Probabilistic Creep Model for Recalibration of Microwave Cavity Flow Meter

Tianyang Fang and Jafar Saniie
ECASP Research Laboratory (http://ecasp.ece.iit.edu)
Department of Electrical and Computer Engineering
Illinois Institute of Technology, Chicago, IL, U.S.A.

Sasan Bakhtiari and Alexander Heifetz
Nuclear Science and Engineering Division
Argonne National Laboratory
Lemont, IL, U.S.A.

***Abstract*—This paper assesses the long-term creep effect on a recently designed Microwave Cavity Flow Meter. This sensor, designed for precise flow rate measurement in nuclear reactor coolant cycles, relies on membrane deformation in the cavity for flow sensing. Operating in challenging conditions with high temperatures of ~650° C and a constant fluid load of ~100 MPa, its long-term performance is impacted by the unpredictable creep effect. Long-term creep deformation and damage exhibit significant variance across multiple parameters. Therefore, a probabilistic creep model would be beneficial for addressing this variability. We propose a probabilistic creep model for recalibration of the microwave cavity flow sensor based on the Monte Carlo method and the Wilshire–Cano–Stewart (WCS) creep model.**

Keywords—microwave resonator, creep, Monte Carlo, Wilshire-Cano-Stewart Model, probabilistic model.

I. INTRODUCTION

Microwave Cavity Flow Meter relies on the deformation of the thin membrane on one side of the cavity caused by the load from the flow to measure the flow rate. This deformation changes the cavity's dimension, which causes a shift in its resonant frequency [1-4]. Therefore, the sensor is designed to be sensitive to membrane deformation to measure the flow rate accurately. The microwave flow meter is intended to operate in the coolant cycles in nuclear reactors and thus will experience high temperatures and constant high load from the coolant fluid flow. This environment causes a long-term creep effect that has proven critical in our previous study [5] and must be considered in the recalibration of the sensor. However, the creep effect is highly dependent on the environmental and material parameters. These parameters exhibit significant variance as shown in past studies, making the deterministic creep model unreliable in the long term [6,7]. Therefore, a probabilistic creep model proves beneficial for addressing this variability.

This research paper examines the creep effect on the microwave cavity flow meter using the probabilistic creep model [6,7]. In Section II, a brief introduction to the microwave resonant cavity flow meter is first presented. Then, the importance of evaluating and recalibrating the creep effect for the sensor's membrane is established. Section III introduces the WCS creep model and develops the analytical equations for Minimum Creep Strain Rate (MCSR), time to Stress Rupture (SR), and accumulated creep strain. Section IV establishes the probabilistic creep model using the Monte Carlo method and the WCS model. Section V discusses the simulations of the creep effect based on the probabilistic model. Finally, Section VI provides concluding remarks about the results.

II. CREEP EFFECT ON MICROWAVE CAVITY FLOW METER

A. Microwave Flow Meter

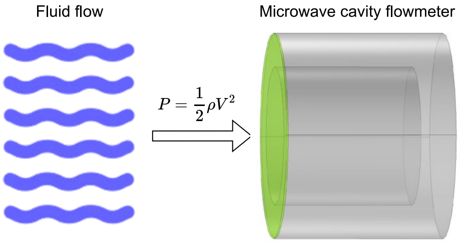

Fig. 1. Microwave cavity flow meter [4]

The microwave cavity flow meter is structured around a cavity resonator, illustrated in Fig. 1. The side of the cavity facing the fluid flow exhibits sufficient flexibility to undergo micron levels of deflections in response to dynamic fluid pressure in accordance with the Bernoulli equation. Deformation of the membrane changes the length of the cavity, resulting in a resonant frequency shift. Previous research [2-4] has confirmed that, within the elastic range, the deformation exhibits a linear correlation with applied pressure, and the frequency shift is proportionate to the deformation:

$$\Delta f \propto \Delta L \tag{1}$$

$$\Delta L \propto P \tag{2}$$

where Δf is the resonant frequency shift, ΔL is the deformation of the membrane and P is the uniform load on the membrane. These two relations facilitate precise flow rate determination when the sensor is immersed in a liquid stream.

