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RESOLUTION LIMIT OF OVERLAPPING ULTRASOUND CHIRPLET ECHOES VIA HYPOTHESIS TESTING

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***Abstract*— The axial resolution capability of an ultrasonic pulse-echo imaging system is known to depend on the bandwidth of the propagating pulse. However, the exact resolution limit, the minimum spacing between two reflectors that a given ultrasound imaging system can resolve, has been lacking. In particular, for a known transducer bandwidth, noise level, excitation frequency, and the dispersion rate, it is desirable to know the exact axial resolution limit. Such a limit is useful for assessing the performance of resolution improvement algorithms such as deconvolution methods. This paper develops a method to estimate the resolution limit in terms of propagating pulse characteristics, signal-to-noise ratio (SNR), and resolution success rate.**

Index Terms - *Resolution limit, time-difference-of-arrival (TDOA), overlapping echoes, hypothesis testing.*

I. INTRODUCTION

The resolution of overlapping echoes is an important problem in ultrasonic NDE as it relates to improving the axial resolution of ultrasound. Due to the bandwidth limitations of the ultrasonic transducer, the echoes measured from closely spaced reflectors (e.g., thin layers, densely spaced point targets) overlap in time making the interpretation difficult. Most often signal processing methods are applied to resolve the overlapping echoes [1], [2]. The performance of these methods is typically assessed through simulations, in terms of their ability to discriminate between two simulated overlapping echoes.

The axial resolution is typically measured as the minimum distance between two closely spaced reflectors that an ultrasound transducer can resolve. For a given propagation medium with known speed of sound, this distance is measured in terms of the time-difference-of-arrival (TDOA) of two echoes due to the two reflectors. The time resolution (i.e., minimum resolvable TDOA) is known to depend on the spectral characteristics (bandwidth and center frequency) of the interrogating pulse. However, the minimum resolvable TDOA for given measurements conditions (signal-to-noise ratio (SNR), excitation frequency, bandwidth and echo dispersion) is lacking. Such a resolution limit is desirable in assessing the performance of high-resolution imaging algorithms. For example, deconvolution techniques are typically evaluated in terms of their ability to resolve the TDOA between two closely spaced echoes [3].

In this paper we propose a method to estimate the resolution limit in terms of the spectral characteristics of the propagating ultrasound pulse. To characterize the ultrasound pulse we use Gaussian Chirplet (GC) model [4]. The parameters of the GC model are intuitive and provide a means to explicitly analyze the effect of bandwidth, center frequency, dispersion rate, and phase on the resolution limit. The resolution limit is sought by formulating a hypothesis testing problem [5]. Considering two echoes measured from two closely spaced reflectors, the problem is posed as the discrimination capacity of an observer to see one echo or two echoes. More specifically the observer has to make a decision between the null hypothesis which assumes that just one GC echo is observed and the alternative hypothesis which assumes that two GC echoes are observed. As such we determine the probability of error in making a decision between the test signal composed of two overlapping echoes from the test signal composed of just one echo. If the reflectors are sufficiently separated, the two echoes can be discriminated with a very small probability of error. However, if the reflectors are too close, the two echoes overlap to a degree that it resembles just one echo, causing a high probability of error in making a decision. In this paper we propose a method to estimate the limit (lower bound) on TDOA below which the two echoes cannot be discriminated with a small probability of error (e.g., 0.01).

The remainder of the paper is organized as follows. Section 2 introduces the hypothesis testing problem and develops a method to estimate the resolution limit in terms of spectral parameters of the GC pulse, SNR, and resolution success rate. Section 3 presents simulation results and demonstrates how the spectral parameters influence the resolution limit. Finally, discussions and conclusions are given in Section 4.

We use conventional notations throughout the paper. Unless otherwise stated, lowercase italic letters denote scalars, and lowercase bold letters denote vectors.

