A Comparative Study of Singular Spectrum Analysis and Empirical Mode Decomposition for Ultrasonic NDE

Yufeng Lu* and Jafar Saniie**

*Electrical and Computer Engineering Department
Bradley University
Peoria, Illinois 61625

** Embedded Computing and Signal Processing Laboratory
Electrical and Computer Engineering Department
Illinois Institute of Technology, Chicago, Illinois 60616

Abstract— **In this investigation, a comparative study of two time-series analysis techniques, singular spectrum analysis (SSA) and empirical mode decomposition (EMD), is carried out for ultrasonic NDE applications. Unlike transform-based approaches, the SSA and EMD are fully driven in the time domain by the data itself. The SSA method captures the trend of signals via eigenvalues from a trajectory matrix. Whereas EMD searches intrinsic mode functions (IMF) via local maxima and minima, then the instantaneous frequency of IMFs is used to disclose the signal trends. Simulation shows that the trend tracking becomes problematic for defect assessment when signal trends become too complicated. As a remedy, a technique combing these two methods is proposed. Experimental and simulation results reveal the effectiveness of the collective effort from EMD and SSA. The study demonstrates that the data-driven nature is unique and attractive to characterize non-stationary ultrasonic signals where scattering, absorption and dispersion effects become dominant.**

Keywords*– *Singular spectrum analysis, empirical mode decomposition, Ultrasonic NDE

I. Introduction

In the ultrasonic NDE of materials, the returning ultrasonic signal carries the scattering, absorption and dispersion effects along the propagation path, and these physical phenomena impact the quality of echoes backscattered from the discontinuities and defects. The random and highly complex nature of the backscatter echoes makes it challenging to extract and quantify the necessary diagnostic information for quality assessment. Therefore, it is of great interest to devise ultrasonic signal processing methods for detecting and characterizing defects in materials. The non-stationary feature of ultrasound echoes makes classic signal processing methods impractical. Various methods have been developed to tackle the problem. In [1-4], different transforms such as short-time Fourier transform, Wigner-Ville distribution, Gabor transform, and discrete Wavelet transform, are utilized to analyze ultrasonic signals in the joint time-frequency domain. They all have shown their own strength and effectiveness in ultrasonic signal processing for certain applications. Additionally, signal modeling, parametric estimation and detection techniques have been studied to achieve better quantitative signal analysis [4-5]. Nevertheless, the challenge remains as of choosing an appropriate transform kernel with desirable resolution to unravel signal information.

Recent research progress in time-series analysis has drawn a lot of attention. Unlike transform-based algorithms where signal space expansion is done in the transform domain, time-series analysis is fully driven by the signal itself. Two mostly discussed methods are empirical mode decomposition (EMD) and singular spectrum analysis (SSA). Both methods fall in the category of data driven analysis. However, they deal with signal analysis from different angle. EMD is carried out as a part of Hilbert Huang transform. It searches intrinsic mode functions (IMF) via local maxima and minima. The instantaneous frequency of IMFs is used to disclose the signal trends. EMD has been utilized in numerous applications such as climate data analysis, medical imaging, underwater acoustic feature extraction, vibration signal analysis, and ultrasonic NDE [6-9]. In the SSA, a four-step processing, which is embedding, SVD, grouping and diagonal averaging, is used. Eigenvalues from a trajectory matrix derived from the signal are used to capture the signal trends. The SSA has also been explored in many applications such as biomedical, geophysical, speech and ultrasonic NDE [10-13].

In this investigation, EMD and SSA are comparatively studied in trend tracking for defect assessment in the context of ultrasonic NDE applications. The simulation result shows that this tracking is problematic when signal trends become too complicated. For instance, multiple trends are captured in a single IMF, where no further signal decomposition can be done. The grouping of eigenvalues from the trajectory matrix turns to be difficult and uncertain. As a remedy, a new technique combing EMD with SSA is proposed to take advantage of collective efforts from both methods. The proposed technique is evaluated through simulation and experimental study.

This paper is organized as follows: Section II briefly reviews both SSA and EMD. Section III presents results of a comparative study through simulation. Section IV discusses the proposed technique which combines SSA and EMD. It also includes simulation and experimental results for ultrasonic NDE applications. Section V concludes the paper.