B. Membrane Creep Effect

By design, the microwave cavity flow meter is intended to function within the elastic deformation regime of the membrane, ensuring a responsive and proportionate correlation between deformation and fluid flow rate. Nonetheless, the elevated operating temperature coupled with the sustained load conditions leads to plastic deformation of the membrane, even within the elastic range, due to the creep effect. This creep-induced deformation has been proven critical to the functionality of the sensor, as it introduces an offset to the resonant frequency shift. Fig. 2 shows that under typical fluid flow pressure, the creep effect starts to introduce significant membrane deformation from ~103 hours.

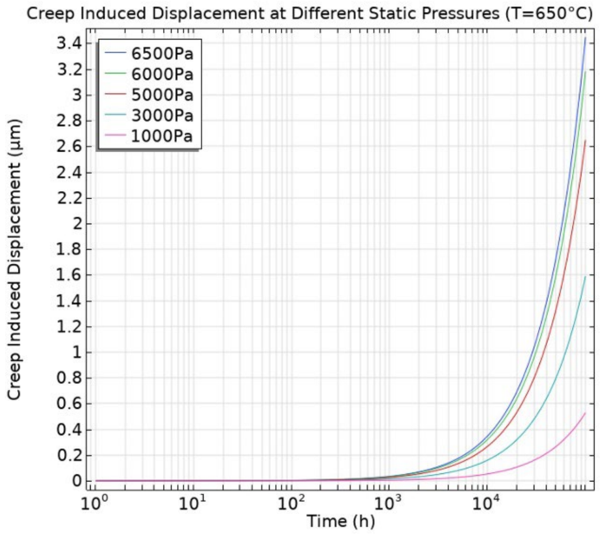

Fig. 2. The creep-induced membrane displacement. Typical loads from fluid flow [5] are applied to the membrane for 10^5 hours. The results are obtained by COMSOL simulation.

III. WILSHIRE–CANO–STEWART MODEL

To accurately assess the long-term creep effect in the scenario demonstrated in Fig. 2, a robust creep model with high-temperature sensitivity and creep modeling capability—the Wilshire–Cano–Stewart Model (WCS)—is adapted in this study. The WCS model is regulated by the creep strain rate and the damage rate equations [7,8]:

$$\dot{\varepsilon}_{cr} = \left[-ln\left(\frac{\sigma}{\sigma_{TS}}\right)\frac{1}{k_2}\right]^{\frac{1}{v}} \cdot e^{-\frac{Q_c}{R\cdot T}} \cdot e^{\lambda\omega} \tag{3}$$

$$\dot{\omega} = \frac{[1-e^{-\phi}]}{\phi}\left[-ln\left(\frac{\sigma}{\sigma_{TS}}\right)\cdot\frac{1}{k_1}\right]^{-\frac{1}{u}} \cdot e^{-\frac{Q_c}{R\cdot T}} \cdot e^{\phi\omega} \tag{4}$$

where $\dot{\varepsilon}_{cr}$ is the creep strain rate, $\dot{\omega}$ is the damage rate, σ is the load, σ_{TS} is the ultimate tensile strength, k_1, k_2, v, u are the Wilshire constants, Q_c is the creep activation energy, R is the gas constant, T is the temperature, λ is the strain trajectory constant and ϕ is the damage trajectory constant. Lastly, ω is the accumulated damage.

