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Echo Parameter Estimation for Ultrasonic NDE Applications via a Two-step Compressed Sensing

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***Abstract*— The recently developed compressed sensing (CS) framework has been applied to various data intensive imaging applications. It provides significant reductions in sampling rate with minimal loss in image quality. In this investigation, a CS-based sampling scheme is introduced for ultrasonic nondestructive evaluation (NDE) signals. The feasibility of parameter estimation based on the new sampling method is explored. Especially, a two-step compressed sensing with Gaussian chirplet model is proposed for data acquisition and echo parameter estimation. The CS framework is utilized to estimate the time-of-arrivals (TOAs) of chirplets. With the estimated TOAs, another iteration of data acquisition is performed to acquire the echo amplitude accurately. It has been shown that the two-step CS can estimate the TOAs of ultrasonic echoes with high accuracy in the presence of noise with SNR as low as -5 dB. Furthermore, the estimation is robust to inaccurate knowledge of the transducer bandwidth.**

Keywords– Ultrasonic NDE, compressed sensing, echo parameter estimation.

I. INTRODUCTION

In ultrasonic nondestructive evaluation (NDE) applications, the ultrasonic signal contains many interfering echoes due to the complex physical properties of the propagation path. The pattern of the signal is dependent on irregular boundaries, and the size and random orientation of material microstructures.

Various analysis methods such as Gabor signal decomposition, chirplet signal decomposition, empirical decomposition, and Fractional Fourier transform have been utilized to examine ultrasonic signals [1-6]. As a common model, Gaussian chirplet (GC) is used for echo estimation in these methods. GC represents a broad range of ultrasonic signals, narrowband or wideband; symmetric or skewed; nondispersive or dispersive. Ultrasonic quantitative evaluation of structures can be achieved through echo parameter estimation of GCs. Nevertheless it is still challenging to assess outsized structures due to a large volume of data collection. The ultrasonic NDE process, including data collection and feature extraction, becomes computationally expensive for in-situ NDE characterization. Consequently, an integrated system with efficient data acquisition and echo parameter estimation is highly desirable.

Recently, compressed sensing (CS) has been successfully applied to data intensive imaging applications such as photography, magnetic resonance imaging, tomography, medical ultrasound imaging [7-10] etc. CS provides significant reductions in sampling rate with minimal loss in image quality. Through CS, a signal can be sparsely represented in a certain domain, where most of the coefficients are null or close to zero. R. Tur, Y. C. Eldar and Z. Friedman first introduced CS into the area of biomedical ultrasound imaging [10-11]. To link ultrasound imaging to the latest progress in CS, they assume that the returning ultrasound signal consists of a finite number of known-shape pulses, i.e., the ultrasound signal has finite rate of innovation. As a result, innovation rate sampling, a sub-Nyquist sampling scheme, is developed and applied to ultrasound imaging.

The aim of this study is to design a compressed sampling scheme for ultrasound NDE signals and explore the feasibility of echo parameter estimation based on the new sampling method. Especially, the Gaussian chirplet model is introduced to combine lower rate sampling and annihilating filter in the CS framework with parameter estimation. The integrated method is used to acquire ultrasonic data and estimate the time-of-arrival (TOAs) and amplitudes of chirplet type echoes. Moreover, simulated ultrasonic signals are utilized to evaluate performance of the proposed compressed sensing method.

This paper is organized as follows: Section II briefly reviews the background and current status of compressed sensing for ultrasound imaging, then describes a two-step compressed sensing method for ultrasonic NDE. Section III presents the simulation results. Section IV concludes the paper.

II. COMPRESSED SENSING FOR ULTRASOUND IMAGING

Donoho, Candès and Tao originated the compressed sensing research [12-13]. They have demonstrated that for a sparse signal, it is possible to design an undersampled measurement scheme which can be used to reconstruct the signal with high accuracy. In [14-16], Vetterli *et al* showed sparse sampling of signal innovation. Their main findings are briefly described as follows [14]:

For a periodic signal with period T

$$x(t)=\sum_{m\in Z}\sum_{k=1}^{K}a_k\delta\left(t-\tau_k-mT\right), \tag{1}$$

where $\tau_k\in[0,T]$ and $\delta(\bullet)$ is the Dirac function, the signal can be fully determined by the $2K$ parameters, i.e., K amplitudes a_k and K time-of-arrivals (TOAs) τ_k .

That is, $x(t)$ has a rate of innovation, $\frac{2K}{T}$. Vetterli *et al* devised a finite rate of innovation (FRI) sampling scheme to estimate and recover these parameters with the number of samples close to the rate of innovation, $\frac{2K}{T}$. Furthermore, the FRI sampling scheme is connected and compared with the CS work done by Donohue *et al* [12].

