Model-based parameter estimation for defect characterization in ultrasonic NDE applications

Yufeng Lu* and Jafar Saniie**

*Department of Electrical and Computer Engineering, Bradley University, Peoria, IL 61625

**Department of Electrical and Computer Engineering, Illinois Institute of Technology, Chicago, IL 60616

Abstract—In this study, a model-based parameter estimation algorithm is applied to characterize defects for ultrasonic NDE applications. Chirplet, a signal model representing a broad range of ultrasound echoes, is utilized for signal decomposition and parameter estimation. The model-based parameter estimation algorithm and its results are discussed in the context of system modeling. Through simulation and benchmark data analysis, the effectiveness of model-based signal processing has been successfully demonstrated. This method can be used as a quantitative method for applications in flaw detection, defect estimation, pattern recognition and material characterization.

Keywords—ultrasound model, parameter estimation, NDE application

I. INTRODUCTION

In ultrasonic NDE applications, modeling is one of the most important aspects to improve our understanding of ultrasound physics and develop better techniques for material characterization. There have been tremendous efforts in modeling of ultrasound system and ultrasonic signal. A general electromechanical reciprocity is derived to calculate elastic wave scattering coefficients [1-2]. Based on the assumption that the reflector (flaw) is sufficiently small and the condition of liquid-solid interface, a further simplified model, i.e., quasi-plane wave measurement model, is established explicitly in [3]. It has been shown that ultrasound system model generally includes three important components [4-5]. They are beam model which characterizes the wave from the transducer, flaw model which represents how the flaw interacts with ultrasound, and a lumped system transfer function which includes the behaviors of all electrical and mechanical components in the system [4-5]. In addition, the transducer geometry, material properties, and wave type have to be taken into account. The beam model would be different for circular, spherical or rectangular probe, planar or focused [5-7]. Out of many beam models, the multi-Gaussian beam model is commonly used [8-11]. [5,10] shows an example of how to compute multi-Gaussian beam coefficients. [4-5] reveals that the plane wave transmission coefficients and frequency-dependent material attenuation terms can be calculated based on the physical parameters of medium interface. In addition, there are various methods to mathematically describe flaw scattering properties. Two commonly used methods are Kirchhoff approximation and separation of variables [4-5]. There are many other factors such as the medium interface (planar or curved), the size of reflectors (small or large), the distance of reflectors (near field or far field), and the shape of reflectors (spherical, circular, or cylindrical) to name a few [5]. Despite the model simplification, it still remains as an extremely challenging problem to model ultrasound measurement system precisely since there are many factors and variables which strongly impact the accuracy of modeling. It is worthy to mention that there have been benchmark studies aiming to quantitatively evaluate manufactured flaws such as side-drilled hole, flat-bottom hole, etc since 2002. Both planar and focused transducers are used for contact or immersion ultrasonic testing [12-14, 21].

Instead of focusing on the mechanism of ultrasound wave generation in NDE systems, ultrasonic signal analysis concentrates on another facet of modeling. The received signal carries the important physical information of reflectors along the propagation path of ultrasound. The information such as the size, location and orientation of reflectors is critical to characterize material. Unfortunately conventional signal processing technique fails to unravel the desired information from these highly overlapped echoes. Therefore, signal processing techniques which can isolate these echoes and reveal the valuable information are highly sought after. Signal modeling and parameter estimation is one of effective ways to decompose ultrasonic signal [15-20]. Furthermore, signal modeling facilitates time-frequency representation with high resolution [17-19]. Chirplet is a signal model commonly used to represent a broad range of ultrasonic echoes. Six parameters in the model can be tuned to account for non-dispersive, dispersive, symmetric, skewed, narrow or broad band echoes [17-18]. To quantitatively characterize defects in ultrasonic NDE applications, it is interesting to connect the effort in model-based signal analysis with ultrasound system modeling. This study aims to present model-based parameter estimation algorithm in the context of ultrasound system modeling for ultrasonic NDE applications.