II. RESOLUTION LIMIT VIA HYPOTHESIS TESTING

We consider the following hypothesis testing problem where in the null hypothesis the data vector ($\mathbf{x}$) of length N contains one GC echo and in the alternative hypothesis the data vector contains two GCs separated by a TDOA of δ, i.e.,

$$\begin{aligned} &\mathrm{H}_0:\ \mathbf{x} = \beta_{00}\mathbf{s}(\boldsymbol{\theta}; t - t_0) + \mathbf{w} \\ &\mathrm{H}_1:\ \mathbf{x} = \beta_{11}\mathbf{s}(\boldsymbol{\theta}; t - t_0 - 0.5\delta) + \beta_{12}\mathbf{s}(\boldsymbol{\theta}; t - t_0 + 0.5\delta) + \mathbf{w} \end{aligned} \quad (1)$$

The GC located at time zero $\mathbf{s}(\boldsymbol{\theta}; t = 0)$ with a unit amplitude is generated by sampling the following continuous model,

$$\mathbf{s}(\boldsymbol{\theta};t) = \exp(-\alpha t^2)\cos(2\pi f_c t + \gamma t^2 + \phi) \quad (2)$$

where the parameter vector $\boldsymbol{\theta} = [\alpha \; f_c \; \gamma \; \phi]$ contains spectral parameters (bandwidth factor, center frequency, chirp rate, and phase) and governs the shape of the echo. The term $\mathbf{w}$ represents the measurement noise vector of length N and is characterized as an additive zero-mean white Gaussian noise (AWGN) with variance σ^2.

The above hypothesis problem can be seen as a binary hypothesis testing problem of the form

$$\begin{aligned} \mathrm{H}_0 &: \mathbf{x} = \mathbf{z}_0 + \mathbf{w} \\ \mathrm{H}_1 &: \mathbf{x} = \mathbf{z}_1 + \mathbf{w} \end{aligned} \quad (3)$$

where $\mathbf{z}_0 = \beta_{00}\mathbf{s}(\boldsymbol{\theta};t_0)$ represents the signal containing one GC echo and $\mathbf{z}_1 = \beta_{11}\mathbf{s}(\boldsymbol{\theta};t_0 - 0.5\delta) + \beta_{12}\mathbf{s}(\boldsymbol{\theta};t - t_0 + 0.5\delta)$ represents the test signal containing two overlapping GC echoes. The optimal decision rule for the above binary hypothesis problem is obtained by the Likelihood Ratio Test (LRT) [6], i.e.,

$$\frac{p(x/\mathrm{H}_1)}{p(x/\mathrm{H}_0)} > \frac{p(\mathrm{H}_1)}{p(\mathrm{H}_0)} \quad (4)$$

where $p(\mathrm{H}_0), p(\mathrm{H}_1)$ denotes the prior probabilities of the null and alternative hypothesis, and $p(x/\mathrm{H}_i)$ denotes the conditional probability of observing $\mathbf{x}$ while hypothesis H_i is true, i.e.,

$$p(x/\mathrm{H}_i) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left[-\frac{1}{2\sigma^2}\left\|\mathbf{x} - \mathbf{z}_i\right\|^2\right] \quad (5)$$

Plugging (5) into (4) and ignoring constant terms yield the test statistics for the hypothesis H_i as

$$T_i(\mathbf{x}) = \mathbf{x}^T\mathbf{z} - \frac{1}{2}\mathbf{z}_i^T\mathbf{z} \quad (6)$$

Assuming equal prior probabilities for both hypothesis, $p(\mathrm{H}_0) = p(\mathrm{H}_1)$, the probability of error can be defined as

$$P_e = 0.5\Pr\left\{T_1(\mathbf{x}) > T_0(\mathbf{x})/\mathrm{H}_0\right\} + 0.5\Pr\left\{T_0(\mathbf{x}) > T_1(\mathbf{x})/\mathrm{H}_1\right\} \quad (7)$$

Evaluating both test statistics (6) in (7), for zero-mean AWGN of variance σ^2, the probability of error in making decisions is given by [6]

$$P_e = Q\left(\frac{1}{2}\sqrt{\frac{\left\|\mathbf{z}_1 - \mathbf{z}_0\right\|^2}{\sigma^2}}\right) \quad (8)$$

where $Q(.)$ denotes the Q-function [6] and $\|.\|$ denotes the norm.