Yonina *et al* extended and applied the FRI sampling to ultrasound imaging [11-13]. A linear combination of Gaussian pulses, $x(t)$, is studied in the context of medical ultrasound:

$$x(t) = \sum_{k=1}^{K} a_k \varphi_k(t)$$

$$\text{where} \quad \varphi_k(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(t-\tau_k)^2}{2\sigma^2}} \qquad (2)$$

a_k is the amplitude of Gaussian pulse

τ_k is the time-of-arrival of Gaussian pulse

σ^2 is the variance of Gaussian pulse

To design a compressed sampling scheme for ultrasound NDE signals and explore the feasibility of echo parameter estimation based on such sampling methods, it is gainful to analyze ultrasonic Gaussian chirp echoes, a type of signal often encountered in ultrasonic measurements containing narrowband, broadband, and dispersive echoes. A linear combination of Gaussian chirplets, $x(t)$, is studied in the context of ultrasonic NDE signal in noise:

$$x(t) = \sum_{k=1}^{K} h_{\Theta_k}(t) + n(t) \qquad (3)$$

where $n(t)$ is a white Gaussian noise, and the Gaussian chirplet (GC) is defined by [3-4],

$$h_{\Theta_k}(t) = a\, e^{-\alpha_1 (t-\tau)^2} \cos\left(2\pi f_c (t-\tau) + \alpha_2 (t-\tau)^2\right) \qquad (4)$$

The parameters of the GC are: τ is the TOA, f_c is the center frequency, a is the amplitude, α_1 is the bandwidth factor, and α_2 is the chirp-rate.

A Gaussian chirplet with varying noise level is simulated to study how noise impacts the estimation results of TOAs and amplitudes in the FRI sampling scheme. The estimation procedures are summarized as follows [11, 14].

1. Simulate a Gaussian chirplet according to Equation 3 and apply Hilbert Transform (HT) to obtain signal envelope,

$$e(t) = abs(Hilbert(x(t))) \qquad (5)$$

In the presence of severe noise, a pre-processing such as low pass filtering or thresholding can be applied before the HT.

2. Pass the signal, $e(t)$, through the sampling kernel, $g(t)$, to get the output, i.e.,.

$$y(t) = e(t) * g(t) \qquad (6)$$

where $*$ denotes the operation of convolution.

Here the sampling kernel, $g(t)$, can be chosen as [11]

$$g(t) = \sum_{m=-K}^{K} b_m e^{j2\pi mt} \qquad (7)$$

where $b_m = 0.54 - 0.46\cos(2\pi \frac{m + \left\lfloor \frac{M}{2} \right\rfloor}{M})$,

$M = 2K + 1$, K is the number of echoes.

From Equation 7, the *M*-length Hamming window coefficients, b_m, can be viewed as truncated Fourier series coefficients used to form a smooth and compactly-support sampling kernel, $g(t)$. The window coefficients, b_m, can be optimized to minimize the mean square error in the sampling. More discussion can be found in [11].

3. Perform the FRI sampling on $y(t)$.

$$c(n) = y(t)\big|_{t=\frac{n}{M}} \qquad (8)$$

where $n = 0 \cdots M-1$.

4. Obtain the calibrated FRI samples,

$$X(m) = S_{M\times M}{}^{-1} C_f(m) \qquad (9)$$

where $C_f(m)$ denotes M-point discrete Fourier transform (DFT) of the sequence $c(n)$.

$$S_{M\times M} = diag\left(b_m e^{-\frac{\omega_m{}^2\sigma^2}{2}}\right),$$

$$\omega_m = 2\pi m, \quad m = -K, \cdots, K$$

According to Equations 2 and 4, the variance satisfies

$$\sigma^2 = \frac{1}{2\alpha_1}$$

5. Find annihilating filter coefficients $A(k)$, $k = 1 \cdots K$, via solving the Yule-Walker equation [14]:

$$\begin{bmatrix} X[0] & X[-1] & \ldots & X[-K+1] \\ X[1] & X[1] & \ldots & X[-K] \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ X[K-1] & X[K-2] & \ldots & X[0] \end{bmatrix} \begin{bmatrix} A[1] \\ A[2] \\ \ldots \\ \ldots \\ A[K] \end{bmatrix} = -\begin{bmatrix} X[1] \\ X[2] \\ \ldots \\ \ldots \\ X[K] \end{bmatrix} \qquad (10)$$

6. Estimate TOAs (τ_k) by finding the roots on unit cycle, u_k , by setting the z-transform polynomial,

$$\mathrm{A}(z) = 0 \tag{11}$$

where $\mathrm{A}(z) = \prod_{k=1}^{K}\left(1 - u_k z^{-1}\right) = 1 + \sum_{k=1}^{K} A(k) z^{-k}$

here $u_k = e^{-j2\pi\tau_k}$ and $A(k)$ are from Step 5. .

