10.1109/ULTSYM.2013.0247

Bilinear Time-Frequency Distributions for Ultrasonic Signal Processing and NDE Applications

Jafar Saniie, Juan Lu and Erdal Oruklu

Department of Electrical and Computer Engineering
Illinois Institute of Technology, Chicago, Illinois, U.S.A.

Abstract- **In this paper, we introduce Singular Value Decomposition (SVD) of Wigner Distribution (WD). The WD is a special and widely adapted method of generalized bilinear time-frequency (GBTF) representation that reveals frequency contents of ultrasonic signals as function of time. The WD offers the best time-frequency (t-f) resolution but significantly suffers because of cross-term artifacts generated due to the inherent bilinear operation of GBTF. In this study, we use SVD to reduce the adverse impact of cross-terms. This approach has been extended to the SVD of Choi-Williams Distribution (CWD), a well-adapted version of GBTF distributions. Gaussian echoes are used to model the ultrasonic backscattered signals and this model is examined to reveal the pitfalls of WD and CWD. For CWD, the range of the exponential kernel parameter is attained based on the impetus to sustain desirable auto-terms, while suppressing the superfluous cross-terms. The SVD of WD and CWD generates basis functions uniquely corresponding to auto-terms and cross-terms. After identifying and discarding the basis functions corresponding to the cross-terms and noise, the desirable basis functions of auto-terms and their singular values are used to reconstruct the t-f distribution. We have applied SVD to WD and CWD of two overlapping echoes with different arrival times and frequency distributions corrupted with noise. We further used this method to detect flaw echoes masked by microstructure scattering noise. The flaw echo in the t-f plane clearly reveals lower frequency distribution at the location of the defect. These numerical results and experimental evaluations confirm that the application of SVD is a desirable approach to improve the accuracy of GBTF.**

I. Introduction

In ultrasonic nondestructive evaluation (NDE) of materials, the frequency-dependent scattering, attenuation, and dispersion make the backscattered signal highly complex and non-stationary. In this study, generalized bilinear time-frequency (GBTF) methods have been applied to the multi-component ultrasonic signals that consist of overlapping echoes in order to display their signal energy on a joint time-frequency (t-f) plane. The t-f analysis of the ultrasonic target echoes reveal time of arrival and center frequency that is often used in assessment of quality and integrity of materials. However, the GBTF distributions inherently contain artifacts known as cross-terms which obscure the important information regarding target echoes. Therefore, signal processing is essential to recognize the cross-terms and then devise methods to alleviate their adverse impact.

The t-f analysis by Cohn [1] views the Wigner Distribution (WD) as a special case of the generalized t-f distribution function which can be written as:

$$P(t,\omega)=\frac{1}{4\pi^2}\iiint_{-\infty}^{+\infty} e^{-j\theta t-j\tau\omega+j\theta\mu}\,\Phi(\theta,\tau)f^*\left(\mu-\frac{\tau}{2}\right)f\left(\mu+\frac{\tau}{2}\right)d\mu d\tau d\theta \quad (1)$$

where the kernel function, $\Phi(\theta,\tau)$ is equal to unity:

$$WD_f(t,\omega)=\int_{-\infty}^{+\infty} e^{-j\omega\tau} f\left(t+\frac{\tau}{2}\right)f^*\left(t-\frac{\tau}{2}\right)d\tau \quad (2)$$

The WD has certain properties that make it attractive in both theoretical analysis and practical implementation [2,3]. It is especially advantageous in analyzing dispersive signals in which different frequency contents arrive at different times. In general, different kernel functions result in different types of t-f distributions. Due to the random and complex nature of backscattered ultrasonic echoes, and due to the fact that the echoes are not exactly Gaussian in shape, the WD of ultrasonic signals are corrupted by the cross-term. It is our objective to analyze the performance of the WD for ultrasonic signal characterization in the presence of cross-term. Based on the properties of the signal's WD, singular value decomposition (SVD) is used to reduce the cross-terms. Furthermore, this approach is verified in ultrasonic nondestructive evaluation.

