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Analysis of Fractional Fourier Transform for Ultrasonic NDE Applications

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Abstract—**In this study, a Fractional Fourier transform-based signal decomposition (FrFT-SD) algorithm is utilized to analyze ultrasonic signals for NDE applications. FrFT, as a transform tool, enables signal decomposition by rotating the signal with an optimal transform order. The search of optimal transform order is conducted by searching the highest kurtosis value of the signal in the transformed domain. Simulation study reveals the relationship among the kurtosis, the transform order of FrFT, and the chirp rate parameter. Furthermore, parameter estimation is applied to the decomposed components (i.e., chirplets) for echo characterization. Simulations and experimental results show that FrFT-SD not only reconstructs signal successfully, but also estimates parameters accurately. Therefore, FrFT-SD could be an effective tool for analysis of ultrasonic signals.**

Keywords – Fractional Fourier transform (FrFT), signal decomposition, kurtosis, ultrasonic NDE

I. Introduction

Ultrasonic signals contain important information pertain to the physical properties of the propagation path. These backscattered echoes often interfere with each other due to limited resolution of the ultrasonic transducer, closed locations, boundaries, orientations, and sizes of random reflectors and may also be corrupted by measurement noise and/or undesired scattering echoes. Consequently, it becomes a challenging problem and conventional signal analysis techniques fail to unscramble the desired signal information for material characterization. Therefore, signal processing methods capable of characterizing the non-stationary behavior of ultrasonic signal for NDE applications are highly desired.

Time-frequency analysis has been widely used to examine signals in joint time-frequency domain and probe how frequency changes with time in those signals. The advantage and disadvantage of time-frequency analysis methods such as short-time Fourier transform, Wigner-Ville distribution, wavelet transform, and chirplet transform are well-known [1-6]. It remains a challenging problem to find a method which is applicable to a broad range of ultrasonic echoes, narrowband or broadband; symmetric or skewed; nondispersive or dispersive.

Recently, there has been a growing attention in Fractional Fourier transform. As a generalized Fourier transform with an additional parameter (i.e., transform order), FrFT has potentials in all areas where Fourier transform can be applied. It has been utilized in different applications such as high resolution SAR imaging, sonar signal processing, blind deconvolution, and beamforming in medical imaging [7-10]. Short term FrFT, component optimized FrFT, and locally optimized FrFT have also been proposed for signal decomposition [11-13]. There are many different ways to decompose a signal. Essentially it is an optimization problem under different criteria. The optimization is conducted either globally or locally. For ultrasonic NDE applications, local responses from scattering microstructure and discontinuities are more interesting for flaw detection and material characterization.

In our previous work [14], FrFT is introduced as a transformation tool for ultrasonic signal decomposition. FrFT is employed to estimate an optimal transform order, which is associated with highest kurtosis value in fractional transform domain. The searching process of optimal transform order is based on a segmented signal for a local optimization. Once the optimal transform order is obtained, the FrFT with the transform order is applied to the entire signal. A signal component is acquired by applying a window in the fractional domain and inverse FrFT. Ultrasonic signal is iteratively decomposed into a group of signal components until a predefined stop criterion such as signal reconstruction error or the number of iterations is satisfied.

In this investigation, analysis of FrFT for ultrasonic nondestructive evaluation applications is studied. FrFT-based signal decomposition (FrFT-SD) leads to an efficient signal decomposition for simulated ultrasonic signals. Furthermore, each decomposed component is modeled using six-parameter chirplet echoes. As a result, parameter estimation is performed for a quantitative analysis of ultrasonic signals. The chirplet covers a board range of signals representing frequency-dependent scattering, attenuation and dispersion effects in ultrasonic applications. Moreover, the relationship among kurtosis, transformation order of FrFT and chirp rate is addressed and studied through simulations.

The outline of the paper is as follows. Section II reviews FrFT-based signal decomposition (FrFT-SD). Section III addresses how kurtosis, transformation order and chirp rate are related through simulations. Section IV discusses performance evaluation of FrFT-SD and parameter estimation for simulated data. Section V shows the results of experimental study. Section VI concludes the paper.

II. FrFT-based Signal Decomposition

The FrFT-SD for ultrasonic signal is reviewed as follows.

