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Singular spectrum analysis for trend extraction in ultrasonic backscattered echoes

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***Abstract*— In this investigation, singular spectrum analysis (SSA) is explored to decompose and analyze ultrasonic signals in nondestructive evaluation (NDE) applications. Unlike transform-based algorithms, SSA is a time-series analysis algorithm which is completely driven by the signal itself. As a result, ultrasonic signal is decomposed through a four-step processing, that is embedding, singular value decomposition (SVD), grouping and diagonal averaging. The decomposition result can be used to characterize ultrasonic signals for the NDE of materials. Simulation results show that SSA reveals signal trend related to defects and grain scatters. The performance of the algorithm is also compared with other algorithms of ultrasonic signal decomposition for signal analysis and feature extraction. Numerical and analytical results demonstrate that SSA is effective in ultrasonic signal analysis. Especially the data-driven nature makes SSA unique for signal analysis. The algorithm can be utilized for flaw detection, signal classification, and pattern recognition in NDE applications.**

Keywords– Singular spectrum analysis, signal decomposition, Ultrasonic NDE, backscattered echoes

I. INTRODUCTION

In ultrasonic imaging applications, the received signal always carries important physical information along the wave propagation path. It includes the location, size, and orientation of discontinuities, as well as attenuation and dispersion characteristics of material. Due to the non-stationary feature of the signal, it is impractical to use classic signal processing techniques for signal analysis and information interpretation. As such, different time-frequency analysis techniques and various transformations have been utilized to examine ultrasonic signal for NDE applications. In [1], short-time Fourier transform (STFT) is utilized to estimate the ultrasonic attenuation and dispersion. In [2-5], discrete wavelet transform and chirplet transform have been used to perform multi-resolution analysis, signal decomposition and parameter estimation on ultrasonic signal. In addition, Wigner-Ville distribution (WVD) is introduced for ultrasonic applications to achieve a higher resolution in time-frequency representation [6]. However, the cross-term property of WVD becomes problematic for complex signal analysis, especially for those signals with multi-components. As a remedy, modified WVDs are used to counteract the disadvantage of cross-term interference in time-frequency analysis [7-10]. Nevertheless, it still remains as a challenging problem to extract the desired signal information for structural health monitoring, nondestructive evaluation, material characterization and quality control.

In recent years, there have been some research progress in time-series analysis. Unlike transform-based techniques, which projects signal into orthogonal or non-orthogonal subspaces under various optimization criteria, time-series analysis is a technique fully driven by the signal itself. Two commonly used algorithms are empirical mode decomposition (EMD) and singular spectrum analysis (SSA). Thanks to their intrinsic adaptive feature, that provide unique advantages in nonlinear and non-stationary signal processing. In EMD, a signal is decomposed into a set of intrinsic mode functions (IMF) directly derived from local minima and maxima of the signal [11-13], whereas an embedded trajectory matrix is used for trend extraction and signal decomposition in SSA. It has been used to analyze non-stationary data in numerous applications such as biomedical, speech, geophysical, meteorology etc [14-18]. In this investigation, SSA is explored to decompose and analyze ultrasonic signal in NDE applications. Specifically, it is implemented through a four-step processing (i.e., embedding, SVD, grouping and diagonal averaging). First, a trajectory matrix is obtained from the signal, then SVD is performed on the trajectory matrix to obtain its eigenvalues and eigenvectors, where eigenvalues are sorted and regrouped. Finally, diagonal averaging is applied to reconstruct time series components corresponding to different groups of eigenvalues.

This paper is organized as follows: Section II reviews singular spectrum analysis and describes how it is applied to extract trend in ultrasonic signals. Section III presents simulation/experimental results and the performance evaluation of SSA. Furthermore, trend extraction and eigenvalue sorting are discussed is section III. . Section IV concludes the paper.

II. SINGULAR SPECTURM ANALYSIS AND TREND EXTRACTION

Given a sequence, $s(n)$, where $0 \le n \le N-1$, the general procedure of SSA is described below [19].