The cross WD of two continuous signals *f(t)* and *g(t)* is defined as:

$$WD_{f,g}(t,\omega)=\int_{-\infty}^{+\infty} e^{-j\omega\tau} f\left(t+\frac{\tau}{2}\right)g^*\left(t-\frac{\tau}{2}\right)d\tau \quad (3)$$

If a signal is the superposition of two signals *f(t)* and *g(t),* then the auto WD of the signal is given by:

$$WD_{f+g}(t,\omega)=WD_f(t,\omega)+WD_g(t,\omega)+2Re[WD_{f,g}(t,\omega)] \quad (4)$$

where $WD_{f,g}(t,\omega)$ is the cross WD of two signals *f(t)* and *g(t).* Often, this cross-term creates new frequency components which make the analysis of multi-component signals difficult.

Authorized licensed use limited to: Illinois Institute of Technology. Downloaded on October 02,2020 at 06:17:07 UTC from IEEE Xplore. Restrictions apply.

II. Singular Value Decomposition of WD

Let's consider an ultrasonic Gaussian echo defined as:

$$y_1(t) = A_1 e^{-a_1(t-t_1)^2} e^{j\omega_1 t} \tag{5}$$

where A_1 is the echo amplitude, ω_1 is the center frequency, t_1 is the arrival time, and α_1 is the bandwidth factor. The WD of the above signal is given by:

$$WD_{y_1}(t,\omega) = \sqrt{\frac{2\pi}{\alpha_1}} A_1^2 e^{-2\alpha_1(t-t_1)^2} e^{-\frac{(\omega-\omega_1)^2}{2\alpha_1}} \tag{6}$$

Next, consider a signal that contains two ultrasonic Gaussian echoes with center frequencies ω_1 and ω_2 at locations t_1 and t_2 respectively:

$$y_2(t) = s_1(t) + s_2(t) = A_1 e^{-a_1(t-t_1)^2} e^{j\omega_1 t} + A_2 e^{-a_2(t-t_2)^2} e^{j\omega_2 t} \tag{7}$$

The WD of this signal can be obtained by using the property stated in Equation (4):

$$WD_{y_2}(t,\omega) = WD_{s_1}(t,\omega) + WD_{s_2}(t,\omega) + 2Re[WD_{s_1+s_2}(t,\omega)] \tag{8}$$

The auto-terms of the $WD_{y_2}(t,\omega)$ are:

$$WD_{y_2}(t,\omega)_{auto-term} = \sqrt{\frac{2\pi}{\alpha_1}} A_1^2 e^{-2\alpha_1(t-t_1)^2} e^{-\frac{(\omega-\omega_1)^2}{2\alpha_1}} + \sqrt{\frac{2\pi}{\alpha_2}} A_2^2 e^{-2\alpha_2(t-t_2)^2} e^{-\frac{(\omega-\omega_2)^2}{2\alpha_2}} \tag{9}$$

The cross-term can be found:

$$WD_{y_2}(t,\omega)_{cross-term} = 2Re[WD_{s_1+s_2}(t,\omega)]$$

$$= \sqrt{\frac{16\pi}{\alpha_1+\alpha_2}} A_1 A_2 e^{-\frac{\alpha_1\alpha_2((t-t_1)+(t-t_2))^2}{(\alpha_1+\alpha_2)} - \frac{\left(\omega-\frac{\omega_1+\omega_2}{2}\right)^2}{(\alpha_1+\alpha_2)}}$$

$$\cdot \cos\left[(\omega_1-\omega_2)t + \frac{2(\alpha_1(t-t_1)-\alpha_2(t-t_2))}{\alpha_1+\alpha_2}\left(\omega - \frac{\omega_1+\omega_2}{2}\right)\right] \tag{10}$$

