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Ultrasonic Chirplet Echo Parameter Estimation using Time-Frequency Distributions

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***Abstract*- Chirp signals are frequently encountered in ultrasonic non-destructive testing (NDT) and imaging applications. Chirplet echo parameters signify valuable information such as size, shape and orientation of the reflectors as well as the physical properties of propagation path in ultrasonic NDT applications. In most cases, detected echoes are overlapped due to microstructure scattering and minimal spacing of reflectors. This imposes challenges in decomposing the echoes and extracting the echo parameters accurately. Hence, it is very important to develop a methodology to efficiently and accurately estimate the chirplet echo parameters. In this paper, we present a method for estimating the parameters of chirp echo by means of ellipse fitting in time-frequency domain. This study uses two different time-frequency (TF) representations of ultrasonic signal to sequentially estimate the chirplet echo parameters: (i) Wigner-Ville Distribution (WVD), and (ii) Short Time Fourier Transform (STFT). To demonstrate the parameter estimation performance of ellipse fitting method, the signal decomposition and echo estimation algorithm is applied to perform decomposition of ultrasonic experimental signals consisting of many interfering echoes acquired from testing a steel block. The comparison between the reconstructed signal and the experimental signal shows that the decomposition has been successfully performed in the presence of measurement noise and strong interference from microstructure scattering.**

I. INTRODUCTION

Estimating the parameters of chirp signals is required in several practical ultrasonic testing applications such as nondestructive evaluation (NDE) methods, medical applications and sonar [1-3]. Chirplet parameters represent a broad range of echo shapes, including narrowband, broad band, symmetric, skewed, non-dispersive, or dispersive. Moreover, the estimated parameters represent the physical properties of the system, such as position and velocity of a target in radar or sonar target detection [4]. Furthermore, target size and orientation can be characterized in ultrasonic imaging. In addition, ultrasonic data compression is achievable via parameter estimation [5]. Both time domain and frequency domain approaches are applied in traditional estimators. Chirp echo parameters can be estimated based on the Time-Frequency Representation (TFR) of the signal in a successive manner [6].

In this study, we use two different time-frequency (TF) representations of a signal to estimate the chirplet echo parameters: (i) Wigner-Ville Distribution (WVD), and (ii) Short Time Fourier Transform (STFT). WVD provides high resolution, however suffers from cross term interference. STFT does not produce cross-terms, whereas the resolution may be poor.

WVD and STFT of a chirplet signal have the form of concentric ellipses in TF plane. Thus, this study utilizes the ellipse fitting method to locate the chirp echoes within elliptical shape in the TF plane. The outcome is several elliptical contours with varying axial lengths and amplitude. The ellipses with larger axial lengths will have smaller amplitude, which will suffer from noise interference, thus SNR will be low. However, these ellipses can more accurately estimate the parameters due to the availability of more number of points within the elliptical contour. On the other hand, the ellipses with smaller axial lengths have higher amplitude. These will have higher SNR; however, the accuracy of estimation may not be as good as compared to the ellipses with larger axial length. By considering multiple ellipses with different amplitudes for parameter estimation, the average of all the estimates can be used for the final estimation.

In this paper, we demonstrate the application of chirplet parameter estimator based on Ellipse Fitting Method (EFM) for chirplet signal decomposition (CSD). This has been evolved from our earlier work on chirplet parameter estimation and signal decomposition [5-9]. The EFM uses STFT in a successive manner to estimate chirplet parameters. The EFM has been successfully tested on both simulated ultrasonic echoes and experimental ultrasonic target echoes in grain scattering. This study has a wide range of applications such as data compression, ultrasonic velocimetry, ultrasonic ranging and target detection.

II. CHIRPLET PARAMETER ESTIMATION USING ELLIPSE FITTING

The ellipses identified from the chirplet signal, is represented in the form of a general elliptic equation as shown below.

$$At^2 + Bt\omega + C\omega^2 + Dt + E\omega + F = 0 \quad (1)$$

The TF representation of the chirplet signal is transformed into the form of a general ellipse equation. Then the parameters of TF representation are compared with the parameters of the ellipse to estimate the chirplet parameters.