FrFT of a signal, $f(t)$, is given by

$$FrFT^{\alpha}(x)=\frac{e^{-i\left(\frac{\pi}{4}-\frac{\pi\alpha}{4}\right)}}{\left(2\pi\left|\sin\frac{\pi\alpha}{2}\right|\right)^{\frac{1}{2}}}e^{\frac{1}{2}ix^2\cot\frac{\pi\alpha}{2}}\int_{-\infty}^{\infty}e^{\left(-i\frac{xt}{\sin\frac{\pi\alpha}{2}}+\frac{1}{2}it^2\cot\frac{\pi\alpha}{2}\right)}f(t)dt \tag{1}$$

where α : denotes transform order of FrFT

x : denotes the variable in transform domain.

The transform order, α, is related to the rotation angle, ϕ, with $\alpha=\frac{\pi\phi}{2}$.

For ultrasonic applications, ultrasonic chirp echo is a type of signal often encountered in ultrasonic backscattered signal accounting for narrowband, broadband, and dispersive echoes. It can be modeled as [3-5]:

$$f_{\Theta}(t)=\beta\exp\left(-\alpha_1(t-\tau)^2\right) + i\,2\pi f_c(t-\tau)+i\theta+i\;\alpha_2(t-\tau)^2 \tag{2}$$

where $\Theta=[\tau\;\; f_c\;\; \beta\;\; \alpha_1\;\; \alpha_2\;\; \theta]$ denotes the parameter vector, τ is the time-of-arrival, f_c is the center frequency, β is the amplitude, α_1 is the bandwidth factor, α_2 is the chirp-rate, and θ is the phase.

The objective of FrFT-SD is to decompose a highly convoluted ultrasonic signal, $s(t)$, into a series of signal components, $FC_j(t)$.

$$s(t)=\sum_{j=1}^{N}FC_j(t)+r(t) \tag{3}$$

where $FC_j(t)$ denotes the jth signal component and $r(t)$ denotes the residue of the decomposition process.

The steps involved in the iterative estimation of an experimental ultrasonic signal are outlined as follows:

1. Initialize the iteration index $i=1$.
2. Get a windowed signal $s\_win(t)$ after applying a window, $w_i(t)$, in time domain.

   $$s\_win(t)=s(t)*w_i(t) \tag{4}$$

3. Obtain FrFT of the signal, $s\_win(t)$, for different transformation order, α.

   $FrFT^{\alpha}(x)_{s\_win(t)}$ denotes FrFT of the signal $s\_win(t)$ with a various of orders α.

4. Calculate kurtosis of $FrFT^{\alpha}(x)_{s\_win(t)}$ for different order, α.

   $$K(\alpha)=\frac{M_4\left(FrFT^{\alpha}(x)_{s\_win(t)}\right)}{\left[M_2\left(FrFT^{\alpha}(x)_{s\_win(t)}\right)\right]^2} \tag{5}$$

   where $M_4(\bullet)$ denotes 4th order central moment and $M_2(\bullet)$ denotes 2nd order central moment.

5. Find the optimal transformation order, α_{est}.

   $$\alpha_{est}=\arg_{\alpha}MAX\left(K(\alpha)\right)$$

   α_{est} corresponds to the FrFT transform order where $K(\alpha)$ has the max value.

6. Apply FrFT with the estimated order α_{est} to the signal $s(t)$ and obtain $FrFT^{\alpha_{est}}(x)_{s(t)}$.
7. Get a windowed signal from $FrFT^{\alpha_{est}}(x)_{s(t)}$.

   $$FrFT\_win(x)=FrFT^{\alpha_{est}}(x)_{s(t)}*win_i(x)$$

8. Get a decomposed component, $FC_i(t)$, by applying the transformation order, $-\alpha_{est}$, to the signal, $FrFT\_win(x)$.

   $$FC_i(t)=FrFT^{-\alpha_{est}}(t)_{FrFT\_win(x)}$$

9. Obtain the residual signal by subtracting the estimated echo, $FC_i(t)$ and update the signal, $s(t)$.
10. Calculate energy of residual signal(E_r) and check convergence: ($E_{\min}$ is predefined convergence condition)

    If $E_r<E_{\min}$, STOP; otherwise, go to step 2.

