Abstract - The ultrasonic signals backscattered from multilayered materials using a pulse-echo method are inherently low-resolution due to the band-limited nature of the interrogating transducer. To improve resolution, the Wiener Filter (WF) based solutions have been used traditionally, although they provide limited improvement and are highly sensitive to noise. More recently, sparse deconvolution techniques (e.g., \(ℓ_1\)-regularized Least Squares Deconvolution (\(ℓ_1\)-LSD)) have proved to be effective in improving the resolution; however, they are highly dependent on pulse invariance. It is common for an ultrasonic wavelet to change its shape as it propagates through the materials due to frequency dependent attenuation, scattering and dispersion. Consequently, these changes in pulse shape limit the application of these algorithms which need a prior knowledge of pulse-wave of the system. In this paper we present an adaptive and highly efficient deconvolution technique, Cepstrum Sparse Deconvolution (CSD), which does not require any prior information of the system or the detected object but compensates for the pulse-wave variation by estimating the fundamental pulse-wave shape directly from the backscattered signal. This pulse-wave estimation is based on cepstrum processing of the signal prior to the deconvolution process. Applications and comparisons are made to evaluate the performance of CSD, WF and \(ℓ_1\)-LSD through experimental and simulated ultrasound data which contain varying-shape wavelets with respect to amplitude, center frequency, arrival time, bandwidth, and phase parameters. The Normalized Mean Absolute Score (NMAS) index, a measure of distance between the actual reflectivity function and the estimated function is used to evaluate these algorithms’ performance. The results show that CSD is the fastest algorithm and offers highest resolution among these three algorithms even though the pulse-wave may have gone through a significant change.

Keywords - Sparse Deconvolution, Cepstrum Processing, Wiener Filter, \(ℓ_1\)-LSD

I. INTRODUCTION

Ultrasonic NDE testing is a cost-effective and practical method for examining the material internal structure. However, the backscattered echoes measured from the material are inherently low-resolution due to the bandlimited nature of the interrogating echo. To improve resolution, classical deconvolution algorithms based on the Wiener Filter (WF) are commonly used. WF based solutions provide only moderate resolution gains and are highly sensitive to noise [1]. More recently sparse deconvolution techniques (e.g., \(ℓ_1\)-LSD) have been proposed to reinforce resolution [2]. These techniques achieve significant resolution improvement by exploiting the sparsity of the Material Reflectivity Function (MRF). They are very effective when the ultrasound pulse is invariant across the propagation distance; however, their performance deteriorates when the pulse varies during propagation. Moreover, these deconvolution methods need to incorporate a priori knowledge of the system’s pulse-wave. The fundamental wavelet of backscattered echoes or the system’s pulse-wave represents basic characteristics of the transducer and the object under detection.

In this paper a new deconvolution method for recovering MRF from the measured signal is proposed and named as Cepstrum Sparse Deconvolution (CSD). Cepstrum itself is a powerful homomorphic processing algorithm for deconvolution. It was widely used in ultrasonic image processing for composite materials inspection, pitch determination of human speech, audio processing, geophysics, radar, medical imaging and others [3-5]. The general cepstrum deconvolution algorithm is very efficient for a signal without noise, however it easily fails even for a signal with a very low level noise [4]. To overcome this shortcoming, a low pass filter and a lifter are incorporated into the CSD. The CSD is highly adaptive to the varying pulse since it does not depend on a nominal pulse-wave knowledge of transducer which is typically required for WF and \(ℓ_1\)-LSD methods. This technique is successfully applied in analyzing the MRF of a reverberant material. The performance of the CSD algorithm is also tested by randomly generated MRFs with varying pulse.

CSD theory and mathematical model for ultrasonic backscattered echoes are introduced in Section II. Section III presents deconvolution performance analysis of sparse signals for WF, \(ℓ_1\)-LSD and CSD algorithms. In Section IV, the conclusions are given.

