

Block Sparse Compressed Sensing in Ultrasonic NDE Echo Analysis and Parameter Estimation

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Abstract— In this investigation, compressed sensing (CS) is utilized to examine the sparsity of ultrasonic NDE signal, which consequently improves the efficacy of data sampling with significant lower rate than the Nyquist rate. The time-of-arrivals (TOAs) and amplitudes of dominant echoes are estimated through annihilating filter in the CS via matrix operation. The matrix size is proportional to the number of echoes. Block sparse representation is introduced to limit the number of echoes, subsequently reduces the computation of signal reconstruction. Furthermore, the echo estimation results are used as a priori information in signal decomposition and parameter estimation. Our experimental study shows that the block sparse CS can accurately estimate the TOAs and amplitudes of ultrasound echoes in the presence of noise with SNR as low as -5 dB. It also demonstrates the boosted sampling efficiency and significant reduction of computation brought by the block sparse scheme in echo estimation and parameter estimation. The study could have a great potential in material evaluation and structural health monitoring, especially real-time defect detection and pattern recognition.

Keywords— *Ultrasound NDE, block sparse, compressed sensing, signal decomposition, parameter estimation*

I. INTRODUCTION

In ultrasonic NDE applications, the received signal carries valuable physical information along the wave propagation path such as location, orientation and size of discontinuities. Various signal processing algorithms, including parametric or nonparametric, time-domain or transformed domain, have been utilized to interrogate ultrasonic echoes [1-4]. These echoes are often highly overlapped and sometimes noise-contaminated. Signal modeling is commonly used to characterize these echoes. Gaussian chirplet model has been applied to analyze ultrasonic signals [3]. It has been proved to be robust in echo analysis and parameter estimation. In particular, the six parameters of chirplet model could be used to epitomize the physical property of reflectors.

In addition to the important aspect of retrieving valuable information from ultrasonic NDE data through signal processing, there is another aspect that should be considered, which is data sampling. As of assessing and monitoring the in-situ oversized structures, it becomes challenging to collect and analyze a large volume of data due to the workload time and computation expense. The Nyquist-Shannon sampling theorem bridges between the continuous-time domain and discrete-time domain for signal reconstruction and data collection. It regulates the amount of data needed in applications including ultrasonic NDE. In recent years, an

emerging low sampling rate scheme, compressed sensing (CS), has attracted considerable attention and research effort in data intensive applications [5-7]. Sparsity has been widely studied in representation or approximation for data compression. It shows that a fractional number of non-zero coefficients in a transformed domain can be used to represent the source information without or with little loss in terms of signal quality.

The CS is creatively introduced to exploit the sparsity of signal in the context of sampling. The findings from M. Vetterli et al have shown that sparsity could be exploited in the sampling scheme of signals with finite rate of innovation (FRI) [8-10]. The effort of collecting data could be greatly less at the first place, which is extremely helpful for high volume data applications such as astronomy, magnetic resonance imaging and tomography [5-7]. The discussion in [9-10] focuses on the sampling of periodic stream of Dirac, splines or pulses with Strang-Fix conditions. The sparse sampling of signals with FRI is introduced to the application of medical ultrasound for sub-Nyquist sampling [11]. A stream of finite Gaussian pulses is used to model the ultrasound signal. The time-of-arrival (TOA) and amplitude of Gaussian pulses are estimated for signal restoration. The experimental study has shown that the TOAs of Gaussian pulse have been estimated with high accuracy. On the other hand, the performance of amplitude estimation is not as great as the TOA estimation. A large estimation error is observed when a strong noise presents in the vicinity of pulses. It is worth to mention that the exact locations of pulses carry more weight than the accurate reflections coefficients in the imaging application discussion in [11]. In contrast, both TOA and amplitude are significant in ultrasonic NDE applications. The amplitude estimation is critical to assess the size of discontinuity for material characterization and structure health monitoring.

In our previous work, echo analysis and parameter estimation is studied under the newly developed sampling scheme. In particular, a model-based signal decomposition algorithm is integrated in the CS framework, where a common model, Gaussian chirplet model, is used in echo estimation [12-13]. The results of experimental study are promising. However, the computation incurred by sparse sample calibration and annihilating filtering are mainly operations with matrices. The matrix size is proportional to the number of echoes. Here we introduce block sparse compressed sensing to limit the number of echoes, subsequently reduces the computation of signal reconstruction. An experimental study is presented to demonstrate and compare the performance of echo analysis and parameter estimation based on the block sparse CS.