1. Embedding

   In this step, the signal, $s(n)$, is mapped to multidimensional vectors by applying a L-length window, where the L-dimensional vectors are used to construct a trajectory matrix $\mathbf{T}$.

$$\mathbf{T}=\begin{bmatrix} s(0) & s(1) & \cdots & s(K-1) \\ s(1) & s(2) & \cdots & s(K) \\ \cdots & \cdots & \cdots & \cdots \\ s(L-1) & s(L) & \cdots & s(N-1) \end{bmatrix} \quad (1)$$

where L is the length of window, N is the length of $s(n)$, and $K=N-L+1$. Without loss of generality, it is assumed that $L\le\frac{N}{2}$ and $L\le K$.

2. Singular value decomposition

For the $L\times L$ matrix, $\mathbf{S}=\mathbf{T}\,\mathbf{T}^T$, find its sorted eigenvalue matrix

$$\boldsymbol{\lambda}=\begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ 0 & 0 & \cdots & \cdots \\ 0 & 0 & \cdots & \lambda_L \end{bmatrix} \quad (2)$$

where $\lambda_1\ge\lambda_2\ge\cdots\ge\lambda_L\ge 0$

and the corresponding $L\times 1$ orthonormal eigenvectors $\mathbf{U}_i$, where $\mathbf{S}\mathbf{U}_i=\lambda_i\,\mathbf{U}_i$ and $\|\mathbf{U}_i\|=1$ for $0\le i\le L$.

Then trajectory matrix, $\mathbf{T}$, can be written as follows

$$\mathbf{T}=\mathbf{U}\begin{bmatrix} \sqrt{\lambda_1} & 0 & \cdots & 0 \\ 0 & \sqrt{\lambda_2} & \cdots & 0 \\ 0 & 0 & \cdots & \cdots \\ 0 & 0 & \cdots & \sqrt{\lambda_L} \end{bmatrix}\mathbf{V}^T \quad (3)$$

where $\mathbf{U}=[\mathbf{U}_1 \quad \mathbf{U}_2 \quad \cdots \quad \mathbf{U}_L]$

$\mathbf{V}=[\mathbf{V}_1 \quad \mathbf{V}_2 \quad \cdots \quad \mathbf{V}_L]$

and $\mathbf{V}_i=\frac{\mathbf{T}^T\mathbf{U}_i}{\lambda_i},\ 0\le i\le L$

To facilitate the next step (i.e., grouping), Equation (3) can be rewritten as

$$\mathbf{T}=\sum_{i=1}^{L}\mathbf{T}_i \quad (4)$$

where $\mathbf{T}_i=\sqrt{\lambda_i}\mathbf{U}\,\mathbf{V}^T,\ 0\le i\le L$

There may exist $\lambda_i=0$ for $d<i\le L$. If that is the case, only those d positive eigenvalues are chosen to form the SVD of $\mathbf{T}$ [19].

3. Grouping

From Equation (4), it can be seen that the $\mathbf{T}_i$ corresponds to the eigenvalue $\sqrt{\lambda_i}$. These matrices can be regrouped into m disjoint subsets.

$$\mathbf{T}=\sum_{i=1}^{m}\mathbf{I}_i$$

There are various strategies to break $\mathbf{T}$ into subsets $\mathbf{I}_i$ [19-20]. A common strategy of obtaining $\mathbf{I}_i$ is to group those $\mathbf{T}_i$ with closest eigenvalues. The way of breaking out eigenvalues is not necessarily unique, especially when the break points of eigenvalues are not clear. Other methods have been proposed in [19-21]. In addition, signal trends in $s(n)$ could be revealed by the reconstructed signals from those subsets. As such, the grouping process is a form of harmonic identification [13].

4. Diagonal average

An N-sample sequence can be obtained from a $L\times K$ matrix through diagonal averaging. Assuming the $L\times K$ matrix to be processed is $\mathbf{Y}$ and $y_{i,j}$ denotes the element (i,j) in $\mathbf{Y}$, the diagonal averaging operation is described below.

$$g(n)=\begin{cases} \dfrac{\sum_{m=1}^{n+1} y_{m,n-m+2}}{n+1} & 0\le n<L-1 \\ \dfrac{\sum_{m=1}^{L} y_{m,n-m+2}}{L} & L-1\le n<K \\ \dfrac{\sum_{m=n-K+2}^{N-K+1} y_{m,n-m+2}}{N-n} & K\le n<N \end{cases} \quad (5)$$

As a result, N-sample sequences can be reconstructed from each grouped matrix in $\mathbf{I}_i$, $0\le i\le m$ through diagonal average.