II. LINEAR SYSTEM MODEL OF MRF & SPARSE DECONVOLUTION ALGORITHMS

The measured backscattered echoes from a material can be modeled (see Figure 1) as the output of a linear system where the MRF \(x\) is convolved with a time-varying transducer pulse \(h\) and corrupted with noise \(w\). The system (material) characteristics is described as MRF, which is tried to be recovered from the backscattered signal through deconvolution algorithms.

Figure 1. Linear system model of material reflectivity function
The ultrasound signal obtained from a material whose output is given by the model in Figure 1 can be represented by series of a number of resolvable echoes,

\[ y(nT) = \sum_{m=1}^{M} \beta_m h_m(t_m - \tau_m) + w(nT), \quad n = 0, 1, \ldots, N - 1 \]  

where the parameter pairs \( \{\tau_m, \beta_m\} \) represent the reflectivity function of the \( m \)-th scatter echoes, and \( h_m(t_m - \tau_m) \) represents the ultrasound pulse at \( t_m \). The term \( w(t) \) is the measurement noise and characterized as zero-mean white Gaussian noise (WGN) with variance \( \sigma_w^2 \). The ultimate objective of this NDE system is to recover the MRF, i.e., \( \{\tau_m, \beta_m\}, m = 1, 2, \ldots, M \) from the measured data. Under pulse invariance assumption, i.e., \( h_m(t_m - \tau_m) = h(t_m - \tau_m) \), the model in (1) can be simplified to,

\[ y = H \cdot x + w \]  

where \( H \) represents the convolution matrix of size \( N \times N \) (whose columns are circularly shifted copies of the transducer pulse \( h \)), and \( x, w \) and \( y \) are vectors of length \( N \) that represent respectively the MRF, noise, and measured data. If no assumption is made on the characteristics of the MRF \( x \), one obtains the most general pseudo-inverse solution via Least Squares estimation, \( \hat{x} = (H^T H)^{-1} H^T y \). Due to the bandlimited ultrasound pulse \( h \), such a solution is highly sensitive to noise [3]. A very low level noise, for instance, SNR \( = 150\text{dB} \) will fail the algorithm [4]. The CSD that we proposed here uses a low pass filter to lower the noise level and a short pass filter to extract the power cepstrum of \( h(t) \) only from the power cepstrum of \( y(t) \). Such procedure significantly improves noise resistance of the CSD as demonstrated via simulations in the subsequent section.

To illustrate the principle of cepstrum deconvolution, we construct the ultrasonic signal as two echoes in discrete-time domain [3],

\[ s(nT) = h(nT) + \beta \cdot h(nT - m_i T) \]  

where \( h(nT) \) is the measuring system impulse response, \( T \) is the sample interval, \( \alpha \) is a normalized amplitude for the second component \( h(nT - m_i T) \). The signal can be represented in a convolutional form as

\[ s(nT) = h(nT) * [\delta(nT) + \beta \cdot \delta(nT - m_i T)] \]  

After doing Fourier transform of power spectrum and logarithm, we can get:

\[ S_{pc}(\omega) = \log|S_{hh}(\omega)| + \log[1 + \beta \cdot e^{-j\omega m_i T}] + \log(1 + \beta \cdot e^{-j\omega m_i T}) \]  

where \( S_{hh}(\omega) \) is the frequency spectrum of \( h(nT) \). By performing inverse Fourier transform of the above equation, we have the power cepstrum of the signal (Eq.5),

\[ S_{pc}(nT) = F^{-1} \left( \log|S_{hh}(\omega)| + \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k}{k} \delta(n - km_i T) + \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\beta k}{k} \delta(n - km_i T) \right) \]  

The first term of the above equation is the power cepstrum of the fundamental wavelet. Since \( \beta \) is a normalized amplitude which is not larger than 1 and the index \( k \) increases as time increasing, the second and third terms will be periodically rapidly decaying. To extract the fundamental wavelet from the backscatter echoes, only the first term is needed. To obtain this wavelet, shortpass lifter or comb lifter is required. By performing the inverse processing of above procedure, the autocorrelation of the fundamental wavelet can be fetched [3]. The whole procedure for extracting \( h(nT) \) is shown in Figure 2. with solid line.