The SVD can be used to decompose the auto-terms and to obtain the corresponding basis functions which represent the true t-f information of ultrasonic echoes. For a multi-component signal (i.e., signal contains several Gaussian echoes):

$$y_N(t) = \sum_{j=1}^{N} s_j(t) = A_1 e^{-a_1(t-t_1)^2} e^{j\omega_1 t} + A_2 e^{-a_2(t-t_2)^2} e^{j\omega_2 t} + \cdots + A_N e^{-a_N(t-t_N)^2} e^{j\omega_N t} \tag{11}$$

The SVD theorem gives the following equality [4]:

$$\lambda_{ij} U_{ij}(\omega) = \int_{-\infty}^{+\infty} WD_{s_j}(t-\tau,\omega) V_{ij}(\tau) d\tau \tag{12}$$

and

$$\int_{-\infty}^{+\infty} WD_{s_j}(t,t) dt = \sum_{i=1}^{N} \lambda_{ij} \tag{13}$$

where λ_{ij} is the i^{th} singular value, U_{ij} is i^{th} basis function in frequency and V_{ij} is i^{th} basis function in time, for $s_j(t)$ component of the ultrasonic signal.

As we mentioned earlier, the WD of the multi-component signal contains the cross-terms. By applying the singular value decomposition to the WD, the cross-terms can be significantly reduced and in certain instances, eliminated (see Figure 1 as an example).

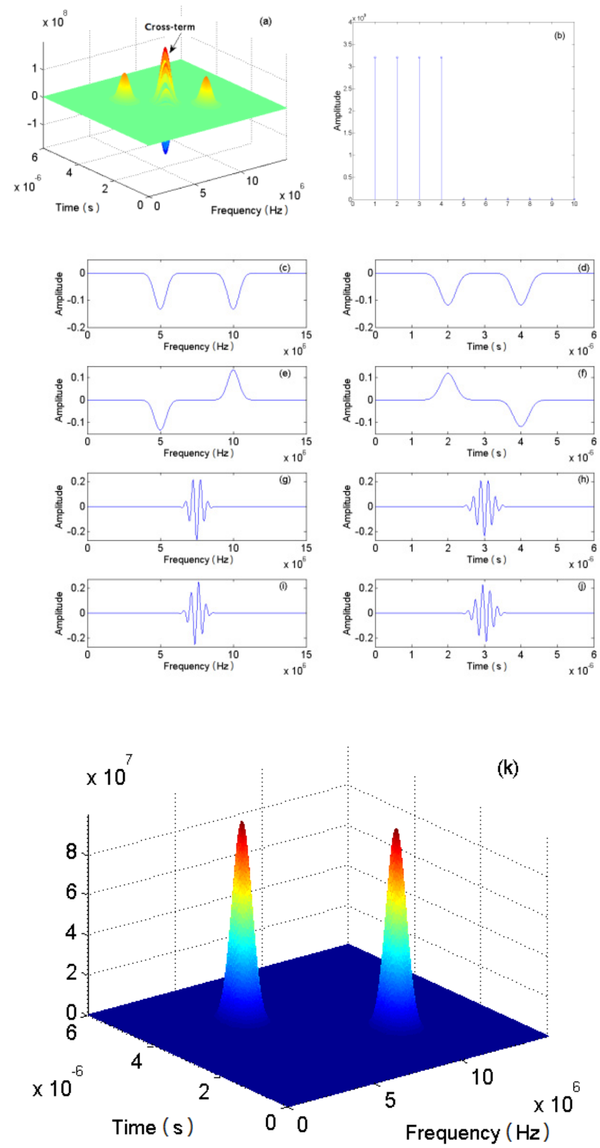

Figure 1. Cross-term elimination of two wavelets WD using basis functions. (a) The WD of two Gaussian echoes, (b) The first 5 estimated singular values, (c, e, g, i) Frequency basis functions, (d, f, h, j) Time basis functions, (k) The WD after using SVD algorithm

Authorized licensed use limited to: Illinois Institute of Technology. Downloaded on October 02,2020 at 06:17:07 UTC from IEEE Xplore. Restrictions apply.