II. Review for SSA and EMD in Trend Tracking

SSA and EMD shares a similar goal, which is to decompose a signal into a linear combination of functions in time domain, where these functions ideally embody the intrinsic trends of signal.

$$s(t) = \sum_{i=1}^{m} g_i(t) + r(t) \quad (1)$$

where $s(t)$ denotes the given signal, $g_i(t)$ denotes the ith trend functions, and $r(t)$ denotes the residue.

As mentioned in the Introduction section, a four-step processing, which is embedding, SVD, grouping and diagonal averaging, is applied in the SSA.

The procedures of SSA are briefly described below [13]. Without losing generality, $s(t)$ can be represented as a sequence, $s(n)$, where $0 \leq n \leq N-1$.

1. **Embedding**

   A $L \times K$ trajectory matrix, $\mathbf{T}$, is formed from multidimensional vectors derived from the sequence $s(n)$, where L denotes the length of segmented data of each column vector and $K = N - L + 1$

$$\mathbf{T} = \begin{bmatrix} s(0) & S(1) & \cdots & s(K-1) \\ \vdots & \vdots & \ddots & \vdots \\ s(L-1) & S(L) & \cdots & S(N-1) \end{bmatrix} \qquad (2)$$

2. **Singular value decomposition (SVD)**

   The trajectory matrix, $\mathbf{T}$, is decomposed as

$$\mathbf{T} = \mathbf{U} \begin{bmatrix} \sqrt{\lambda_1} & 0 & \cdots & 0 \\ 0 & \sqrt{\lambda_2} & \cdots & 0 \\ \vdots & \vdots & \cdots & \cdots \\ 0 & 0 & \cdots & \sqrt{\lambda_L} \end{bmatrix} \boldsymbol{V}' \qquad (3)$$

   where $\mathbf{U} = [\mathbf{U_1} \quad \mathbf{U_2} \quad \cdots \quad \mathbf{U}_L]$

   $\|\mathbf{U}_i\| = 1$ for $1 \leq i \leq L$

   $\boldsymbol{V}' = [\mathbf{V_1} \quad \mathbf{V_2} \quad \cdots \quad \mathbf{V}_L]'$

   λ_i denotes the sorted eigenvalues of the

   $L \times L$ matrix, $\mathbf{TT}'$ $(\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_L \geq 0)$

   and $\mathbf{T}'$ denotes the transpose operation of $\mathbf{T}$

   Then Equation (3) can be rewritten as

$$\mathbf{T} = \textstyle\sum_{i=1}^{L} \mathbf{T_i} \qquad (4)$$

   where $\boldsymbol{T_i} = \sqrt{\lambda_i}\mathbf{U}\boldsymbol{V}'$

3. **Grouping**

   From Equation (4), those matrices, $\mathbf{T_i}$, can be regrouped into m disjoint subsets

$$\mathbf{T} = \textstyle\sum_{i=1}^{m} \mathbf{Y_i} \qquad (5)$$

   A common way of obtaining subsets is to group those $\mathbf{Y_i}$ whose eigenvalues are close with each other. There are different strategies to identify subsets [14]. The grouping process can be viewed as a form of harmonic identification.

4. **Diagonal averaging**

   Time sequences can be obtained from those subset matrices, $\mathbf{Y_i}$ (see Equation 5) through diagonal averaging. The ith time sequence can be written as

$$g_i(n) = \begin{cases} \dfrac{\sum_{l=1}^{n+1} y_{l,n-l+2}}{n+1} & 0 \leq n < L-1 \\ \dfrac{\sum_{l=1}^{L} y_{l,n-l+2}}{L} & L-1 \leq n < K \\ \dfrac{\sum_{l=n-K+2}^{N-K+1} y_{l,n-l+2}}{N-n} & K \leq n < N \end{cases} \qquad (6)$$

   where $y_{l,k}$ denotes the element (l,k) in $\mathbf{Y_i}$

   As a result, through the four steps in SSA, the sequence, $s(n)$, can be decomposed into m components.

$$s(n) = \textstyle\sum_{i=1}^{m} g_i(n) + r(n) \qquad (7)$$

   where $g_i(n)$ denotes the ith component, $r(n)$ denotes the residue.