Figure 2. Extraction of the fundamental wavelet \( h(nT) \) via modified cepstrum algorithm (solid line) and \( x(nT) \) via deconvolution (dashed line).
The MRF, as Figure 1 and Eq. 1 or Eq. 2 indicates, after doing Fourier transform, can be represented in frequency domain as,

\[ \hat{X}(\omega) = \frac{Y(\omega)}{H(\omega)} \quad (10) \]

The division operation is sensitive for frequency values \((w)\) for which \(H(\omega)\) is zero. To resolve this problem, we added a small constant number to \(H(\omega)\) in denominator. After doing an inverse Fourier transform, the MRF, \(\hat{x}(nT)\) will be recovered. The whole procedure from extracting fundamental pulse-wave to the deconvolution through spectral division to obtain \(\hat{x}(nT)\) is called as MRF as shown in Figure 3.

III. PERFORMANCE EVALUATION OF CSD AND APPLICATIONS

To evaluate the performance of CSD in comparison to WF and \(l_1\)-LSD in reconstructing the MRF, numerous sparse signals are simulated according to Eq. 1. White Gaussian Noise is added to the signal to further evaluate the algorithm’s qualities in presence of noise. Furthermore, the CSD is applied to an experimental ultrasonic signal to verify its performance. In order to quantitatively evaluate the algorithms in recovering the MRF we used a performance metric, the Normalized Mean Absolute Score (NMAS) defined as,

\[ NMAS = \frac{\|x-x\|}{\|x\|} \quad (11) \]

This metric is a matching index between the restored and actual MRF. Lower values of NMAS indicate a better performance. We simulated backscattered ultrasonic signals composed of Gaussian Chirplet (GC) echoes [9] whose parameters, bandwidth, chirp rate, center frequency, phase and amplitude comply with the normal distribution with mean value \(\mu_\omega = [20(MHz), 0.0, 4MHz, 0.5\pi, 1]\) and covariance matrix \(C_\omega\) where \(\text{diag}(C_\omega) = [2, 0.1, 0.2, 1, 1]\) [9]. These echoes are distributed in time such that the time differences between consecutive echoes abide Poisson or uniform distribution to generate sparse signals. Zero-mean white Gaussian noise is added to the signals to simulate noise. Using the above procedure, two examples of simulated ultrasound signals are shown in Figure 3 and Figure 5 without noise and with noise (SNR = 12dB) respectively. Such simulated signals were deconvolved by WF, \(l_1\)-LSD and CSD algorithms. For WF and \(l_1\)-LSD, the prior pulse-wave was based on the mean GC echo (constructed from the parameter vector \(\mu_\omega\)) to form the degradation matrix \(H\).

Figure 4. is the CSD result for the signal shown in Figure 3. From this figure, we see that the reconstructed MRF by CSD (blue spikes) exactly represent the positions of the real MRF (red spikes). We also can see that two highly overlapping echoes at the rightest side of the signal at 950 in time axis. These two echoes, by visible observation, are seen as one echo. However, CSD has capability to separate and identify them as two echoes.

To test the performance of the CSD with noise, WGN is added to simulate signal with 12 dB SNR as shown in Figure 5. From this figure, it is seen that the CSD has the best capability to extract the MRF with highest accuracy compared to the other two algorithms. To obtain the best \(l_1\)-LSD result, the best sparsity controlling parameter \(\lambda\) is selected between \(\lambda = 3\sigma\sqrt{\sum n|h(n)|^2}\) and \(\lambda_{\text{max}} = \arg \min \|\hat{x} - x\|\) to obtain the minimum NMAS or the best qualitative result [9]. Similarly, to obtain the best performance with WF, noise variance parameter \(\sigma_w\) (see Eq.3) is also optimized.

