Optimization of Microwave Resonant Cavity Flowmeter Design for High Temperature Fluid Sensing Applications

Tianyang Fang and Jafar Saniie
ECASP Research Laboratory (http://ecasp.ece.iit.edu)
Department of Electrical and Computer Engineering
Illinois Institute of Technology, Chicago, IL, U.S.A.

Sasan Bakhtiari and Alexander Heifetz
Nuclear Science and Engineering Division
Argonne National Laboratory
Lemont, IL, U.S.A.

***Abstract*— This paper investigates optimization of parameters to enhance performance of a microwave resonant cavity transducer for high temperature fluid flow sensing in advanced reactors. The cylindrical microwave cavity flowmeter is a novel sensor that measures fluid flow velocity through deflection of the cavity wall due to dynamic fluid pressure. This sensor is designed to operate in harsh environments of a nuclear reactor because sensor material is resilient to high temperatures, ionizing radiation, and corrosion. Parameters that determine performance characteristics of the transducer include dimensions of the microwave cavity, cavity electromagnetic excitation method, and cavity material. We investigate transducer design through computer simulations, with the objective of maximizing cavity Q-factor.**

Keywords—Microwave Resonator, Resonance, High-temperature Fluid Flow Measurement，Propagation, Coupling.

I. INTRODUCTION

In advanced high-temperature fluid reactors, where molten fluoride salt or liquid sodium are used as the primary coolant, the flow velocity provides critical information about reactor power [1-4]. Common approach to molten salt flow measurement involves external clamp-on ultrasonic flowmeters which are resistant to radiation and high temperature [5][6]. However, the ultrasonic transducers are limited to flow sensing in pipes. We have developed an immersion flow sensing transducer based on a cylindrical microwave cavity resonator, which is resilient to the harsh environment and allows for in-core flow sensing in a nuclear reactor [1][2].

In this research, the key parameters of the microwave cavity flowmeter design are investigated to achieve optimized flow sensing performance. The paper's outline is as follows: Section II introduces the flow sensing theory of the cylindrical microwave cavity flowmeter and discuss the two factors that determine the performance of the sensor, the flowmeter sensitivity and the Q factor of the cavity resonator as a flowmeter. In Section III, parameters that affect the two performance factors are investigated and optimized choices for each parameter are discussed. Section IV provides concluding remarks about the results of this research.

II. MICROWAVE CAVITY RESONATOR SENSING THEORY

The theory of the flow sensor based on microwave cavity resonator was discussed in previous publications [1][2]. This section briefly summarizes the main concepts of model of the flow sensor.

A. *Microwave Cavity Flowmeter Design*

The microwave cavity flowmeter design is based on a cavity resonator, as shown in Fig. *1*. The cavity resonator is excited through an aperture on the sidewall of the cylinder, and the surrounding area of the excitation aperture is machined flat to interface with a waveguide. The whole sensor is made of a metal with good electrical conductivity, and the inner wall of the hollow cavity can be silver coated to obtain higher conductivity. The top side is made of a thin membrane ($d = 203.2\mu m$). The bottom and the sidewall of the cavity are much thicker than the top to provide the necessary mechanical strength. The thinness of the membrane is selected such that when a pressure change is applied to the sensor the membrane will deform measurably. Within the elastic deformation range, this deformation is linearly dependent on the pressure applied to the membrane. Thus, when inserted into a liquid stream the flow rate near the sensor can be determined if the deformation of the membrane can be measured accurately. Next, the connection between the flow rate, the membrane deformation, and the resonant frequency shift of the cavity resonator caused by the deformation will be discussed.

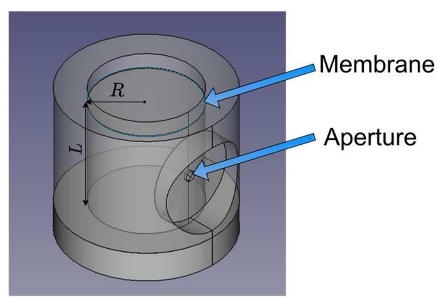

Fig. 1. Microwave Cavity Flowmeter. The cavity walls are set transparent to show the hollow cavity inside.