From above, it can be seen that the decomposition results, $g_i(n)$, are computed in a single step through diagonal averaging. Whereas the decomposition process in EMD is completed through sifting process in an iterative fashion. The procedures of EMD for a given signal, $s(t)$, are described below [9].

1. **Initialization:** Set $x(t) = s(t)$ and the iteration index $i = 1$
2. **Locate local extremes:** search and find all local maxima and minima of $x(t)$
3. **Envelop forming:** Form maxima envelop, $h_{max}(t)$, and minima envelop, $h_{min}(t)$, by interpolating the local maxima and minima, respectively. The mean envelop, $m(t)$, can be obtained by averaging $h_{max}(t)$ and $h_{min}(t)$.
4. **IMF checking:** Let $h(t) = x(t) - m(t)$, check if $h(t)$ satisfies the conditions to be an IMF.

   The common conditions are

   a) $|N_{extreme} - N_{zero-crossing}| \leq 1$
      where $\mathrm{N_{extreme}}$ denotes the number of local extreme points in $h(t)$ and $\mathrm{N_{zero-crossing}}$ denotes the number of zero-crossing points

   b) *The mean envelop of $h(t)$ is close to zero.*

   If $h(t)$ is an IMF, move on to step 5. Otherwise, update $x(t)$ with $h(t)$ and go back to step 2.
5. **IMF saving**: save $g_i(t) = h(t)$, then update the signal residue with $x(t) = s(t) - g_i(t)$
6. **Sifting condition checking:** Check the signal residue from previous step. If it is a constant or monotonic function, save all IMFs and stop the sifting process. Otherwise, update the iteration index $i = i + 1$, repeat steps 2 to 6.

After the iterative sifting process, the signal can be represented as Equation (1) where $g_i(t)$ denotes the ith IMF instead, and $r(t)$ is the residue.

III. A Comparative Simulation Study

In [9, 13], EMD and SSA have been used for ultrasonic NDE applications. It has been reported that both of them can effectively track the trends in ultrasonic echoes, especially when these echoes are well separable or lightly overlapped. In this study, EMD and SSA are comparatively studied through simulation. In particular, ultrasonic signal with highly-overlapped echoes and severe noise is utilized for performance evaluation. In the context of ultrasonic NDE application, chirplet echoes are often encountered in ultrasonic backscattered signals. They represent a vast variation of echoes including dispersive or non-dispersive, narrow or broad band echoes. As such, an ultrasound signal, $s(t)$, can be modeled as [5].

$$s(t) = \sum_{i=1}^{l} h_i(t;\Theta_i) + n(t) \qquad (8)$$

where $n(t)$ denotes White Gaussian noise;

$h_i(t;\Theta_i)$ denotes ith chirplet echo

$$h_i(t;\Theta_i) = a_i e^{-\alpha_{1i}(t-\tau_i)^2} cos\left(2\pi f_{ci}(t-\tau_i)+\alpha_{2i}(t-\tau_i)^2+\theta_i\right)$$

and the parameter vector $\Theta_i = [\tau_i,\ f_{ci},\ a_i,\ \alpha_{1i},\ \alpha_{2i},\ \theta_i]$, whereτ_i denotes time-of-arrival, f_{ci} denotes center frequency,a_i denotes amplitude, α_{1i} denotes bandwidth factor, α_{2i} denotes chirp rate, and θ_i denotes phase.

An ultrasonic signal including multiple heavily overlapped echoes and noise is simulated, where these echoes can be viewed as representations of intrinsic trends. The parameter vectors of echoes are listed below.

$$\Theta_1 = [2.0\,\mu s \quad 8\,MHz \quad 1.0 \quad 20\,MHz^2 \quad 25\,MHz^2 \quad 0\ rad/s]$$
$$\Theta_2 = [2.5\,\mu s \quad 5\,MHz \quad 0.8 \quad 15\,MHz^2 \quad 15\,MHz^2 \quad 1\,rad/s]$$
$$\Theta_3 = [3.0\,\mu s \quad 4\,MHz \quad 1.0 \quad 25\,MHz^2 \quad 25\,MHz^2 \quad 0\ rad/s]$$

