisting multiple chirplet echoes is introduced. Analytical results show that the algorithm is efficient and successful in signal representation.

**Keywords**- Chirp echoes, ellipse fitting, ultrasonic signal decomposition, computation

## I. INTRODUCTION

Estimating the parameters of chirp signals is needed in several practical applications such as Nondestructive Evaluation (NDE) methods, medical applications and sonar [1-3]. The parameters of a chirplet represent a broad range of echo shapes, including the narrowband or broadband; symmetric or skewed; non-dispersive or dispersive. Moreover, the estimated parameters represent the physical properties of the system, such as position and velocity of a target in radar or sonar target detection [4]. Furthermore, target size and orientation can be characterized in ultrasonic imaging. Ultrasonic data compression is achievable via parameter estimation [5]. Both time domain and frequency domain approaches are applied in traditional estimators. Chirp echo parameters can be estimated based on the Time-Frequency Representation (TFR) of the signal in a successive manner [6]. In Matching Pursuit (MP) methods, parameters are chosen from a dictionary of functions in which the parameters are discretized [7-9]. Parameter estimation can be achieved via Maximum Likelihood Estimation (MLE) [10-12]. Fractional Fourier Transform (FRFT) can be used as an alternative method for parameter estimation [13-14].

By using chirplet excitation in NDE and flow measurement applications, the reflected echo will also be a chirplet governed by the velocity of target within the propagation path. The two major time frequency representation i.e. Wigner-Ville Distribution (WVD) and Short-Time Fourier Transform (STFT) of a chirplet signal will be in the form of concentric

ellipses in the Time-Frequency (TF) plane. While in [15] a single chirplet echo parameter estimator using WVD was introduced, in this letter we show the application of chirplet parameter estimator based on ellipse fitting method (EFM) for Chirplet Signal Decomposition (CSD). Moreover, we present a realization of chirplet parameter estimator running on an FPGA development board.

## II. THE PARAMETER ESTIMATION PROCEDURE

The model for a single chirp echo is:

$$f_{\circ}(t) = A \exp[-\alpha(t-\tau)^2/2 + j2\pi f_c(t-\tau) + j\beta(t-\tau)^2/2] \quad (1)$$

where

$$\Theta = [\alpha, \beta, A, f_c, \tau] \quad (2)$$

is the index of the chirp echo,  $\tau$  is the time of arrival,  $f_c$  is the center frequency,  $\alpha$  is the bandwidth factor,  $\beta$  is the chirp-rate and  $A$  is the amplitude. In order to have a normalized signal that has unit energy, the term  $\sqrt[4]{\alpha/\pi}$  is used as amplitude.

The STFT window function is:

$$h(t) = \sqrt[4]{a/\pi} \exp[-a(t)^2/2] \quad (3)$$

It can be shown [15] that Short-Time Fourier Transform of the chirplet signal is in the form of concentric ellipses that their slope, center, major and minor axis correspond to the parameters  $\Theta$ .

The steps of proposed method for estimating the chirplet parameters are as follows. First, the STFT of the signal is evaluated. Two main parameters i.e.  $\tau$  and  $f_c$  for each component are obtained from maxima locations in the TF plane of the signal. The remaining three parameters are estimated using ellipse fitting.

Among conic fitting approaches, direct least squares ellipse-fitting method satisfies our objective of chirplet parameters estimation [16]. Since the contour lines corresponding to the STFT of the signal in TF plane are in elliptical shape, the proposed ellipse fitting approach shows proper performance. A general conic curve is generally represented by:

$$F(\boldsymbol{\gamma}, \mathbf{z}) = \boldsymbol{\gamma} \mathbf{z} = ax^2 + bxy + cy^2 + dx + ey + f = 0 \quad (4)$$