This paper is organized as follows: Section II reviews ultrasound system modeling including multi-Gaussian beam model, material attenuation and flaw scattering mode. It also reviews chirplet-based signal decomposition and parameter estimation. Section III shows simulation and benchmark study. Section IV concludes the paper.
II. ULTRASOUND SYSTEM MODEL AND CHIRPLET SIGNAL DECOMPOSITION IN ULTRASOUND

The general equation for a received A-scan voltage in ultrasound pulse-echo system can be written as [5]. (assuming P-wave and liquid-solid medium here).

\[ V_r(\omega) = H(\omega) \left\{ \frac{4\pi\rho_x c}{-ik_z Z_{\text{in}}} \right\} \sum \hat{V}^{(i)}(X, \omega) \hat{V}^{(j)}(X, \omega) A(X, \omega) e^{i\beta \cdot X} dS \]  

(1)

where \( H(\omega) \) denotes system function; \( \hat{V}^{(i)}(X, \omega) \) denotes the normalized velocity in medium \( \alpha \), which is mainly described by the beam model. For instance, in the case of liquid-solid interface, \( \alpha=1 \) stands for liquid, \( \alpha=2 \) stands for solid; \( X \) denotes the coordinates on the surface \( S \); \( A(X, \omega) \) denotes the flaw scattering function; \( k_z \) denotes the wave number; \( Z_{\text{in}}^{(\alpha)} \) denotes the equivalent acoustic impedance; \( \rho_z \) denotes the density of medium, \( c_z \) denotes the sound speed and \( e \) denotes the unit vector in the direction of incident wave propagation.

Depending on the type of flaw (spherical, circular, or cylindrical) and the size of flaw (small, large), further simplification can be applied to Equation (1). In this investigation, a benchmark data from the World Federation of NDE centers is utilized for signal analysis, with two different sizes of side drilled holes (SDH) are examined. The ultrasound system setup is shown in Figure 1.

![Figure 1](Image)

**Figure 1.** Experiment setup for SDH in Aluminum (Benchmark 2004)

1. Flaw scattering amplitude: \( A(X, \omega) \)

   For the cylindrical reflectors (i.e., SDH here) shown in Figure 1, the \( A(X, \omega) \) can be described using far-field scattering models. It is computed using separation of variables [5].

2. Beam model \( \hat{V}^{(i)}(X, \omega) \) and \( \hat{V}^{(j)}(X, \omega) \)\(^{[5,10]}\)

   The multi-Gaussian beam model is adopted in the simulation. An integrated beam velocity term is calculated using the Wen-and-Breazeale coefficients.

3. System function \( H(\omega) \)

   The system function, \( H(\omega) \), can be modeled as a simple Gaussian function with the center frequency \( f_c \), and bandwidth of the transducer [5]. However, the cruel approximation could not accurately describe the system. With the help of Wiener filter, the system function can be obtained based on an experimental reference signal from the front surface.

\[ H(\omega) = \frac{V_r(\omega) e^{\text{acoustic}}(\omega)}{V^{\text{acoustic}}(\omega) + \xi^2 M^2} \]  

(2)

where \( M = \max|e^{\text{acoustic}}(\omega)| \), and the constant, \( \xi \) stands for the noise level, \( e^{\text{acoustic}}(\omega) \) denotes acoustic/elastic transfer function,

\[ t^{\text{acoustic}}(\omega) = 2R_1 e^{2\pi f_c D} \left[ J_0 \left( \frac{k_1 a^2}{2D} \right) - iJ_1 \left( \frac{k_1 a^2}{2D} \right) \right] \]  

(3)

where \( J_0(\bullet) \) and \( J_1(\bullet) \) denote 0-order and 1st-order Bessel functions, \( D \) is the water path from the transducer to the front surface, \( a \) is the radius of transducer, \( k_1 \) denotes the wave number, and \( R_1 \), the ratio of pressure on the interface, which is determined by the densities and sound speeds.

As discussed in section I, a model-based parameter estimation algorithm will be exploited in the environment of ultrasound system modeling. In particular, the chirplet signal decomposition and parameter estimation algorithm is summarized as follows [17]. It is assumed that a complex ultrasound signal, \( s(t) \), can be represented by a linear combination of chirplets \( f_{\Theta_j}(t) \) with various parameter vectors \( \Theta_j \)

\[ s(t) = \sum_{j=0}^{N-1} f_{\Theta_j}(t) + n(t) \]  

(4)

where \( f_{\Theta_j}(t) = \beta e^{-\alpha_1(t-\tau_1)^2} e^{i2\pi f_c(t-\tau_1)+i\phi+\alpha_2(t-\tau_1)^2} \)

\[ \Theta_j = [\tau \ f_c \ \beta \ \alpha_1 \ \alpha_2 \ \phi] \]