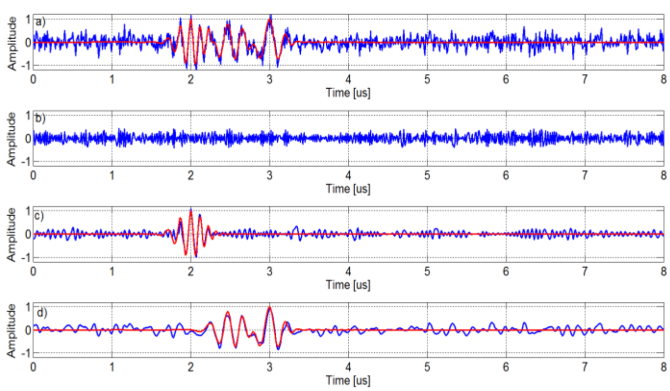

Figure 1. a) Simulated noisy ultrasonic signal superimposed with the simulated noise-free ultrasound echoes, b) The first IMF, c) The second IMF, and d) Residue

EMD is applied to process the simulated signal. The results are shown in Figure 1. Figure 1a shows the simulated noisy signal superimposed noise-free ultrasound echoes. Figure 1b illustrates the first IMF, which is mainly the decomposed noise. It shows that EMD is very effective in terms of denoising. Figure 1c shows the second IMF, which captures the first ultrasound echo. However, the other two ultrasound echoes are not identified individually in Figure 1d. It demonstrates that trend tracking becomes much more challenging in the screnarios of high interfering ultrasound echoes.

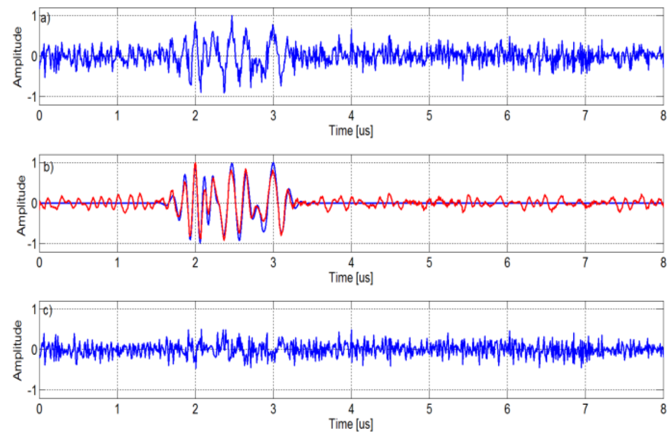

Figure 2. a) Simulated noisy ultrasonic signal, b) The noise-free signal superimposed with the signal recovered from the first 12 eigenvalues, and c) Signal residue

SSA is applied to the same set of simulated signal, where the window size L is set to 60. The results are shown in Figure 2. Figure 2b displays the noise-free signal superimposed with the signal reconstructed from the 12 dominant eigenvalues. Figure 2c shows that the SSA process reduces noise in a certain degree. It is noticed that SSA does not show the same level of effectiveness in terms of signal decomposition as the EMD does. The ultrasound echoes are not fully separable in this case.

IV. Experimental and Simulation Study of the Proposed Method

From the simulation from Section III and [13], it is reasonable to conclude that the trend tracking becomes problematic in the scenarios of highly overlapped and noisy ultrasound echoes. Neither EMD nor SSA can successfully obtain desired results alone due to their own limitation. As such, we propose a remedy by combing EMD with SSA to take advantage of collective efforts from both methods. The procedures are described below. First, EMD is applied to the simulated noisy ultrasonic signal (See Figure 1a). Based on the result of EMD, the signal in Figure 1d is selected to be further tuned using SSA. To facilitate the SSA, a windowing approach is utilized to prepare the data. The EMD process is mainly based on local extremes through interpolation. Applying a window generates an incomplete echo, which breaks the intrinsic behavior of signal and triggers the boundary problem in EMD. Whereas SSA tracks signal statistically. An incomplete echo does not fully disrupt the statistical behavior of signal. Figure 3 shows the result of the proposed method on the simulated ultrasonic echoes. It can be seen that the signal in Figure 1d is further decomposed into two parts (see Figure 3c and 3d). It shows that the proposed method is effective in terms of trend tracking and denoising.