7. Estimate amplitudes (a_k) using u_k and the FRI samples $X(m)$,

$$\begin{bmatrix} a_1 \\ a_2 \\ \dots \\ \dots \\ a_K \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ u_1 & u_2 & \dots & u_K \\ u_1^{\,2} & u_2^{\,2} & \dots & u_K^{\,2} \\ \dots & \dots & \dots & \dots \\ u_1^{\,K-1} & u_2^{\,K-1} & \dots & u_K^{\,K-1} \end{bmatrix} \begin{bmatrix} X[0] \\ X[1] \\ \dots \\ \dots \\ X[K] \end{bmatrix} \tag{12}$$

Figure 1 shows the estimated TOA and amplitude for a simulated ultrasound GC under varying signal-to-noise ratio (SNR). It can be seen that the estimation of TOAs is much accurate compared with the estimation of amplitudes. The estimation of amplitude has a large error for a noisy ultrasound signal.

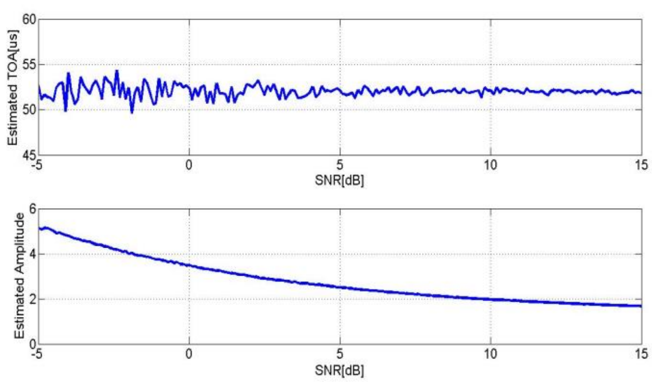

Figure 1. a) Estimated TOA (true value: 51.95 us), and b) Estimated amplitude (true value: 1.3) of a simulated GC in varying SNR.

While TOA estimation is satisfactory, it is observed that amplitude estimation is sensitive to the noise and sparseness of sampling rate. For accurate amplitude estimation, we propose a second iteration in data acquisition with sparse sampling utilizing the estimated TOA to acquire the actual echo amplitude. With the estimated TOAs from the first step and a priori knowledge of the transducer bandwidth, a second data acquisition with windowing is used to isolate and acquire the dominant chirplets.

III. RESULTS

To illustrate the advantages of the proposed two-step CS in ultrasonic signal processing, a signal consists of multiple ultrasonic chirp echoes is simulated and processed. The simulated signal, $s(t)$, can be written as follows

$$s(t) = \sum_{k=1}^{K} h_{\Theta_k}(t) + n(t) \tag{13}$$

It is worth pointing out that a priori information of the bandwidth (α_1), or variance, (σ^2), is utilized in the FRI sampling process (see Equation 9). In a practical system, the bandwidth information of ultrasound transducer can be used as a reference. However, due to the frequency dependent absorption and scattering, the bandwidth of each chirplet varies in the returned ultrasound signal. A simulation with various σ is conducted to evaluate the estimation accuracy of TOAs. Here, a 5-echo signal is simulated. From Table 1, it can be seen that the TOA estimation is robust with the inaccurate knowledge of bandwidth.

Table 1. Estimation accuracy under varying bandwidth (σ)

Actual TOAs / Variance	**51.95** [us]	**58.17** [us]	**72.72** [us]	**93.50** [us]	**128.82** [us]
σ	51.95	58.17	72.72	93.50	128.82
1.05 σ	51.95	57.97	72.72	93.50	128.82
2 σ	51.95	56.52	74.80	93.91	128.82
4 σ	50.49	53.20	80.41	95.37	128.82

Ultrasonic chirp echoes with different degree of overlaps are simulated, where the chirp rate models the dispersion effect in ultrasonic testing. Figure 2 shows slightly-overlapped ultrasound chirplets with SNR of 6.5 dB.