\( n(t) \) denotes the reconstruction error and system noise

III. SIMULATION AND EXPERIMENTAL STUDY

To have a fair comparison, following the experimental setting shown in Figure 1, a planar transducer with 5 MHz nominal center frequency and 12.7 mm diameter is utilized in the simulation. Water-aluminum interface is adopted for the ultrasound pulse-echo system. The water path from the transducer to front surface is 50.8 mm. A series of SDH flaws with their diameters ranging from 1 mm to 4 mm with 0.1 mm
increment are used for simulation. Based on ultrasound system modeling (see Section II), the ultrasound echoes from these SDH flaws are simulated, then analyzed by the chirplet signal decomposition algorithm for parameter estimation and signal reconstruction.

From Equation 3, it can be seen that the simulated ultrasound echo is represented in the format of spectrum in the frequency domain. It becomes difficult to directly estimate the TOA of echoes through Equation 3. To overcome the problem, the TOA from the front surface reference signal is estimated first. Then the reference TOA is utilized as a baseline to calibrate all estimated results for flaw echoes. It is also noticed that the estimated center frequency is lower than the nominal center frequency of transducer. As such, the estimated center frequency of front surface echo and its 3-dB bandwidth are used in the simulation of ultrasound system modeling.

The estimation results for the most important parameters such as time-of-arrival (TOA), center frequency and amplitude are reported in Figure 2. It shows that the model-based parameter estimation can successfully track the TOA and center frequency. In addition, it can be seen that the estimated value of amplitudes is almost proportional to the size of SDH. Therefore, the parameters could be utilized to characterize flaws quantitatively.

For the experimental study, the SDH data from the 2004 benchmark has been analyzed [21]. The estimated parameters have been compared with those from simulation. Table 1 and 2 list the estimated parameters from the simulated ultrasound signals and the benchmark data. It can be seen that those parameters are estimated accurately. Furthermore, for the 1-mm SDH, Figure 3 shows the reconstructed signals for the experimental reference signal, the simulated 1-mm SDH ultrasound signal, and the experimental 1-mm SDH ultrasound signal. The similar results for the 4-mm SDH are plotted in Figure 4. The model-based parameter estimation algorithm estimates and reconstructs the signals with high accuracy. Furthermore, it can be seen that the simulated ultrasound signals are well aligned with those experimental data.

![Figure 2](image1.png)

**Figure 2** a) Estimated time-of-arrival (TOAs) changes with the diameter of SDH  b) Estimated center frequency (fc) changes with the diameter of SDH  c) Estimated amplitude changes with the diameter of SDH

![Figure 3](image2.png)

**Figure 3** a) Experimental front surface reference signal superimposed with the estimated signal using model-based algorithm  b) Simulated flaw signal (1mm SDH) superimposed with the estimated signal using model-based algorithm  c) Experimental flaw signal (1 mm SDH) superimposed with the estimated signal using model-based algorithm

![Figure 4](image3.png)

**Figure 4** a) Experimental front surface reference signal superimposed with the estimated signal using model-based algorithm  b) Simulated flaw signal (4mm SDH) superimposed with the estimated signal using model-based algorithm  c) Experimental flaw signal (4mm SDH) superimposed with the estimated signal using model-based algorithm

<table>
<thead>
<tr>
<th>Time-of-arrival [us]</th>
<th>Simulated data</th>
<th>Experimental data</th>
</tr>
</thead>
<tbody>
<tr>
<td>76.52</td>
<td>76.57</td>
<td></td>
</tr>
<tr>
<td>Center frequency [MHz]</td>
<td>4.52</td>
<td>4.49</td>
</tr>
<tr>
<td>Amplitude [Volt]</td>
<td>0.0238</td>
<td>0.0227</td>
</tr>
</tbody>
</table>

Table 1. Estimated parameter comparison of simulated flaw signals and experimental flaw signals (1 mm SDH)
IV. CONCLUSION

In this paper, chirplet model is used for signal decomposition and parameter estimation in the context of ultrasound system modeling. Simulation and benchmark results indicate that the model-based algorithm offers an efficient way of characterizing defects. This method successfully associates the estimated chirplets and their parameters as a quantitative technique to characterize defects. Additionally, this type of study can be utilized in ultrasonic NDE to assess the structural integrity of materials.

V. ACKNOWLEDGMENT

The authors would like to thank the World Federation of NDE centers for the ultrasonic benchmark data 2004 used in this paper.

REFERENCES