III. Kurtosis, transform order and chirp rate

Kurtosis is commonly used in statistics to evaluate the degree of peakedness for a distribution. It is defined as the ratio of 4th central moment and square of variance [see Equation 5 in section II]. A signal with high Kurtosis means that it has a distinct peak around the mean. In the literatures of FrFT [11-14], Kurtosis is mostly used as a metric to search the optimal transform order of FrFT. Different transform order directs the degree of signal rotation caused by FrFT.

For ultrasonic signals that can be modeled by chirplets, the chirp rate parameter can be interpreted as an indicator of rotation in time-frequency representation. To show the link between these two types of rotations, a simulation study is performed. A single chirp echo is simulated with the parameters $\Theta=[3.6us\;\; 5MHz\;\; 1\;\; 0\;\; 25MHz^2\;\; \alpha_2]$ where the chirp rate, α_2, varies from 1 to 100.

FrFT-SD is applied to the chirp echo. Kurtosis and the corresponding optimal transform order are recorded. The connections among kurtosis, optimal transform order of

FrFT and chirp rate of chirp echoes are shown in Figures 1, 2 and 3. It can be seen that the optimal transform order is a monotonic function of chirp rate; meanwhile kurtosis is a monotonic function of transform order. Most importantly, kurtosis is almost a linear function of chirp rate in the simulation range. The simulation results confirm that kurtosis can be used in FrFT-SD for ultrasonic signal processing.

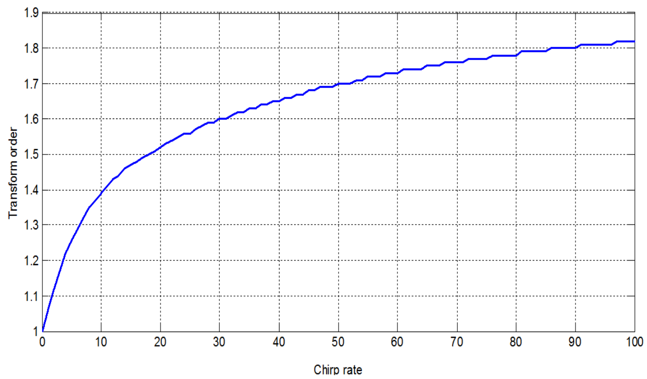

Figure 1. Chirp rate v.s. transform order

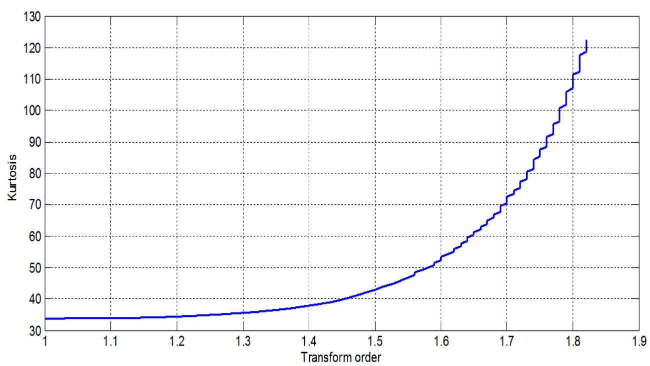

Figure 2. Transform order v.s. kurtosis

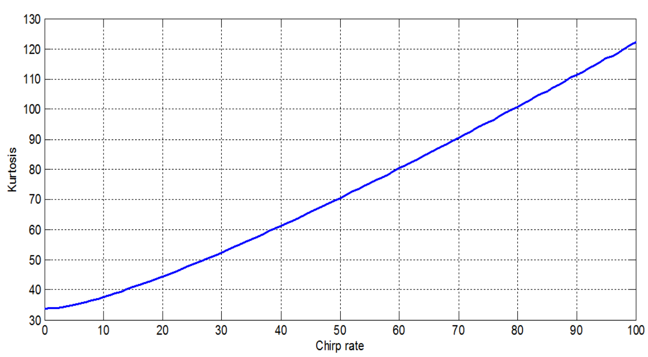

Figure 3. Chirp rate v.s. kurtosis

IV. SIMULATION RESULTS OF FRFT-SD

To illustrate the effectiveness of FrFT-SD for signal decomposition, parameter estimation of ultrasonic signals is perfomed. A signal consisting of two ultrasonic chirp echoes is simulated and processed using FrFT-SD. The parameters of the two echoes embedded in the signal are

$$\Theta_1 = [3.6us \quad 8MHz \quad 1 \quad pi/6 \quad 25MHz^2 \quad 25MHz^2]$$

$$\Theta_2 = [3.9us \quad 5MHz \quad 1 \quad 0 \quad 25MHz^2 \quad 30MHz^2]$$

The FrFT-SD results of the signal are shown in Figure 4. Furthermore, the estimated parameters are listed as follows.