It is obvious from the above equation that for a given noise level P_e depends on the Euclidian distance, $d = \left\|\mathbf{z}_1 - \mathbf{z}_0\right\|^2$, between the two test signals one of which contains only one echo and the other one contains two closely spaced echoes. The P_e can be also expressed in terms of the two test signal energies and correlation [6] as

$$P_e = Q\left(\sqrt{\frac{\bar{E}(1-\rho_s)}{2\sigma^2}}\right) \quad (9)$$

where $\bar{E} = \frac{1}{2}(\mathbf{z}_0^T\mathbf{z}_0 + \mathbf{z}_1^T\mathbf{z}_1)$ denotes the average energy and $\rho_s = \frac{\mathbf{z}_0^T\mathbf{z}_1}{\bar{E}}$ denotes the correlation coefficient. Without loss of generality we constrain the average energy of the two test signals to a constant and define Energy-to-Noise Ratio (ENR) as

$$ENR\,[\mathrm{dB}] = 10\log_{10}\left(\frac{\bar{E}}{\sigma^2}\right) \quad (10)$$

Furthermore, we constrain the energy of the test signal with two GCs to be equal to that of the test signal with one GC. This is necessary because the total energy of the two overlapping echoes fluctuates due to the constructive and destructive interferences as the TDOA between them is varied. Under these constraints, for a given ENR, the probability of error will depend on the correlation coefficient between the test signals (see Equation (9)). This correlation coefficient depends on the TDOA between the echoes as well as the parameter vector of the GC echo, i.e.,

$$\rho_s(\delta,\boldsymbol{\theta}) = \beta_0\beta_1\mathbf{s}^T(\boldsymbol{\theta};t)\mathbf{s}(\boldsymbol{\theta};t-0.5\delta) + \beta_0\beta_2\mathbf{s}^T(\boldsymbol{\theta};t)\mathbf{s}(\boldsymbol{\theta};t+0.5\delta) \quad (11)$$

The above equation relates δ and $\boldsymbol{\theta}$ to the correlation coefficient, which is in turn related to probability of error (see Equation (9) above). For a given resolution success rate ($\eta = 1 - P_e$), ENR, and a particular setting of the spectral parameters $\boldsymbol{\theta}$, ρ_s can be computed from Equation (11). The resolution limit δ_ℓ is defined as the smallest TDOA δ that attains the resolution success rate. Next section presents simulation results that estimate δ_ℓ for varying pulse shapes.

III. SIMULATION RESULTS

In order to assess the resolution limit, we simulate two GCs and adjust the degree of overlap by varying the TDOA (δ) between the echoes. GCs are generated by sampling the continuous model in Equation (2) with 100 MHz sampling rate. Fig.1 shows two GCs corrupted with AWGN (ENR = 30 dB) for a particular setting of the GC spectral parameters (e.g., $\boldsymbol{\theta} = [10(MHz)^2 \quad 7.1MHz \quad 10(MHz)^2 \quad 0]$) when the two echoes are completely separated (see Fig1.a), and completely overlapped (see Fig1.b). The TDOA is varied in small steps (one tenth of the sampling interval) and each time the correlation between the test signals ρ_s (11) and the probability of error P_e (9) are computed. The resolution success rate (η) is set to 0.99. Hence, when this success rate is attained (i.e., when P_e calculated via Equation (9) has dropped below 0.01) we recorded the TDOA and called it resolution limit (δ_ℓ) for this parameter setting and noise level (e.g., ENR).

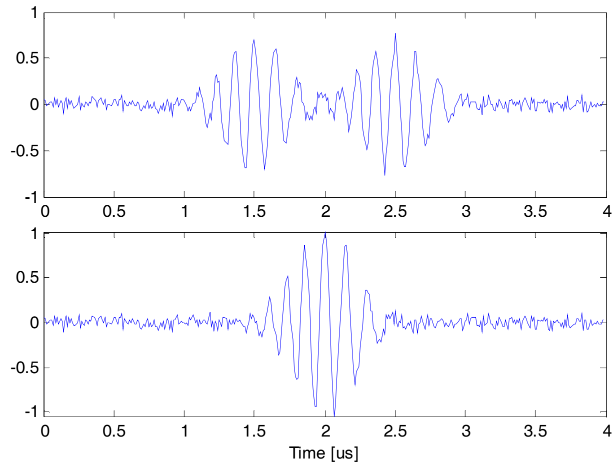

Fig. 1. Simulation of overlapping GC echoes by varying the TDOA between the echoes:
a) Two sufficiently separated GCs,
b) Two completely overlapped GCs.

Following the above procedure we estimated the resolution limit in terms of the spectral parameters of the pulse (bandwidth, center frequency, chirp rate, and phase) for 3 different noise level (ENR 10, 20, and 30 dB). Each time we varied one spectral parameter in a meaningful range in small steps while keeping others fixed. Fig.2 shows how the resolution limit varies with bandwidth factor parameter when other parameters are fixed. (Note that bandwidth factor parameter has a linear relationship with echo bandwidth.) The resolution limit is expressed in term of the sampling interval. As shown in the figure, resolution capacity increases (i.e., resolution limit gets smaller) as the bandwidth increases. The improvement in resolution limit with respect to bandwidth is more prominent when the noise level increased.