Therefore, through the four steps in SSA, the signal, $s(n)$, can be decomposed into m components.

$$s(n)=\sum_{i=1}^{m}g_i(n)+r(n) \quad (6)$$

Here $g_i(n)$ denotes the ith component, $r(n)$ denotes the residue.

III. SIMULATION STUDY

To introduce the SSA for ultrasonic pulse-echo system, it is useful to analyze the SSA on ultrasonic chirp echoes, commonly encountered in ultrasonic backscattered signal representing a broad range of echoes such as narrow-band, broad-band, dispersive and non-dispersive echoes. In the ultrasonic NDE application, an ultrasound echo signal, $x(t)$, can be modeled as [5]

$$x(t) = s(t) + n(t) = \sum_{k=1}^{K} h_{\Theta_k}(t) + n(t) \qquad (7)$$

where $s(t)$ denotes a reflected ultrasound signal,

$n(t)$ denotes a white Gaussian noise.

$h_{\Theta_k}(t)$ denotes Gaussian Chirplet (GCs) represented by

$$h_{\Theta_k}(t) = a_k\, e^{-\alpha_{1k}(t-\tau_k)^2} \cos\left(2\pi f_{ck}(t-\tau_k) + \alpha_{2k}(t-\tau_k)^2\right) \qquad (8)$$

Here the parameter vector of the kth GC $\Theta_k = [\tau_k \quad f_{ck} \quad a_k \quad \alpha_{1k} \quad \alpha_{2k}]$ consists of the time-of-arrival τ_k, the center frequency f_{ck}, the amplitude a_k, the bandwidth factor α_{1k}, and the chirp-rate, α_{2k}.

An ultrasonic signal including multiple interfering echoes is simulated for this study. In Figure 1a, a noisy ultrasonic signal includes two chirplets with parameters $\Theta_1 = [4\mu s \quad 5MHz \quad 1 \quad 25MHz^2 \quad 20MHz^2]$ and $\Theta_2 = [5\mu s \quad 6MHz \quad 1 \quad 35MHz^2 \quad 15MHz^2]$.

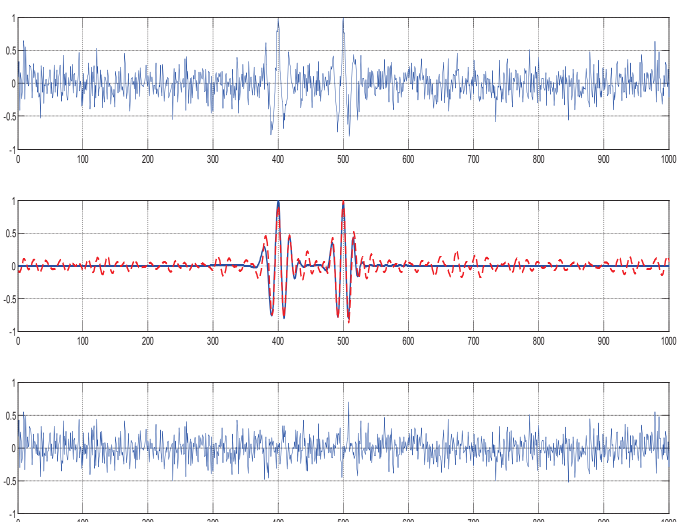

Figure 1. a) Simulated noisy ultrasonic signal b) The noise-free signal superimposed with the signal reconstructed by using the first 16 dominant eigenvalues c) Residue

Figure 1b displays the noise-free signal superimposed with the reconstructed signal with first 16 dominant eigenvalues. It shows that the SSA process is effective in reducing noise and catching trends in the signal. To examine the impact of eigenvalues on the trend extraction, Figure 2 shows the plot of sorted eigenvalues. It can be seen that eigenvalues drops rapidly after the first 16 values. Furthermore, if the signal is decomposed into 3 components for the first 30 eigenvalues, the related spectral information is shown in Figure 3. The spectrum of signal based on the reconstruction from the 1st and 2nd most dominant eigenvalues has a peak at 4.98 MHz, which is very close to 5 MHz , the true value of 1st simulated chirp echo. The spectrum of signal based on the reconstruction from the 3rd and 4th dominant eigenvalues has a peak at 5.96 MHz, which is also close to 6 MHz, the true value of the 2nd simulated chirp echo.