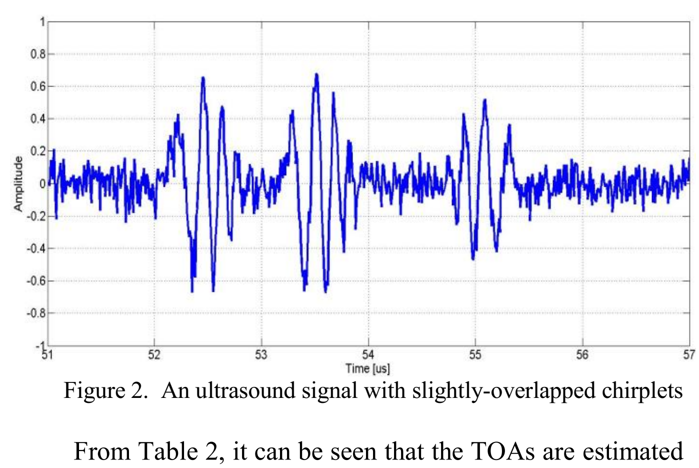

Figure 2. An ultrasound signal with slightly-overlapped chirplets

From Table 2, it can be seen that the TOAs are estimated with high accuracy. The estimated TOAs are used as a guide to perform another iteration of data acquisition with windowing. The parameters of chirplets are estimated via successive chirplet parameter estimation technique [4]. Table 3 compares the estimated parameters with the actual parameters used in the simulation. Moreover, an ultrasonic signal with chirplets of dispersive nature is simulated with SNR of 6.5 dB (see Figure 3). Table 4 compares the estimated and actual parameters of chirplets.

Table 2. Estimated TOAs for the simulated signal in Figure 2

Actual TOA [us]	**52.46**	**53.51**	**55.08**
Estimated TOA [us]	52.51	53.48	55.01

Table 3. Parameter estimation results via two-step compressed sensing for the signal in Figure 2

Echo #		α_1 [MHz]^2	α_2 [MHz]^2	τ [us]	f_c [MHz]	a
1	**Actual**	**12.50**	**29.07**	**52.46**	**5.00**	**0.60**
	Estimated	10.96	29.08	52.46	5.04	0.61
2	**Actual**	**12.50**	**24.71**	**53.51**	**5.00**	**0.70**
	Estimated	13.44	24.89	53.51	5.08	0.71
3	**Actual**	**12.50**	**-21.80**	**55.08**	**5.00**	**0.50**
	Estimated	14.00	-18.11	55.07	4.99	0.49

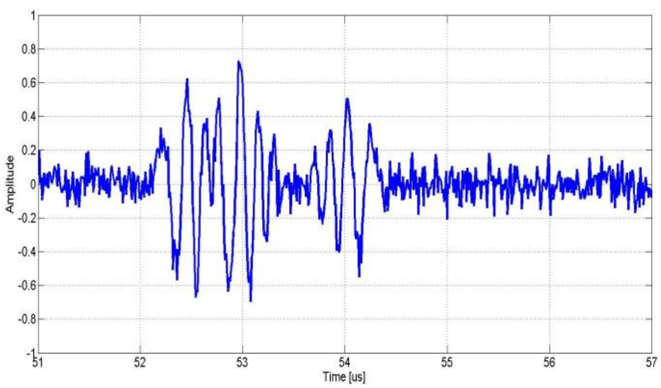

Figure 3. An ultrasound signal with heavily-overlapped dispersive chirplets.

Table 4. Parameter estimation results via two-step compressed sensing for the signal in Figure 3

Echo #		α_1 [MHz]^2	α_2 [MHz]^2	τ [us]	f_c [MHz]	a
1	**Actual**	**12.50**	**29.07**	**52.46**	**5.00**	**0.60**
	Estimated	19.74	24.97	52.44	4.84	0.64
2	**Actual**	**12.50**	**24.71**	**52.98**	**5.00**	**0.70**
	Estimated	11.78	24.99	52.97	4.96	0.69
3	**Actual**	**12.50**	**-21.80**	**54.03**	**5.00**	**0.50**
	Estimated	12.47	-20.85	54.03	4.98	0.48

From the simulation study, it is shown that the TOAs of ultrasonic pulses can be estimated with high accuracy in the presence of noise with SNR as low as -5 dB. Furthermore, the estimation is robust to inaccurate knowledge of the transducer bandwidth. The two-step CS approach is successfully applied for signal decomposition and parameter estimation for a typical NDE signal. CS promises very efficient data acquisition schemes in ultrasonic sensing systems.

IV. CONCLUSION

In this study, the CS framework is incorporated to conventional ultrasonic signal processing for NDE applications. In particular, the two-step CS enables signal decomposition and parameter estimation with a minimal effort in data collection. The study may have broad applications in imaging, material characterization, target detection and pattern recognition.

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