B. *Flow Measurement Theory*

The relation between the flow rate and the pressure applied to the membrane is given by the Bernoulli equation:

$$P = \frac{1}{2}\rho V^2 \quad (1)$$

where ρ is the liquid density, P is the pressure, and V is the flow velocity magnitude near the sensor. When pressure is applied to the membrane, it deforms following Timoshenko's model for radially constrained circular plates [7]:

$$w(r) = \frac{PR^4}{64D}\left(1 - \frac{r^2}{R^2}\right) \quad (2)$$

where R is the radius of the membrane, $r \in [0, R]$ is the radial distance from the central point of the membrane, and D is the material flexural rigidity:

$$D = \frac{Ed^3}{12(1 - v^2)} \quad (3)$$

where E and v are the membrane material's Young's modulus and Poisson Ratio, respectively, and d is the thickness of the membrane. Calculating the area-average of $w(r)$ and substituting Equation 1 and Equation 3 into Equation 2 yields the average displacement of the membrane:

$$\Delta L = -\frac{1}{\pi R^2}\int_0^R w(r) \cdot 2\pi r\, dr = \frac{(1 - v^2)}{32E}\frac{R^4}{d^3}\rho \cdot V^2 \quad (4)$$

Next, we establish the connection between the displacement ΔL and the resonant frequency change, denoted Δf. The resonant frequency for a cylindrical cavity excited in TE_{nml} mode is [1]:

$$f_{nml}^{TE} = \frac{c}{2\pi\sqrt{\mu_r \varepsilon_r}}\sqrt{\left(\frac{X'_{nm}}{R}\right)^2 + \left(\frac{l\pi}{L}\right)^2} \quad (5)$$

In this equation, c is the speed of light, μ_r is the permeability and ε_r is the permittivity of the medium inside the cavity. X'_{nm} is the n^{th} root of the derivative of the m^{th} order Bessel function. All these parameters are determined for a given medium and coupling mode. Finally, the R and L are the radius and the height of the hollow cavity, as shown in Fig. 1. The height of the cavity is selected to be $L = 2R$. There are two advantages in selecting this radius-to-height ratio. The first is that when $L = 2R$, the Q factor is near the maximum value for specific coupling modes, including the TE_{011} mode [8]. The other advantage of this ratio is that the two fractions inside the square root in Equation 5 can be merged. Doing so yields:

$$f = L^{-1} \cdot \frac{c\sqrt{4{X'_{nm}}^2 + (l\pi)^2}}{2\pi} \quad (6)$$

Then, using Newton's Method and combining Equations 4 and 6 gives:

$$\Delta f = \Delta L \cdot f'(L) = C_{nml}\frac{(1 - v^2)R^2\rho}{Ed^3}V^2 \quad (7)$$

$$C_{nml} = \frac{c\sqrt{4{X'_{nm}}^2 + (l\pi)^2}}{256\pi} \quad (8)$$

Equation 7 shows the direct relationship between the resonant frequency shift Δf and the flow velocity V. Note that this equation only applies when Newton's Method is still relevant. Using common materials such as steel, aluminum, and copper alloys (considered options for the sensor design), we can build membranes that are rigid enough when the membrane is pressured within the elastic limit, $\Delta L \ll L$.

C. Resonant Frequency Shift Detection and Q factor

After establishing the relationship between Δf and V, the flow velocity can be determined by measuring the resonant frequency shift from the S_{11} parameter frequency sweep of the microwave system. In an unideal situation, the baseline of the sweep is not flat, as shown in Fig. 2. Thus, filtering techniques are required to locate the resonance accurately. Fig. 3 shows the filtered trace with the resonant frequency located at 17.774GHz. To quantify how recognizable the resonant frequency f is on the frequency sweep, the Q-factor is used. By definition [8]:

$$Q = 2\pi f\frac{Average\ energy\ stored}{Energy\ loss\ per\ second} = 2\pi f\frac{W_m + W_e}{P_{loss}} \quad (9)$$

In Equation 9, W_m is the average magnetic energy, W_e is the average electrical energy stored, and P_{loss} is the rate of power loss in the system. The energies are not easily evaluated experimentally. Thus, the bandwidth definition of Q is used instead, which is defined as:

$$Q = \frac{f}{f_{BW}} \quad (10)$$

The bandwidth definition of the Q of the cavity can be easily evaluated. The resonance bandwidth f_{BW} is the distance between the two -3dB (relative to the resonance dip) points near the resonance on the frequency sweep. The resonant frequency f can be determined, and verified experimentally, once the dimensions of the cavity resonator are selected.