The rest of the paper is organized as follows: Section II describes the block sparse compressed sensing in ultrasound NDE echo analysis. Section III presents experimental results and discusses the performance of proposed method. Section IV concludes the paper.

II. BLOCK SPARSE COMPRESSED SENSING IN ULTRASOUND NDE ECHO ANALYSIS

As we discussed earlier, the number of echoes in compressed sensing is limited to reduce the computation load. To do so, an ultrasonic signal, $x(t)$, is decomposed into N blocks, which can be written as

$$\begin{aligned} x(t) &= \sum_{j=1}^N x_j(t) \\ &= \sum_{j=1}^N (s_j(t) + n_j(t)) \end{aligned} \quad (1)$$

where $x_j(t)$ denotes the j th block signal, $s_j(t)$ denotes the noise-free reflected ultrasound signal in the j th block and $n_j(t)$ denotes a white Gaussian noise.

In addition, the reflected ultrasound signals, $s_j(t)$, can be modeled as

$$s_j(t) = \sum_{i=1}^{M_j} h_{\theta_i}(t) \quad (2)$$

where

M_j denotes the number of Gaussian chirplets in the j th block $h_{\theta_i}(t)$ denotes the i th returned Gaussian chirplet echo from reflectors, which can be written as [2]

$$h_{\theta_i}(t) = a_i e^{-\alpha_i(t-\tau_i)^2} \cos(2\pi f_{ci}(t - \tau_i) + \alpha_{2i}(t - \tau_i)^2) \quad (3)$$

where the parameter vector $\theta_i = [\tau_i \ f_{ci} \ a_i \ \alpha_{1i} \ \alpha_{2i}]$ including parameters TOA, center frequency, amplitude, bandwidth factor, and chirp rate respectively.

The procedures of CS-based signal decomposition for the ultrasound signal of each block (see Equation 2) is described as follows [12-13].

1. Signal preparation

In the CS of signal with FRI, the TOA and amplitude of pulses are estimation [11]. Therefore, Hilbert transform is applied to obtain the envelop signal, $e_j(t)$ which is

$$e_j(t) = \text{Hilbert}(s_j(t)) \quad (4)$$

It is worth to point out that preprocessing with a low pass filter or simple thresholding can be applied to the signal before the envelope detection, when a severe noise is present.

2. Filtering using sampling kernel

Different sampling kernels such as functions satisfying Strang-Fix conditions, exponential Spline

of first order can be used as the sampling kernel. Following the suggestion of [11], a smooth and compact-supported sampling kernel, $g(t)$, is used to filter the envelop signal, $e_j(t)$, where

$$g(t) = \sum_{m=-K}^K b_m e^{2j\pi mt} \quad (5)$$

Here K denotes the number of Gaussian chirplets in $s_j(t)$, that is M_j from Equation (2), and b_m denotes the coefficients of M -length Hamming window

$$b_m = 0.54 - 0.46 \cos\left(\frac{2\pi(m+\frac{M}{2})}{M}\right) \quad (6)$$

here $M = 2K + 1$

The measured signal, $y(t)$, can be written as

$$y(t) = e_j(t) * g(t)$$

where $*$ denotes the convolution operation

3. Sparse sampling and calibration

A sample sequence, $c(n)$, can be obtained by

$$c(n) = y(t)|_{t=\frac{n}{M}} \text{ for } n = 0, \dots, M-1. \quad (7)$$

Furthermore, the calibration sequence, $X(m)$, can be obtained from the measured signal through sparse sampling and calibration.

$$X(m) = S_{M \times M}^{-1} C_f(m) \quad (8)$$

here

$$C_f(m) = \mathcal{DFT}(c(n))$$

$$S_{M \times M} = \Lambda \left(b_m e^{-\frac{\omega_m^2 \sigma^2}{2}} \right)$$

Λ : denotes diagonal matrix

$$\omega_m = 2\pi m, \ m = -K, \dots, K$$

σ^2 can be estimated from the bandwidth of ultrasound transducer [13].

4. Estimation of annihilating filter coefficients [10]

The annihilating filter coefficients, (i) , $1 \leq i \leq K$, are estimated by solving the equation below.

$$\begin{bmatrix} X(0) & X(-1) & \dots & X(-K+1) \\ \vdots & \vdots & \ddots & \vdots \\ X(K-1) & X(K-2) & \dots & X(0) \end{bmatrix} \begin{bmatrix} A(1) \\ \vdots \\ A(K) \end{bmatrix} = \begin{bmatrix} X(1) \\ \vdots \\ X(K) \end{bmatrix} \quad (9)$$

5. TOA and amplitude estimation [12-13]

The estimated TOAs, $\hat{\tau}_i$, $1 \leq i \leq K$ are obtained by solving the Z-domain equation below using annihilating-filter approach [10]

$$A(z) = 0 \quad (10)$$

where $A(z) = \prod_{i=1}^K (1 - u_i z^{-1}) = 1 + \sum_{i=1}^K A(i) z^{-i}$ and $u_i = e^{-j2\pi\hat{\tau}_i}$, and $A(i)$ is from Equation (9).