where  $\boldsymbol{\gamma} = [a \ b \ c \ d \ e \ f]^T$  and  $\mathbf{z} = [x^2 \ xy \ y^2 \ x \ y \ 1]^T$ .  $F(\boldsymbol{\gamma}, \mathbf{z})$  is considered as algebraic distance of a point  $(x_i, y_i)$  to the curve  $F(\boldsymbol{\gamma}, \mathbf{z}) = 0$ . Best fitting to the  $N$  data points is achieved by minimizing the sum of squared algebraic distance:

$$\sum_{i=1}^N [F(\boldsymbol{\gamma}, \mathbf{z})]^2 \quad (5)$$

where  $\boldsymbol{\gamma}$  is constrained to avoid the trivial solution  $\boldsymbol{\gamma} = 0$ . The constraint applied [16] in (5) is  $4ac - b^2 = 1$  which can be expressed in the form  $\boldsymbol{\gamma}^T \mathbf{C} \boldsymbol{\gamma} = 1$  where  $\mathbf{C}$  is:

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (6)$$

The minimization can be solved by considering eigenvalue system [17]:

$$(\mathbf{D}^T \mathbf{D} - \lambda \mathbf{C}) \boldsymbol{\gamma} = 0 \quad (7)$$

where

$$\mathbf{D} = \begin{bmatrix} x_1^2 & x_1 y_1 & y_1^2 & x_1 & y_1 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x_n^2 & x_n y_n & y_n^2 & x_n & y_n & 1 \end{bmatrix} \quad (8)$$

After obtaining the eigenvalues  $\lambda$  by solving,  $\det(\mathbf{D}^T \mathbf{D} - \lambda \mathbf{C}) = 0$ , we can readily calculate the parameter vector  $\boldsymbol{\gamma}$  using equation (7) by solving a 5 by 5 system of linear equations. The proposed fitting ellipse method is computationally efficient compared to the other iterative and general conic fitting methods [18].

### III. CHIRPLET SIGNAL DECOMPOSITION

A complex signal can be decomposed into Gaussian chirplets using ellipse fitting method successively. Since WVD suffers from cross terms [19-20], Short-Time Fourier Transform (STFT) is used. At the end of the decomposition, the original signal is expressed as a linear combination of chirplet echoes.

$$s(t) = \sum_{j=0}^{N-1} f_{\Theta_j}(t) \quad (9)$$

where  $f_{\Theta_j}(t)$  is the chirplet echo and  $\Theta_j$  is the corresponding vector of chirplet in (2).

Decomposition is performed as follows: First, by searching for the maxima point in the STFT of the signal, the most dominant echo is determined and its time of arrival and center frequency are obtained. Second, by fitting the ellipse on the selected contour line in the time-frequency plane of the signal the remaining parameters i.e. bandwidth factor, amplitude and chirp rate are estimated. Third, the estimated echo is subtracted from the reconstructed signal and reconstruction error is calculated. The process is repeated until the reconstruction error falls below a predetermined acceptable error value.

The ellipse fitting method (EFM) algorithm is used to decompose an ultrasonic experimental signal consisting of many interfering echoes acquired from a steel block. Estimated echoes (red ones) for the first two iterations and reconstructed signal after 15 iterations are depicted on top of the original signal (in blue) (see Figures 1-3). The comparison between the reconstructed signal and the experimental one shows that the decomposition has been successfully performed with the presence of measurement noise and interference from microstructure scattering.