Continuing with the WCS model, the creep strain rate (MCSR) is given by:

$$\dot{\varepsilon}_{MCSR} = \left[-ln\left(\frac{\sigma}{\sigma_{TS}}\right)\frac{1}{k_2}\right]^{\frac{1}{v}} \cdot e^{-\frac{Q_c}{R\cdot T}} \tag{5}$$

time to rapture, or Stress-Rapture (SR) is given by:

$$t_{rapture} = \left[-ln\left(\frac{\sigma}{\sigma_{TS}}\right)\cdot\frac{1}{k_1}\right]^{\frac{1}{u}} \cdot e^{\frac{Q_c}{R\cdot T}} \tag{6}$$

Indefinite integration of equation 4 yields the creep damage to the time function:

$$\omega(t) = -\frac{1}{\phi}ln\left(1-\left(1-e^{-\phi}\right)\frac{t}{t_{rapture}}\right) \tag{7}$$

The MCSR, SR, and creep damage are important indicators of the creep effect. Nonetheless, notice that in equations 5, 6, and 7 there is an assumption that the initial creep damage $\omega_0 = 0$. In reality, due to manufacturing procedures, transportation, and storage, materials may have been inflicted by creep damage before being tested. Thus, by considering $\omega_0 \neq 0$, the MCSR, SR, and creep damage become:

$$\dot{\varepsilon}_{MCSR} = \left[-ln\left(\frac{\sigma}{\sigma_{TS}}\right)\frac{1}{k_2}\right]^{\frac{1}{v}} \cdot e^{-\frac{Q_c}{R\cdot T}} \cdot e^{\lambda\omega_0} \tag{8}$$

$$t_{rapture} = \left(\frac{e^{-\phi\omega_0}-e^{-\phi}}{1-e^{-\phi}}\right)\cdot\left[-ln\left(\frac{\sigma}{\sigma_{TS}}\right)\cdot\frac{1}{k_1}\right]^{\frac{1}{u}} \cdot e^{\frac{Q_c}{R\cdot T}} \tag{9}$$

$$\omega(t) = -\frac{1}{\phi}ln\left(e^{-\phi\omega_0}-\left(e^{-\phi\omega_0}-e^{-\phi}\right)\frac{t}{t_{rapture}}\right) \tag{10}$$

Last but not least, the integration of equation 3 yields the accumulated creep strain:

$$\varepsilon = \int_0^t \dot{\varepsilon}_{cr}\,dt \tag{11}$$

$$= \left[-ln\left(\frac{\sigma}{\sigma_{TS}}\right)\frac{1}{k_2}\right]^{\frac{1}{v}} \cdot e^{-\frac{Q_c}{R\cdot T}} \cdot \int_0^t \exp[\lambda\cdot\omega(t)]\,dt \tag{12}$$

This integration can be therefore numerically calculated by substituting equations 9 and 10 into equation 12.

IV. PROBABILISTIC CREEP MODEL

Next, the deterministic WCS model is used in the probabilistic creep model that generates the distributions for MCSR, SR, and creep strain using the Monte Carlo method.

A. Monte Carlo Method

The Monte Carlo method can be used to repeatedly sample a function to get a distribution of numerical results. This distribution is then analyzed to evaluate the target function. In the probabilistic creep modeling, the uncertainty of the parameters in the WCS equations will be the source of variance. In this Monte Carlo simulation, variable parameters are sampled using their probability density function (PDF). The samples are then sent into the WCS model to calculate the MCSR, SR, and ε distributions. These distributions can then be evaluated using statistical models.

B. Variable Parameters

In the WCS model, almost all the parameters are variable to a certain degree. The load σ varies due to the nature of the fluid flow. The temperature T varies due to the fluctuations of the environmental temperature. The Wilshire constants, k_1, k_2, v, u, the trajectory constant λ and ϕ, are subject to the uncertainty of the material's properties. The previously mentioned initial creep damage ω_0 depends on the initial condition of the material at the point of test.

In this research, we used the material properties of SS304 stainless steel listed in [7] with adaptations (see Table 1) for the simulation and also included the load and temperature distribution. The temperature-dependent ultimate tensile strength σ_{TS} is derived from [9] with interpolations (see Table 2).