Equation 10 can be used to quickly evaluate the Q factor of a cavity resonator. However, this Q is only for the resonator as a standalone microwave circuit. Because a microwave resonator requires an excitation source to operate, this inevitably introduces an external load to the system. The Q factor then becomes the Loaded Q:

$$Q_L^{-1} = Q_e^{-1} + Q^{-1} \quad (11)$$

In Equation 11, the Loaded Q is denoted by Q_L, Q is the quality factor of the resonator or unloaded Q, and Q_e is the External Q, determined by the input impedance of the external circuit. While Q_L is an important factor on its own for the resonator design, the loaded Q is a deterministic factor that directly affects the observed S-parameter sweep. For instance, using Equation 10, the quality factor calculated from Fig. 4 is an unloaded Q by nature.

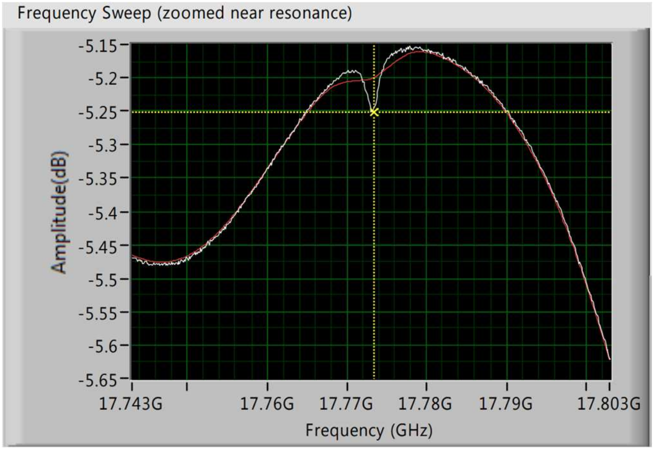

Fig. 2. S-parameter Sweep from experiments conducted in [9]. The white curve indicates that the baseline is not flat, thus requiring filtering. The filtered baseline is marked red and is subtracted from the white curve to get the resonance.

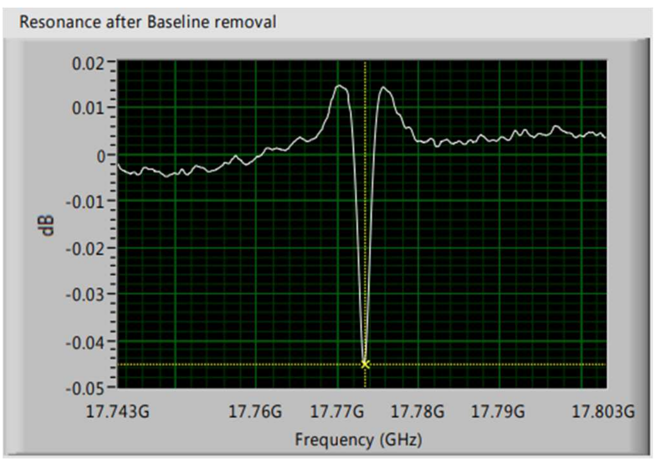

Fig. 3. The resonance after baseline removal. It is acquired by subtracting the red curve from the white curve in Fig. 2.

III. Optimized Microwave Cavity Flowmeter Design

According to the discussion in the previous section, two factors significantly impact the flowmeter's performance. One is the aspect ratio between the resonant frequency shift and the square of the flow velocity:

$$a = \frac{\Delta f}{V^2} = C_{nml} \frac{(1 - v^2) R^2 \rho}{E d^3} \quad (12)$$

which can be obtained from Equation 7. This value can be interpreted as the flow measurement sensitivity. A higher a means a more significant resonant frequency shift at a given flow velocity. The apparent options to achieve this are to (a) choose a coupling mode that maximizes the C_{nml}, (b) use a thinner membrane for a smaller d in the denominator, (c) increase the radius of the membrane, and (d) choose a material with a lower Young's modulus E and a lower Poisson ratio v.