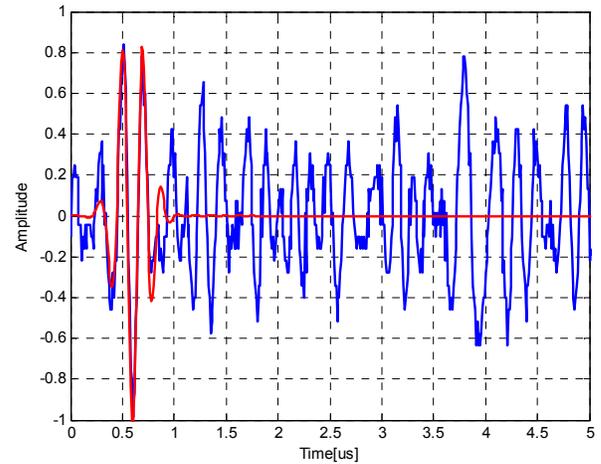


Figure 1. Estimated first echo (red) on top of the original signal (blue)

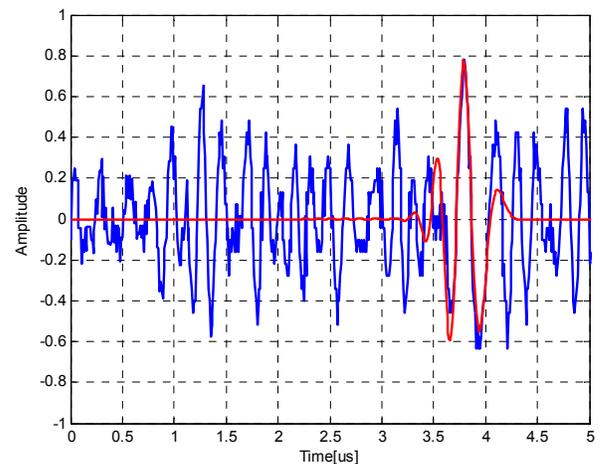


Figure 2. Estimated second echo (red) on top of the signal after the subtraction of the first estimated echo (blue)

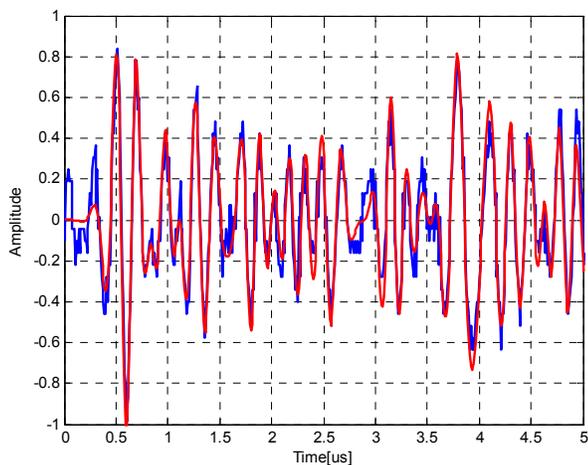


Figure 3. Reconstructed signal (red) on top of the original signal after 15 iterations

EFM estimator consists of two main procedure; STFT and ellipse fitting routine that are computationally more efficient than maximum likelihood estimator. Moreover, for parameter estimation using time-frequency method in successive manner, it is necessary to generate a signal, varying a parameter and then compute the correlation of the generated signal and the measured signal to find the best estimation for that parameter. This process has to be repeated for each parameter separately.

The chirplet parameter estimation based on ellipse fitting has been implemented on Xilinx Virtex-5 (xc5v1x110tff1136-1) FPGA running at 125 MHz. The entire chirplet parameter estimation algorithm in C code is executed on Xilinx Microblaze processor using the Xilinx Embedded Development Kit (EDK). Data and instruction are located in Block RAMs (BRAMs). The algorithm runs extremely fast by using the on chip memory instead of off-chip memory (like DDR RAM) due to faster access rate to BRAMs. While utilizing the on-chip memory speeds up the access to the memory, it needs a careful memory management scheme due to limited memory space. All the required memory has to be released quickly after performing calculations.

#### IV. CONCLUSION

Using both MLE algorithms and time-frequency methods in successive manner are common in ultrasonic signal decomposition and NDE applications. However these techniques require excessive computations. A template matching based approach by using ellipse fitting to the chirp components in the time-frequency plane has been presented. The experimental results show that ultrasonic flaw signal can be estimated efficiently and accurately with presence of noise.

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