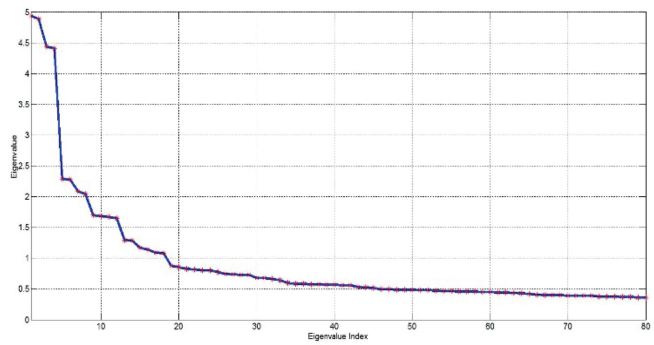

Figure 2 Eigenvalues in singular spectrum analysis (L = 200)

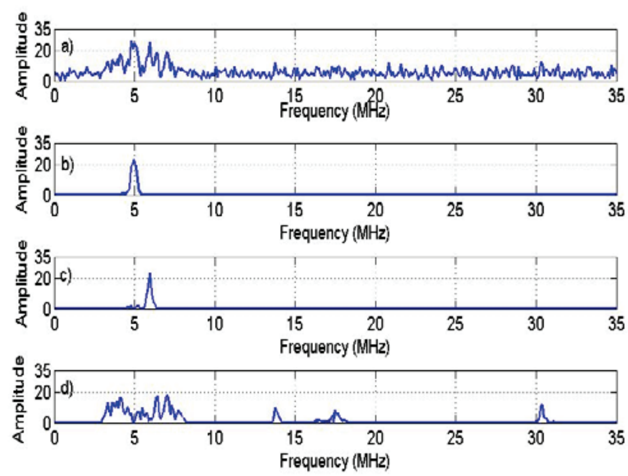

Figure 3. a) Spectrum of the simulated signal b)Spectrum of signals with 1st and 2nd eigenvalues (Peak at 4.98 MHz; True value : 5 MHz) c) Spectrum of signals with 3rd and 4th eigenvalues (Peak at 5.96 MHz; True value: 6 MHz)
d) Spectrum of signals with 5th to 30th eigenvalues

Figure 4 shows the scenario of highly overlapped chirp echoes. It demonstrates that the SSA has its own advantage of noise reduction and tracks the signal trend. Unlike other ultrasonic signal decomposition algorithms [4-5], the SSA algorithm does not provide high-resolution signal representation in time-frequency domain. Due to the nonparametric nature of SSA, quantitative results such as parameter estimations are not available in the SSA. Nevertheless, it still shows its unique feature in terms of trend extraction and noise reduction. It could also be utilized to assist other algorithms to achieve better performance.

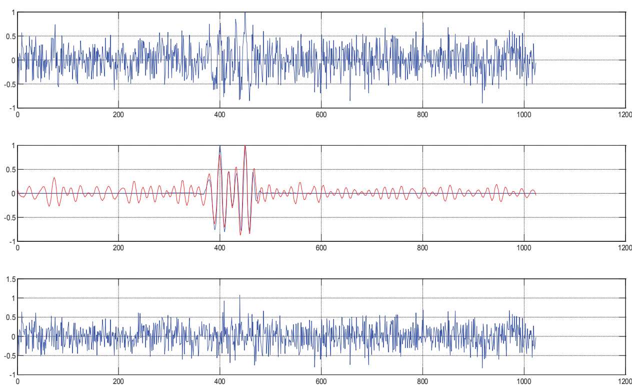

Figure 4. a) Simulated noisy ultrasonic signal b) The noise-free signal superimposed with the reconstructed signal using the first 16 dominant eigenvalues c) Residue

IV. CONCLUSION

In this study, the application of singular spectrum analysis is demonstrated in ultrasonic signal processing. Simulation results show that the singular spectrum analysis reveals signal trend related to defects and grain scatters. The performance of algorithm is discussed and compared with other algorithms of ultrasonic signal decomposition for signal analysis and feature extraction. The study shows that singular spectrum analysis is effective in ultrasonic signal analysis. Especially the data-driven nature makes singular spectrum analysis unique for signal analysis. This algorithm can be utilized for flaw detection, signal classification, and pattern recognition in NDE applications.

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