The Q factor is another critical element in the microwave cavity flowmeter design. A higher Q corresponds to a narrower dip at the resonance in the frequency response trace, which makes it easier to locate the resonant frequency and reduce systematic errors. On the other hand, a lower Q factor correspond to a flatter resonance, making finding the resonance harder and less reliable. In a worst-case scenario, the baseline can mask the resonance dip entirely. In the following sections, several parameters of the microwave cavity resonator that affect those two factors are investigated.

A. Cavity Dimensions and Coupling Mode

According to Equation 5, the dimensions of the cylindrical cavity resonator determine the resonant frequency for a certain coupling mode. In turn, for a chosen resonant frequency range, the dimensions are limited by the operating wavelength. For a K-band resonator, for instance, the height of the cavity should be in the range of 0.75 to 2.4cm.

Moreover, the radius-to-height ratio defined by $\frac{2R}{L}$ dictates what coupling mode is possible for the cylindrical cavity resonator at a given resonant frequency range. Also, for lower-order coupling modes like TE_{011} and TE_{012}, keeping this ratio at 1 is a good choice as it yields the largest Q factor when conductor loss is taken into consideration [8].

B. The Flowmeter Membrane

The flowmeter's membrane is a thin layer of metal on one end of the cylindrical cavity. As shown in Fig. 4, the membrane deforms elastically under the pressure generated by the fluid flow. According to Equation 12, a thinner membrane and a larger radius correspond to a better flow measurement sensitivity. However, care must be taken when choosing d and R, as the pressure must not exceed the yield strength of the membrane. In the worst case, the membrane can be fractured and lead to leakage. Thus, when designing the membrane, the maximum pressure applied must be taken into consideration so that the membrane deflection is well within the elastic range.

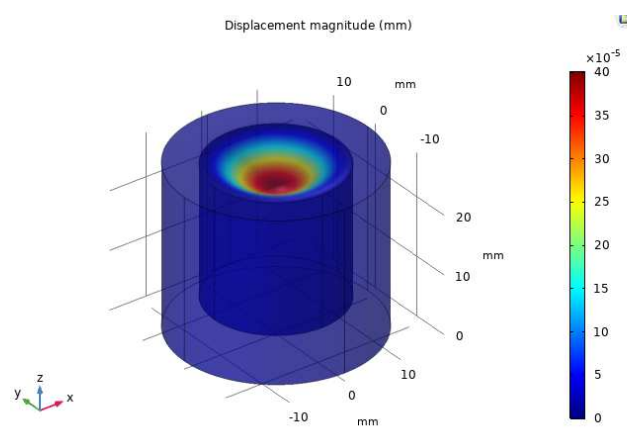

Fig. 4. A demonstration of the membrane displacement under a certain flow pressure. Because the displacment is in micrometer (µm) level, it is magnified by 10^5 times in the plot for observation[9].

C. *Cavity Wall Materials*

The cavity wall material affects the conductor loss of the resonator. Materials with higher conductivity yield higher Q factor, thus are more desirable. Table I lists some metals considered to be used for fabricating the resonator sensor [1][10]:

TABLE I. METALS ELECTRICAL CONDUCTIVITY

Metals	Conductivity
Copper (Cu)	5.96 x 10^7
Silver (Ag)	6.30 x 10^7
Aluminum (Al)	3.77 x 10^7

Fig. 5 shows the S-parameter plot of a cylinder resonator sensor with different inner wall materials. Because aluminum has a much lower conductivity, its resonance is weaker (-0.53dB). Silver wall provides the best resonance, but copper comes very close, with the difference at the resonance center being less than 0.05dB. Considering silver and copper have the best conductivity among the common metals, they are the preferred choices for cavity walls.