$$\Theta_{1\_est} = [3.60us \quad 8.01MHz \quad 0.93 \quad 0.51 \quad 25.29MHz^2 \quad 26.71MHz^2]$$

$$\Theta_{2\_est} = [3.89us \quad 4.99MHz \quad 0.93 \quad 0.51 \quad 24.98MHz^2 \quad 29.89MHz^2]$$

It can be seen that after applying FrFT-SD algorithm, not only these two echoes can be decomposed accurately, but the parameters can be estimated precisely.

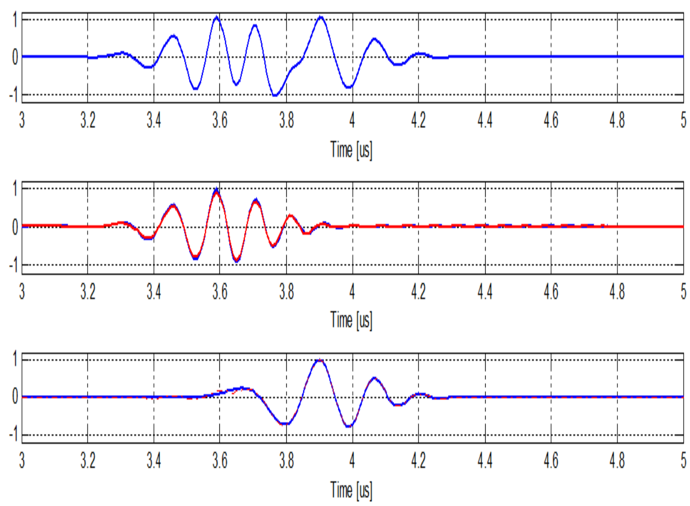

Figure 4. [from top to bottom] a) The simulated ultrasonic signal including two chirp. b) The first chirplet superimposed with the first original chirp echo. c) The second chirplet superimposed with the second original chirp echo.

V. EXPERIMENTAL RESULTS

To evaluate the performance of FrFT-based signal decomposition algorithm, experimental ultrasonic data are utilized to demonstrate the effectiveness of algorithm. The experimental signal is acquired from a steel block with a target embedded using a 5 MHz transducer and sampling rate of 100 MHz. The reconstructed signal using 23 chirplets and the original experimental data are shown in Figure 5. The comparison between the experimental signal and the reconstructed signal clearly demonstrates that the FrFT-SD successfully decomposes the signal and estimates echoes.

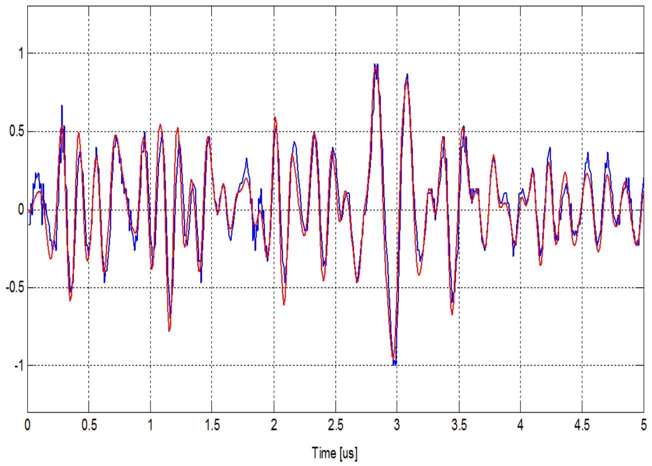

Figure 5. Measured ultrasonic backscattered signal (blue) superimposed with the reconstructed signal consisting of 23 chirplets (red)

VI. CONCLUSION

In this paper, the FrFT-based signal decomposition algorithm is evaluated for analyzing ultrasonic echoes. Simulation study reveals the link among kurtosis, the transform order and the parameters of each decomposed components. Signal decomposition and parameter estimation results show that the algorithm could be an effective tool for signal analysis in NDE applications.

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