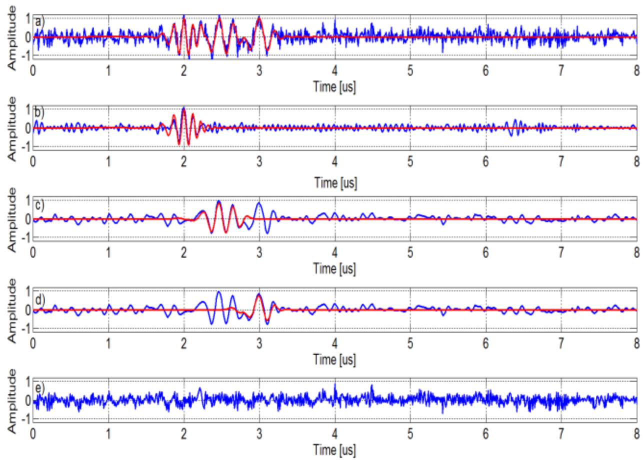

Figure 3. a) Simulated noisy ultrasonic signal superimposed with the simulated noise-free ultrasound echoes, b) The second IMF superimposed with the noise-free echo #1, c) The first recovered echo (in red) from SSA of the signal in Figure 1d (in blue), d) The second recovered echo (in red) from SSA of the signal in Figure 1d (in blue), and e) Signal residue

As an experimental study, the proposed method is utilized to analyze ultrasonic backscattered signal, which is acquired at the sampling rate of 100 MHz with 5 MHz transducer. A steel block with an embedded target (i.e, flat bottom hole) is used as the specimen. Figure 4a shows the experimental signal. After EMD, the signal is decomposed into IMF#1, IMF#2 and signal residue. For the IMF#2, SSA is applied to further tune the trend tracking. It turns out there are about 8 non-zero eigenvalues during the SSA process (See Equation 3). Two components are generated from the IMF#2 (see Figure 4c and 4d), where the target information is well presented in the fine-tuned IMF#2 (Figure 4c)

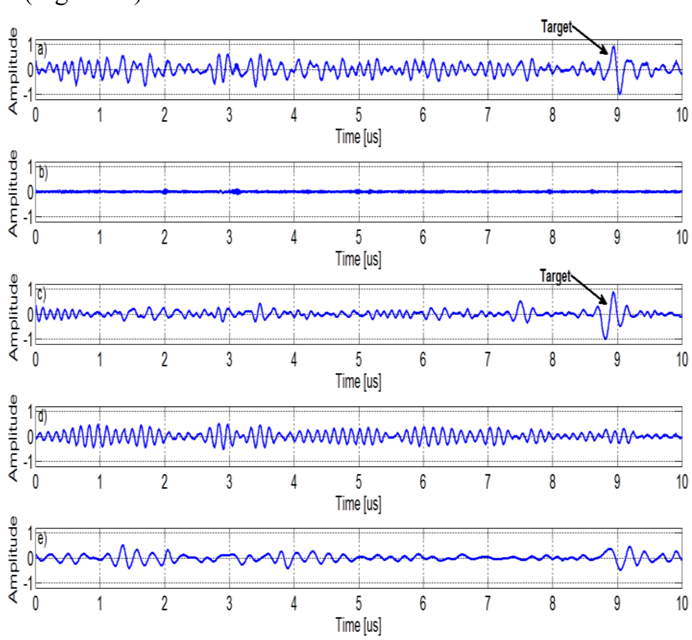

Figure 4. a) Experimental ultrasonic backscattered signal, b) IMF #1, c) Signal recovered using eigenvalues from #3 to #8 in SSA of IMF #2, d) Signal recovered using eigenvalues from #1 to #2 in SSA of IMF #2, and e) Signal residue

V. Conclusion

In this study, both SSA and EMD are utilized for ultrasonic signal processing. Experimental and simulation results show that it is challenging to track trends using SSA or EMD alone for highly overlapped ultrasound echoes in the presence of severe noise. EMD shows better performance in characterizing ultrasonic signal. Our study shows that it could be more effective by taking collective effort from EMD and SSA, where EMD is utilized to obtain IMFs, the SSA is applied to further fine tune the trend extraction from the IMFs. The data-driven nature of techniques discussed in the study make these methods unique and attractive to characterize non-stationary ultrasonic signals where scattering, absorption and dispersion effects becomes dominant.

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