A. Chirplet Parameter Estimation using WVD

Consider a signal

$$s(t) = \eta e^{\frac{-\propto(t-\tau)^2}{2}+\frac{-j\beta(t-\tau)^2}{2}+j\omega_0(t-\tau)+j\varphi}, \quad (2)$$

where $\propto$ is the bandwidth factor, β is the chirp rate, ω_0 is the center frequency, τ is the time of arrival, η is the amplitude, and φ is the phase.

The WVD of a chirplet is given by

$$W(t,\omega) = K\eta e^{-\propto(t-\tau)^2-\frac{(\omega-\beta(t-\tau)-\omega_0)^2}{\propto}}, \quad (3)$$

where K is the normalization constant.

Since the WVD has elliptical shape, we can represent the WVD in the form of ellipse equation as given below.

Any ellipse e^{-h} within the WVD representation of s(t) can be represented as

$$(\alpha^2+\beta^2)t^2 - 2\beta t\omega + \omega^2 + [2\beta\omega_0 - 2(\alpha^2+\beta^2)\tau]t + 2(\beta\tau+\omega_0)\omega + (\alpha^2+\beta^2)\tau^2 - 2\beta\omega_0\tau + {\omega_0}^2 - \alpha h = 0 \quad (4)$$

Comparing (4) with (1),

$$\text{A} = \alpha^2+\beta^2 \quad (5)$$

$$\text{B} = -2\beta \quad (6)$$

$$\text{C} = 1 \quad (7)$$

$$\text{D} = 2\beta\omega_0 - 2(\alpha^2+\beta^2)\tau \quad (8)$$

$$\text{E} = 2(\beta\tau+\omega_0) \quad (9)$$

$$\text{F} = (\alpha^2+\beta^2)\tau^2 - 2\beta\omega_0\tau + {\omega_0}^2 - \alpha h \quad (10)$$

From Equations (5) through (10), we can estimate the chirplet echo parameters $\propto$, β, ω_0, and τ

From (6), $\beta = -B/2$ (11)

From (5), $\alpha = \sqrt{A-\beta^2} = \sqrt{A-\frac{B^2}{4}}$ (12)

From (8) and (9),

$$\tau = \frac{\beta E - D}{2(\alpha^2+2\beta^2)} \quad (13)$$

$$\omega_0 = \frac{E-2\beta\tau}{2} \quad (14)$$

Substituting the values for $\propto$, β, ω_0, and τ, the phase φ can be determined from Equation (2) by identifying the amplitude of s(t) at a particular time instance t.

B. Chirplet Parameter Estimation using STFT

Consider the same signal s(t) given in Equation (2) with a Gaussian window $h(t) = \lambda e^{-at^2}$.

The STFT of a chirplet is presented as the power spectrum of the signal,

$$P_{SP}(t,\omega) = \frac{\sqrt{a\alpha}}{\pi\sqrt{(\alpha+a)^2+\beta^2}} e^{-[\frac{a\propto}{\propto+a}(t-\tau)^2+\frac{[\omega(\alpha+a)-a\beta(t-\tau)-\omega_0(\alpha+a)]^2}{(\propto+a)^2+\beta^2}]+j\varphi} \quad (15)$$

Since the STFT has elliptical shape, we can represent the STFT in the form of ellipse equation as given below.

Any ellipse e^{-h} within the STFT representation of s(t) can be represented as

$$[\frac{a\alpha}{\alpha+a}+\frac{a^2\beta^2}{(\alpha+a)^2+\beta^2}](t-\tau)^2 - \frac{2a\beta(\alpha+a)}{(\alpha+a)^2+\beta^2}\omega(t-\tau) + \frac{(\alpha+a)^2}{(\alpha+a)^2+\beta^2}\omega^2 + \frac{2a\beta\omega_0(\alpha+a)}{(\alpha+a)^2+\beta^2}(t-\tau)$$

$$-\frac{2\omega_0(\alpha+a)^2}{(\alpha+a)^2+\beta^2}\omega + \frac{(\alpha+a)^2}{(\alpha+a)^2+\beta^2}{\omega_0}^2 - h = 0 \quad (16)$$