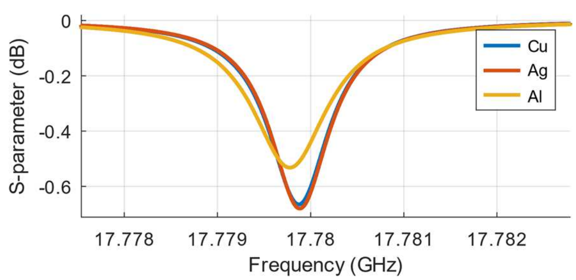

Fig. 5. S-parameter sweep for different wall materials. Note that because of the similar conductivity of Cu and Ag, their S-parameter and therefore Q factor is almost overlapped with each other when compared to the Aluminum.

D. *Excitation Aperture*

The excitation aperture is used to couple the resonator to an external microwave circuit as the power source. The equivalent circuit of an aperture in the transverse waveguide wall is a normalized inductive susceptance defined by:

$$jB = -j\frac{ab}{2\beta\alpha_m} \tag{13}$$

In this equation, a and b are the height and width of the waveguide, β is the propagation constant, and α_m is the magnetic polarization, which is a function of the aperture's dimensions. For a round-hole aperture with a radius r_0, the formula for α_m is:

$$\alpha_m = \frac{4r_0^3}{3} \tag{14}$$

To achieve the highest Q, the external circuit and the resonator must be in the state of critical coupling. The critical coupling is achieved when the coefficient of coupling satisfies:

$$g = \frac{Q}{Q_e} = 1 \tag{15}$$

To achieve critical coupling, the required normalized aperture reactance should be [8]:

$$x_L = \sqrt{\frac{\pi^2 k_0 f}{Q\beta^2 c}} \tag{16}$$

in which, k_0 is the wavenumber and c is the speed of light. Then, to find the aperture size that satisfies critical coupling, we substitute Equation 16 into Equation 13, which yields:

$$r_0 = \sqrt[6]{\frac{9a^2b^2Qc}{64\pi^2 k_0 f}} \tag{17}$$

Equation 17 gives the optimal aperture size that satisfies the critical coupling condition. However, due to the physical boundary of the waveguide transverse wall, the ideal aperture size may not be practical if $r_0 > a$ or $r_0 > b$. Fig. 6 shows different aperture diameters, denoted as φ, that have different coupling efficiencies, and therefore different Q factors. Noticing that at $\varphi_{1.1} = 2.4$mm, the resonance is the strongest. However, for this diameter, the aperture hole is very close to the edge of the boundary of the waveguide transverse wall (10.7x4.3mm), which should be avoided [8]. Also, note that the resonant frequency shifts toward left while the aperture size is increased. This is due to the fact that the coupling condition is changing. Considering the resonator as an equivalent shorted waveguide of length l, for the unloaded resonator, the resonant frequency is found at:

$$\tan \beta l = 0 \tag{18}$$

$$\beta l = \frac{2\pi f}{v_p} \cdot l = k\pi \tag{19}$$

However, when the resonator is loaded, Equation 18 changes to:

$$\tan \beta l + x_L = 0 \tag{20}$$

In this equation, $0 < x_L \ll 1$, thus, compared to Equation 18, the solution for Equation 20 is closer to the origin, which leads to a decrease in resonant frequency.

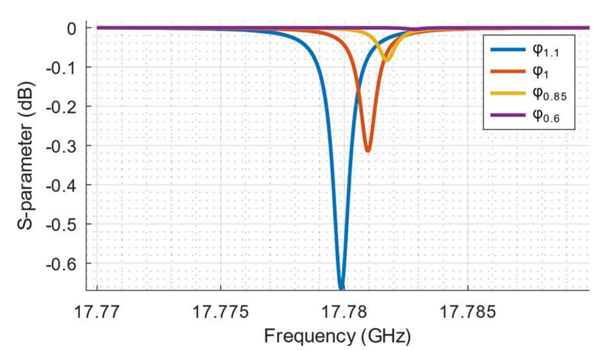

Fig. 6. S-parameter sweep for different aperture diameters. $\varphi_1 = 2.2mm$, $\varphi_{1.1} = 2.4mm$, $\varphi_{0.85} = 1.85mm$, $\varphi_{0.6} = 1.31mm$

IV. CONCLUSION

In this paper, using computer simulations, we have investigated optimization of the microwave cavity parameters for maximizing Q-factor and sensitivity to fluid flow sensing. Future work will consist of experimental validations of the findings in this work.

V. ACKNOWLEDGMENT

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