Fig. 3 shows the relationship between the center frequency and resolution limit when the other parameters are fixed. Improving the center frequency of the pulse provides greater resolution improvement than that of the bandwidth parameter. Furthermore, as the noise level increases, the center frequency plays a bigger role in resolution improvement. Fig. 4 shows the relationship between the resolution and the chirp rate (dispersion rate of a pulse). Increasing the chirp rate improves the resolution as expected. Similarly, Fig. 5 shows how the resolution limit changes with varying the phase of the pulse in range $[0\ 2\pi]$. It is seen that the resolution is independent of the phase. This is expected since the phase of the echo does not play a role in discriminating two overlapping echoes from one.

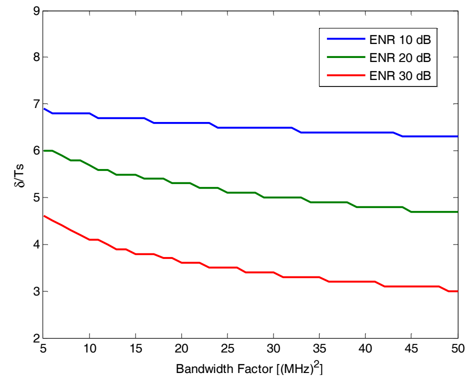

Fig. 2. Resolution limit vs bandwidth factor.

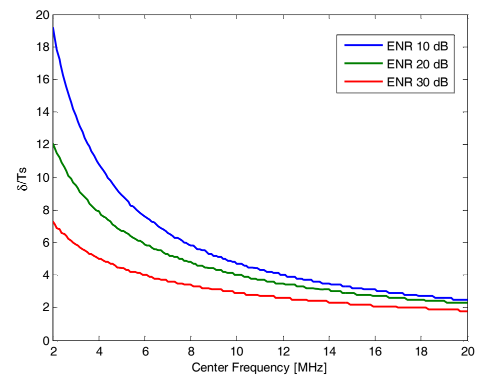

Fig. 3. Resolution limit vs center frequency.

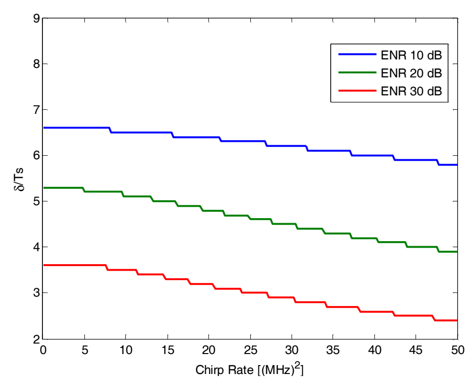

Fig. 4. Resolution limit vs chirp rate.

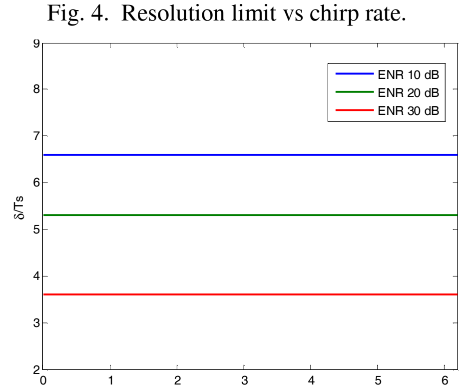

Fig. 5. Resolution limit vs phase.

IV. CONCLUSIONS

In this paper we proposed a numerical method to estimate the axial resolution limit of two closely spaced echoes. This limit is shown to depend on the noise level and spectral characteristics (bandwidth, center frequency, chirp rate) of the propagating echo. It is observed that the resolution limit is improved with increased frequency, bandwidth and chirp rate. It is also observed that resolution limit deteriorates with increased noise level.

It is important to note that in deriving the resolution limit we considered two overlapping echoes with equal amplitudes (i.e., two equal strength reflectors) for convenience. Generalization for unequal strength reflectors is possible within the proposed framework. It is expected that the resolution limit will deteriorate as one of the overlapping echoes gets weaker relative to the other one.

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