Figure 1(a) is a signal that consists of two Gaussian echoes, with two distinct center frequencies. The WD and the singular values with corresponding basis functions are shown in Figure 1(b) to 1(k). Notice that Figure 1(c), 1(d), 1(e) and 1(f) are Gaussian shaped basis functions, whereas the remaining basis functions (Figures 1(g), 1(h), 1(i), 1(j)) are not, and they correspond to cross-terms. Therefore, only the Figure 1(c) to 1(f) can be used to reconstruct the WD, which should only contain the signal information, not the cross-terms. The reconstructed WD is displayed in Figure 1k, wherein the cross-terms are totally removed.

A more severe case in ultrasonic signal processing is when the signal has been corrupted by noise. Figure 2 presents two ultrasonic echoes corrupted by uniformly distributed random noise, where the SNR is 3 dB. The WD of the signal is highly degraded by the cross-terms and the presence of the noise. However, SVD recovers the desirable echoes with improved SNR (see Figure 2(d)).

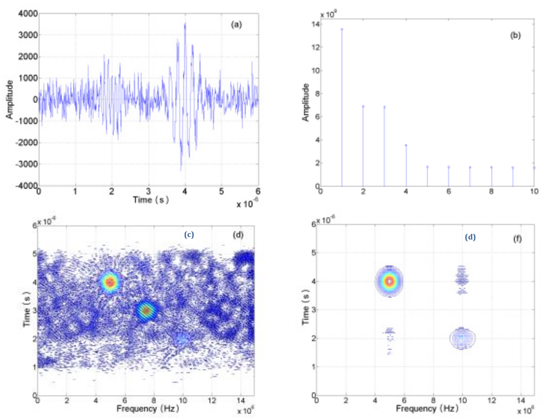

Figure 2. Signal enhancement using basis functions (SNR=3 dB). (a) Two echoes with noise, (b) The singular values, (c) The WD image, (d) The WD image after using SVD algorithm.

The SVD of WD is applied to the experimental ultrasonic signal for spectral representation and the flaw detection as shown in Figure 3. Figure 3(a) displays an experimental ultrasonic signal which includes an echo from a defect. Figure 3(b) shows the singular values applied to the WD and Figure 3(c) presents the WD of the signal. Figure 3(c) clearly shows that the flaw echo is masked by the cross-terms generated from the grain noise, and the location and frequency of the flaw cannot be detected. Figure 3(d) shows the reconstructed WD, by just using the first singular value and the corresponding basis function. The reconstructed WD clearly shows the information of the location and frequency of the flaw echo.

III. SVD OF CHOI-WILLIAMS DISTRIBUTION

As discussed above, SVD of WD works well to extract auto-terms from cross-terms and noise. However, it might fail to extract auto-terms if auto-terms' basis function are contaminated by cross-terms, where the auto-terms of the signal cannot be recovered from these basis functions. A well-known method for cross-terms suppression of WD is using Choi-Williams Distribution (CWD)

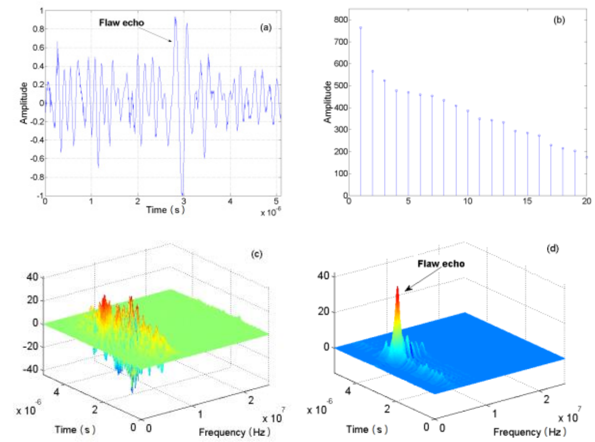

Figure 3. Flaw enhancement using experimental data. (a) Ultrasonic signal with one flaw echo, (b) The singular values, (c) The WD of the signal, (d) The WD after using SVD algorithm