The estimated amplitudes, \hat{a}_i , $1 \leq i \leq K$, can be estimated by solving the equation below.

$$\begin{bmatrix} a_1 \\ \vdots \\ a_K \end{bmatrix} \begin{bmatrix} u_1^0 & u_2^0 & \cdots & u_K^0 \\ \vdots & \vdots & \ddots & \vdots \\ u_1^{K-1} & u_K^{K-1} & \cdots & u_K^{K-1} \end{bmatrix} = \begin{bmatrix} X(0) \\ \vdots \\ X(K) \end{bmatrix} \quad (11)$$

where $u_i = e^{-j2\pi\hat{\tau}_i}$ which is from Equation 10.

6. Signal decomposition with initial estimation parameters from CS

As stated in [11], the estimation of amplitude are not as accurate as the estimation of TOAs. It also can be observed from our experimental results in Section III. Nevertheless, both estimated parameters can be refined in the model-based signal decomposition algorithm [3]. Principally signal decomposition becomes a matter of matching pursuit on incomplete ultrasound block signals for parameter estimation.

Therefore, for each $\hat{s}_j(t)$, $1 \leq j \leq N$, we can have

$$\hat{s}_j(t) = \sum_{i=1}^{M_j} h_{\hat{\theta}_i}(t) \text{ where } \hat{\theta}_i \text{ is the estimated parameter vector for } h_{\theta_i}(t).$$

III. EXPERIMENTAL STUDY

To evaluate the performance of block sparse CS scheme, experimental data from an ultrasonic pulse-echo system have been collected. In the experiment, the sample is a steel block with embedded defects. A 5 MHz broadband transducer is used to receive the ultrasound microstructure scattering signal. The signal is sampled at 100 MHz, then down-sampled for the CS sampling scheme. The number of signal blocks for CS is preset to 4.

It is worth to point out that the concept of block sparse CS here is different from the concept of block-sparse signal in references [14-15]. The block-sparse signal essentially is a signal whose transformed coefficients are not only sparse, but the nonzero entries to be in some fixed blocks. The signal can be represented by only a few blocks (i.e., subspaces), while the overall number of blocks is very large. In our study, the sense of block sparse is introduced for the purpose of computation reduction. Certainly there exists an optimization problem of choosing a proper number of blocks and the size of each block, fixed or variable. Nevertheless, this study demonstrates that the block sparse CS greatly reduces the computation without degrading the performance in the sense of parameter estimation and echo analysis.

Table 1. Comparison of initial TOA and amplitude estimation results via CS-based with and without (w/o) block partition

#	Block sparse CS		CS w/o block partition	
	Estimated TOA [us]	Estimated Amplitude	Estimated TOA [us]	Estimated Amplitude
1	0.58	1.00	0.58	0.97
2	1.06	0.75	1.12	0.63
3	2.04	0.75	2.03	0.80
4	2.87	1.00	2.84	1.00
5	3.50	0.83	3.48	0.97
6	3.85	0.62	4.01	0.55
7	5.10	0.49	5.00	0.66
8	5.59	0.69	5.55	0.99
9	6.16	0.45	6.19	0.59
10	7.43	0.54	7.24	0.58
11	n/a	n/a	7.76	0.73
12	8.15	0.63	8.26	0.70
13	8.93	0.27	n/a	n/a

Table 1 shows the initial TOA and amplitude estimation results via CS-based with and without block partition side by side. In addition, Figure 1a shows the original experimental ultrasonic signal (in Blue) superimposed with the reconstructed signal (in Red). In addition, the estimated TOAs and amplitudes in the sparse sampling are highlighted (in Green). As a comparison, the similar results from the CS without block partition for the same data set are shown in Figure 1b. In addition, the results of parameter estimation and echo analysis without CS are shown in Figure 1c.

A few observations from the experimental study are summarized below.