Figure 5. Simulated sparse ultrasound signal with SNR = 12dB, and CSD, WF, \(l_1\)-LSD deconvolution results.

The average NMAS values and elapsed times with the three algorithms based on 50 different simulated sparse signals without noise were calculated and listed in Table 1. The
average NMAS values and elapsed times of algorithms based on 50 different sparse signals with SNR 12 dB noise were also calculated and listed in Table 2. From these tables, it can be seen that CSD provides a higher fidelity recovery with a significantly faster speed than ℓ1-LSD and WF.

Table 1. Performance Comparison of Algorithms in Terms of NMAS and Processing Times Without Noise

<table>
<thead>
<tr>
<th></th>
<th>CSD</th>
<th>WF</th>
<th>ℓ1-LSD</th>
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<tbody>
<tr>
<td>NMAS</td>
<td>2.1117</td>
<td>4.202</td>
<td>3.6937</td>
</tr>
<tr>
<td>Processing Time</td>
<td>0.005625</td>
<td>0.27912</td>
<td>3.9958</td>
</tr>
</tbody>
</table>

Table 2. Performance Comparison of Algorithms in Terms of NMAS and Process Times With Noise (SNR = 12 dB)

<table>
<thead>
<tr>
<th></th>
<th>CSD</th>
<th>WF</th>
<th>ℓ1-LSD</th>
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<tbody>
<tr>
<td>NMAS</td>
<td>2.4897</td>
<td>6.5366</td>
<td>4.7622</td>
</tr>
<tr>
<td>Processing Time</td>
<td>0.01476</td>
<td>0.31354</td>
<td>3.7297</td>
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</table>

In Table 1, it is indicated that CSD is 710 times faster than ℓ1-LSD when the signal does not contain noise. In Table 2, CSD is 252 times faster than ℓ1-LSD for noisy signal. Since a low pass filter is used to deal with a noisy signal, therefore the CSD becomes slower than dealing with noise free signal although the CSD is still much faster than the other two algorithms. Furthermore, there is still room to optimize the algorithm to improve its computational speed when signal is noisy. By qualitatively observing the result in Figure 5, CSD has much better recovery than the other two algorithms; however, NMAS index does not reflect this visual observation well. Therefore, it may be necessary to create a new quantitative criterion for better evaluation of performance.

Furthermore, we have tested the CSD algorithm for backscattered reverberation echoes obtained from a highly reverberant material, a steel block with multiple cracks, which was interrogated by 2.5 MHz transducer with 100MHz sampling rate. Even though we do not know information about the interrogation system, it will not affect the implementation of the algorithm. The signal and deconvolution results are displayed in Figure 6. From the deconvolution results, we see that the MRF of the reverberant block is accurately identified although the signal is contaminated by heavy noise and reverberation echoes exhibit visible changes in their shapes. The blue unique centralized peak signal is the result from the CSD directly and the black spike is the result from the blue unique centralized peak after it was set a mean value threshold. Based on this result, various properties of the material (e.g., thickness, sound velocity, Young’s module) can be determined. The MRF of this material is obtained from the measured signal only, without using any information about transducer pulse or assuming any statistics on the MRF.

IV. CONCLUSION

In this paper we modeled the material NDE system and represented ultrasound backscattered signal as a sequence of varying shape echoes whose arrival time and amplitude parameters represent the reflectivity of the scatterers. These parameters (i.e., MRF) are estimated from the measured ultrasound data utilizing the CSD algorithm. Through applications it is shown that the proposed method has a number of advantages compared to Wiener Filter and ℓ1-LSD. First, it is totally independent on any prior information, thus the varying pulse shape does not affect the deconvolution result. Second, it provides a high resolution reconstructed MRF as those simulated or experimental results shown. Third, compared to the ℓ1-LSD and the Wiener Filter, the CSD has much faster processing speed and better NMAS values because of the simplest deconvolution principle. Finally, the CSD has shown the best noise resistant ability. All these good qualities are verified through statistical evaluations as presented in Table 1 and 2.

References