TABLE I. LIST OF VARIABLE PARAMETERS FOR SS304

Variable	Distribution	Means and Std
σ	Normal	$\mu = Load\ (MPa)$, $\sigma_{std} = (0.46Load + 0.006)\ (MPa)$
T	Normal	$\mu = T\ (K)$, $\sigma_{std} = 0.5\ (K)$
ω_0	Exponential	$\mu = 1.15 \times 10^{-4}$
k_1	Lognormal	$\mu = 5.108$, $\sigma_{std} = 0.038$
k_2	Lognormal	$\mu = 3.696$, $\sigma_{std} = 0.027$
v	Logistic	$\mu = -0.286$, $\sigma_{std} = 0.001$
u	Logistic	$\mu = 0.376$, $\sigma_{std} = 0.002$
λ_0	Logistic	$\mu = 8.422$ $\sigma_{std} = 0.235$
ϕ_0	Logistic	$\mu = 2.177$, $\sigma_{std} = 0.111$

TABLE II. ULTIMATE TENSILE STRENGTH IN VARIOUS TEMPERATURES

Temperature (K)	550	600	650	700	750
σ_{TS} (MPa)	382	326	270	214	158

V. SIMULATION OF EXPERIMENT

With the established probabilistic model and the distributions of variable parameters, we can simulate the predictions of the minimum creep strain rate (MCSR), time to rapture (SR), and accumulated creep strain. Fig. 3 shows the creep strain accumulation distribution over time, up to 2×10^6 hours, at a set mean Load of 100 MPa, and temperature of 650 °C. Noticing the significant variance in time to creep in different runs of the simulation. Fig. 4 shows the MCSR distributions over various temperatures and loads, matching the research in [7]. The MCSR distribution trajectory always stops at the ultimate tensile strength, consistent with Equation 3. Fig. 5 and 6 show the MCSR and SR probabilistic distribution at a mean Load of 100 MPa, and temperature of 650 °C. The mean of the distribution is marked in red in the plots. Fitting methods and other statistical analyses are planned in the future to evaluate the analytical model of the distribution.

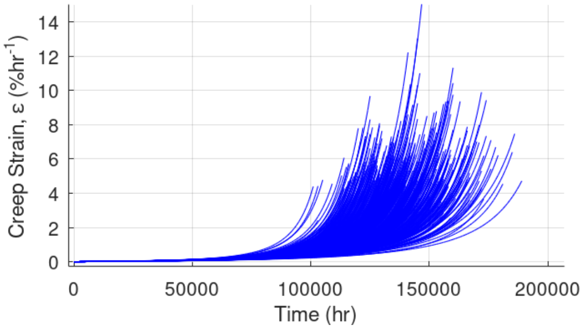

Fig. 3. Creep Strain accumulation over time. The simulation stops at $t = 0.9\ t_{rapture}$

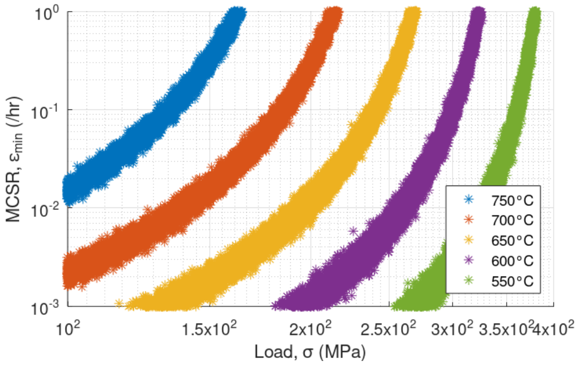

Fig. 4. Probabilistic prediction of MCSR vs Various Loads and Temperatures.

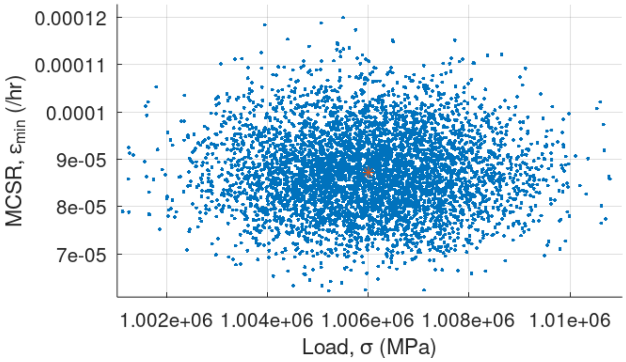

Fig. 5. MCSR probabilistic distribution.