1. Most of estimated TOAs and amplitudes from the CS, with or without block partition, are matched. The estimated parameters from the CS provide a good reference for signal decomposition.
2. The reflected echo from a discontinuity (i.e., target) is around 8 us in Figure 1. It seems that the block CS better tracks the target due to the locality of partition. It is in coincide with the physical nature of reflectors in ultrasonic NDE, that is, reflectors are local.
3. The matrix size involved in the computation is 3 in the block CS versus 12 in the CS without block partition. The block CS shows a better computation efficiency.
4. All three schemes can reconstruct the signal well and track the target successfully in presence of highly scattered noise.

IV. CONCLUSION

In this work, block sparse compressed sensing has been studied for ultrasonic NDE application. The experimental study shows that it can greatly improve the computation efficacy of sparse sampling and parameter estimation in the signal recovery. Additionally, the parameter estimation and signal decomposition demonstrates the robustness of the proposed approach in echo analysis for material characterization. The study may have a great potential in structural health monitoring and material evaluation especially real-time defect detection and pattern recognition.

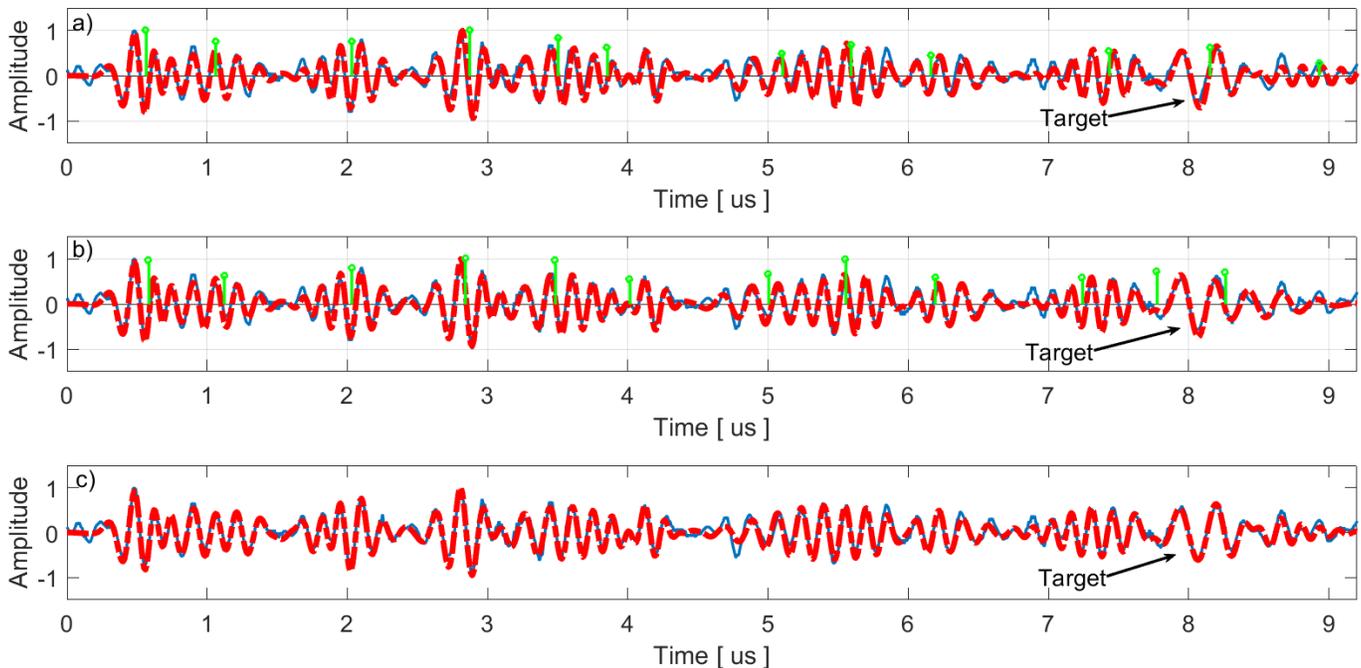


Figure 1. a) Experimental results using **block sparse** CS-based signal decomposition: Original experimental ultrasound echo signal (in Blue), the reconstructed signal with 12 dominant GCs from 4 blocks (in Red), and the estimated TOAs and amplitudes using CS (in Green) b) Experimental results using CS-based signal decomposition: Original experimental ultrasound echo signal (in Blue), the reconstructed signal with 12 dominant GCs (in Red), and the estimated TOAs and amplitudes using CS (in Green) c) Experimental results directly using chirplet signal decomposition: Original experimental ultrasound echo signal (in Blue) and the reconstructed signal with the first 8 dominant GCs (in Red)

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