which is also known as the exponential distribution [5]. The definition of CWD is as follows:

$$P(t,\omega)=\frac{1}{4\pi^2}\iiint_{-\infty}^{+\infty} e^{-j\theta t-j\tau\omega+j\theta\mu}\,\Phi(\theta,\tau) f^*\left(\mu-\frac{\tau}{2}\right) f\left(\mu+\frac{\tau}{2}\right) d\mu d\tau d\theta \quad (14)$$

where

$$\Phi(\theta,\tau)=e^{-\theta^2\tau^2/\sigma} \quad (15)$$

Then, CWD is:

$$E_f(t,\omega)=\iint_{-\infty}^{\infty}\frac{1}{\sqrt{4\pi\tau^2/\sigma}}e^{-\frac{(\mu-t)^2}{4\tau^2/\sigma}-j\omega\tau} f^*\left(\mu-\frac{\tau}{2}\right) f\left(\mu+\frac{\tau}{2}\right) d\mu d\tau \quad (16)$$

The characteristics of the t-f representation using SVD are demonstrated in the Figure 4. Figure 4(a) shows a four-component Gaussian echoes with both time and frequency overlaps. Figure 4(b) displays the undesirable cross-terms of the t-f distribution using the WD. As shown in Figure 4(c), the CWD with σ =1 eliminates certain cross-terms; however, it failed to suppress the cross-terms between the two neighboring echoes which either have the same arrival times or have the same center frequencies. But, as shown in Figure 4(d), these remaining cross-terms in CWD are eliminated totally by SVD method.

The SVD of the CWD has been used to estimate time of arrival and center frequency of two overlapping ultrasonic echoes with different SNR. The overlap of this two echoes is governed by the difference of their arrival times and echo's spreading parameters, σ_1 and σ_2, which are proportional to the echo's time-width. Therefore, we define percentile echo overlap index μ as:

$$\mu=\left(1-\frac{|t_1-t_2|}{3(\sigma_1+\sigma_2)}\right)\times 100 \quad (17)$$

where we assume that $|t_1-t_2|\le 3(\sigma_1+\sigma_2)$ and when $|t_1-t_2|>3(\sigma_1+\sigma_2)$, there is no overlap.

Authorized licensed use limited to: Illinois Institute of Technology. Downloaded on October 02,2020 at 06:17:07 UTC from IEEE Xplore. Restrictions apply.

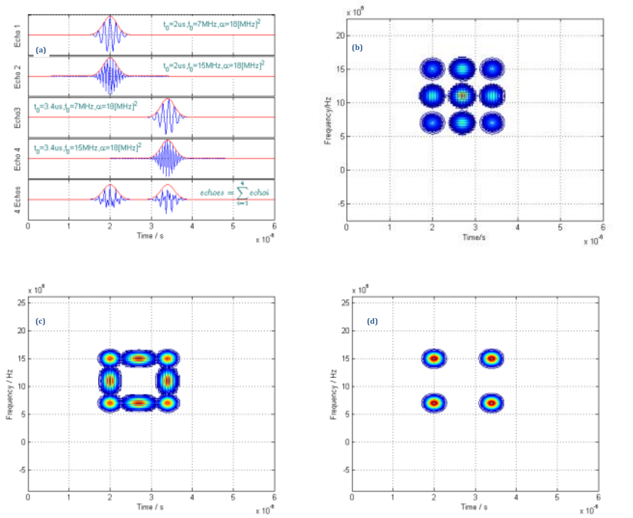

Figure 4. (a) Four-component Gaussian echoes, (b) The contour image of WD, (c) The contour image of CWD, (d) The contour image of CWD followed by SVD

Figure 5 shows the relative percent error of arrival time and center frequencies of two echoes as a function of degree of overlap for different SNRs. The arrival time of the two echoes is less than 3.5% and the relative error of center frequency of the two echoes is less than 1.5%. This means that the error is rather small. Furthermore, as the SNR value decreases, the mean value of relative error increases, which indicates the presence of noise that reduces the estimation accuracy.