Comparing (16) with (1),

$$\text{A} = [\frac{a\alpha}{\alpha+a}+\frac{a^2\beta^2}{(\alpha+a)^2+\beta^2}] \quad (17)$$

$$\text{B} = -\frac{2a\beta}{(\alpha+a)^2+\beta^2} \quad (18)$$

$$\text{C} = \frac{(\alpha+a)^2}{(\alpha+a)^2+\beta^2} \quad (19)$$

$$\text{D} = \frac{2a\beta\omega_0(\alpha+a)}{(\alpha+a)^2+\beta^2} - 2\tau\left[\frac{a\alpha}{\alpha+a}+\frac{a^2\beta^2}{(\alpha+a)^2+\beta^2}\right]$$

$$= -B\omega_0(\alpha+a) - 2\tau A \quad (20)$$

$$\text{E} = \frac{2(\alpha+a)[a\beta\tau-\omega_0(\alpha+a)]}{(\alpha+a)^2+\beta^2} \quad (21)$$

$$\text{F} = [\frac{a\alpha}{\alpha+a}+\frac{a^2\beta^2}{(\alpha+a)^2+\beta^2}]\tau^2 + \frac{(\alpha+a)^2{\omega_0}^2-2a\beta\omega_0(\alpha+a)\tau}{(\alpha+a)^2+\beta^2} - h \quad (22)$$

The peak of $P_{SP}(t,\omega)$ can be used to estimate the center frequency ω_0 and time of arrival τ. The maximum can be obtained by taking the partial derivatives of $P_{SP}(t,\omega)$ with respect to t, which corresponds to the time of arrival τ, and ω, which corresponds to center frequency ω_0 [See Equations (23) and (24].

$$\frac{\partial P_{SP}(t,\omega)}{\partial t} = P_{SP}(t,\omega).(-2).$$

$$\{\left[\frac{a\alpha}{\alpha+a} + a^2\beta^2\right](t-\tau) - a\beta(\alpha+a)(\omega_0-\omega)\} = 0 \quad (23)$$

$$\frac{\partial P_{SP}(t,\omega)}{\partial \omega} = P_{SP}(t,\omega).(-2).$$

$$[(\alpha+a)^2(\omega-\omega_0) - a\beta(\alpha+a)(t-\tau)] = 0 \quad (24)$$

The solutions of (23) and (24) provide an estimation of time of arrival and center of frequency:

$$t = \tau \quad \text{and} \quad \omega_0 = \omega \quad (25)$$

Now, from Equation (20), $\propto$ can be estimated as

$$\propto = -a - \frac{D+2\tau A}{B\omega_0} \quad (26)$$

From Equations (18) and (19),

$$\frac{B}{C} = -\frac{2a\beta}{(\alpha+a)^2} \quad (27)$$

Thus, β can be estimated as

$$\beta = -\frac{B}{C}\frac{(\alpha+a)^2}{2a} \quad (28)$$

The peak value of $P_{SP}(t,\omega)$ is proportional to the amplitude of the actual echo. We are interested in the relative amplitude between the echoes, not the actual amplitude of individual echoes. Thus, the peak of $P_{SP}(t,\omega)$ can be considered as the estimated amplitude η as given below.

$$\eta = \frac{\sqrt{a\alpha}}{\pi\sqrt{(\alpha+a)^2+\beta^2}} \quad (29)$$

Now, from Equation (15), we can find the phase φ, since we know every other parameter for the peak value.

III. ELLIPSE FITTING ALGORITHM

The parameters of the ellipse are estimated by using direct least squares ellipse fitting method [10]. Since the contour lines corresponding to the chirplet components in TF plane have elliptical shape, direct least squares ellipse-fitting method provides accurate estimation of the chirplet parameters. The parameter estimation procedure for this method is described below.

A general conic curve can be represented by:

$$\mathrm{F}(\boldsymbol{\gamma},\mathbf{z}) = \boldsymbol{\gamma}.\mathbf{z} = ax^2 + bxy + cy^2 + dx + ey + f = 0 \quad (30)$$

where $\boldsymbol{\gamma} = [a\ b\ c\ d\ e\ f]^T$ and $\mathbf{z} = [x^2\ xy\ y^2\ x\ y\ 1]^T$. $\mathrm{F}(\boldsymbol{\gamma},\mathbf{z})$ is considered as algebraic distance of a point (x_i, y_i) to the curve $\mathrm{F}(\boldsymbol{\gamma},\mathbf{z}) = 0$. Best fitting to the N data points is achieved by minimizing the sum of squared algebraic distance:

$$\sum_{i=1}^{N}[\mathrm{F}(\boldsymbol{\gamma},\mathbf{z})]^2 \quad (31)$$

where $\boldsymbol{\gamma}$ is constrained to avoid the trivial solution $\boldsymbol{\gamma} = 0$. The constraint applied in Equation (30) is $4ac - b^2 = 1$ which can be expressed in the form $\boldsymbol{\gamma}^T\mathbf{C}\boldsymbol{\gamma} = 1$ where $\mathbf{C}$ is:

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (32)$$

The minimization can be solved by considering eigenvalue system [11]:

$$(\mathbf{D}^T\mathbf{D} - \lambda\mathbf{C})\boldsymbol{\gamma} = 0 \quad (33)$$

where

$$\mathbf{D} = \begin{bmatrix} x_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ x_n^2 & x_ny_n & y_n^2 & x_n & y_n & 1 \end{bmatrix} \quad (34)$$

After obtaining the eigenvalues λs by solving $\det(\mathbf{D}^T\mathbf{D} - \lambda\mathbf{C}) = 0$, we can calculate the parameter vector $\boldsymbol{\gamma}$ using Equation (31) by solving a 5 by 5 system of linear equations. The proposed ellipse fitting method is computationally efficient compared to the other iterative and general conic fitting methods [12].

IV. CHIRPLET SIGNAL DECOMPOSITION

A complex signal can be successively decomposed into Gaussian chirplets using EFM. Since WVD suffers from cross terms, STFT is used for the decomposition. At the end of the decomposition, the original signal is expressed as a linear combination of chirplet echoes as shown in Equation (35).

$$s(t) = \sum_{j=0}^{N-1} f_{\Theta_j}(t) \quad (35)$$

where $f_{\Theta_j}(t)$ is the chirplet echo and Θ_j is the corresponding chirplet parameter vector.

The process of decomposition is detailed below.

1) Initially, by identifying the maxima point in the STFT of the signal, the most dominant echo along with its time of arrival and center frequency are determined.

2) After step1, the ellipse is placed on the selected contour line in the TF plane of the signal, and the bandwidth factor, amplitude and chirp rate are estimated.

3) Finally, reconstruction error is calculated by subtracting the estimated echo from the reconstructed signal.

Steps 1-3 are repeated in order to reach an acceptable reconstruction error.

Estimated echoes for two iterations are depicted in Figure 1 (shown in red color).

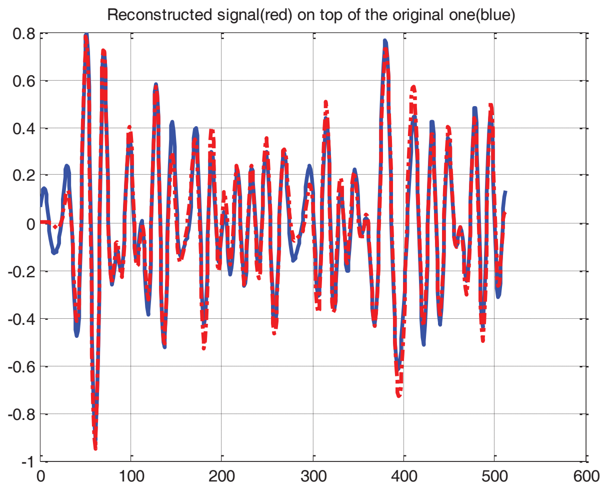

Figure 1. Reconstructed signal superimposed with the original

In this study, EFM is applied to ultrasonic measurements consisting of overlapping echoes. These measurements are acquired from a steel block specimen using a 5 MHz ultrasonic transducer and a 100 MHz sampling rate. The comparison between the experimental and reconstructed signal (see Figure 1) indicates that the EFM is able to efficiently decompose the original signal in the presence of measurement noise.

V. Conclusion

Chirplet echo parameter estimation is significant for many ultrasonic NDT applications. This study utilizes ellipse fitting method for sequentially estimating the parameters of chirp echo using two different time-frequency (TF) representations (WVD and STFT) of ultrasonic signal. The ultrasonic experimental signal consisting of many interfering echoes are decomposed and the echoes are accurately estimated to demonstrate the efficiency of the proposed echo estimation method. The experimental results indicate that ultrasonic flaw signal can be efficiently and accurately estimated even in the presence of noise.

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