IV. Conclusion

Gaussian echoes are used to model the ultrasonic backscattered signals and this model is examined to reveal the pitfalls of WD and CWD. To broaden the usefulness of this approach, singular value decomposition of WD and CWD are analyzed and also verified experimentally as a mean to extract high intelligibility of auto-terms and to further facilitate elimination of the residual cross-terms. The SVD of WD and CWD generates basis functions uniquely corresponding to auto-terms and cross-terms. This method has been evaluated using both simulated and experimental results. In particular we have applied SVD to WD and CWD of two overlapping echoes with different arrival times and frequency distributions corrupted with noise. The overlap ranges from 0% to 100% and the SNR values chosen are 20 dB, 10dB, 3dB and 0dB. Then, the SVD based, reconstructed t-f is used to estimate the arrival times and center frequencies. This approach resulted in accurate arrival-time and center frequency estimates with no more than 4% estimation error where the mean values of the relative errors are about 2% (this includes the estimation results of signals with 0dB SNR). We further used this method to detect flaw echo which is submerged by scattering noise.

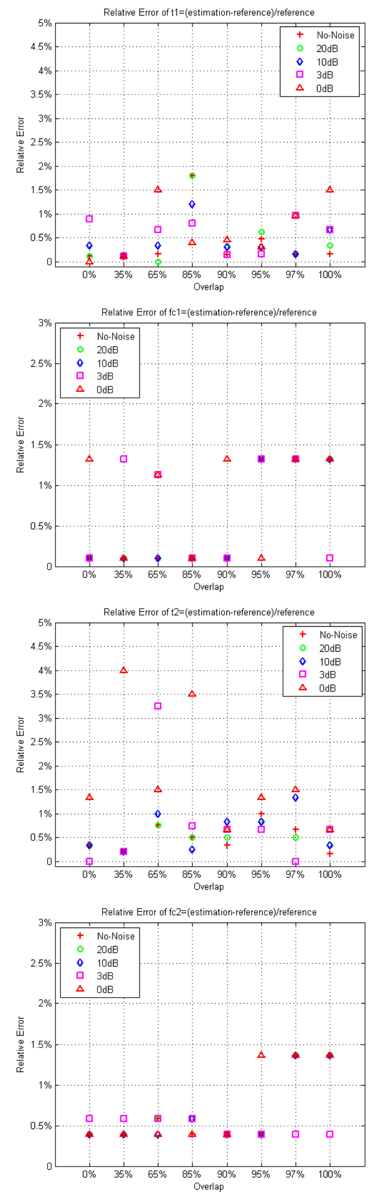

Figure 5. The relative estimation error of echo parameters. (a) The percent error of first echo arrival time (t1), (b) The percent error of first echo center frequency (fc1), (c) The percent error of second echo arrival time (t2) (d) The percent error of second echo center frequency (fc2)

References

[1] L. Cohen, *"Time Frequency Analysis: Theory and Applications"*, Prentice Hall, 1994

[2] E.P. Wigner, "On the Quantum Correction for Thermodynamic Equilibrium," *Phys.* Rev.40, pp. 749-759, 1932.

[3] M.A. Malik, J. Saniie, "Evaluation of Exponential Product Kernel for Quadratic Time-Frequency Distributions Applied to Ultrasonic Signals," *IEEE Ultrasonics Symposium*, pp. 643-648, 1997.

[4] H.L. Van Trees, "Detection Estimation and Modulation Theory," John Wiley and Sons, 1968.

[5] H.I. Choi, W.J. Williams, "Improved Time-Frequency Representation of Multicomponent Signals Using Exponential Kernels," *IEEE Trans. on Acoustics, Speech, Signal Processing*, Vol. ASSP-37 1989.

Authorized licensed use limited to: Illinois Institute of Technology. Downloaded on October 02,2020 at 06:17:07 UTC from IEEE Xplore. Restrictions apply.