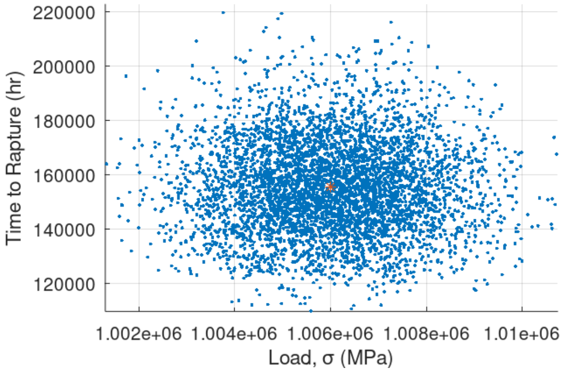

Fig. 6. Time to Rapture distribution.

VI. CONCLUSION

In this research, we established a probabilistic creep effect model for the long-term creep effect on the microwave cavity flow meter. The simulation predictions match those reported in contemporary studies. Future work includes an analytical (in place of numerical) prediction model for the creep effect.

ACKNOWLEDGMENT

This work was supported in part by the US Department of Energy, Office of Nuclear Energy, Nuclear Energy Enabling Technology (NEET) Advanced Sensors and Instrumentation (ASI) program, under contract DE-AC02-06CH11357.

REFERENCES

[1] A. Heifetz, V. Ankel, D. Shribak, S. Bakhtiari, “Microwave Resonant Cavity-Based Flow Sensor for Advanced Reactor High-Temperature Fluids,” ANS Annual Meeting Embedded Conference 12th Nuclear Plant Instrumentation, Control and Human-Machine Interface Technologies (NPIC-HMIT2021) (2021).

[2] A. Heifetz, S. Bakhtiari, E.R. Koehl, D. Shribak, D. Aronson, T. Fang, J. Saniie, “First Annual Report on Development of Microwave Resonant Cavity Transducer for Fluid Flow Sensing,” Argonne National Laboratory ANL-21/49, 2021. doi:10.2172/1827701.

[3] A. Heifetz, G. Carrel, T. Fang, J. Saniie, E. R. Koehl, and S. Bakhtiari, ‘Second Annual Report on Development of Microwave Resonant Cavity Transducer for Fluid Flow Sensing’, Argonne National Laboratory ANL-22/77. Doi: 10.2172/1897183

[4] T. Fang, J. Saniie, S. Bakhtiari and A. Heifetz, "Frequency Shift Baseline Removal for Improved Flow Measurement using Microwave Cavity Resonator," 2022 IEEE International Conference on Electro Information Technology (eIT), Mankato, MN, USA, 2022, pp. 436-439, doi: 10.1109/eIT53891.2022.9813984.

[5] G. Carrel, A. Heifetz, ‘Analysis of Microwave Resonant Cavity Transducer Degradation Mechanisms’ Argonne National Laboratory ANL-22/55, 2022. Doi: 10.2172/1887267.

[6] Hossain, Md A., Jaime A. Cano, and Calvin M. Stewart. "Probabilistic Creep with the Wilshire–Cano–Stewart Model." Fatigue & Fracture of Engineering Materials & Structures 46, no. 10 (2023): 4001-4019. Accessed March 1, 2024. 10.1111/ffe.14120.

[7] S.-S. Guo, J.-G. Gong, P. Zhao, and F.-Z. Xuan, ‘A probabilistic framework of creep life assessment of structural components at elevated temperatures’, Engineering Fracture Mechanics, vol. 281, p. 109162, Mar. 2023.

[8] J. A. Cano and C. M. Stewart, ‘A continuum damage mechanics (CDM) based Wilshire model for creep deformation, damage, and rupture prediction’, Materials Science and Engineering: A, vol. 799, p. 140231, 2021.

[9] J. Moteff, R. K. Bhargava, and W. L. McCullough, ‘Correlation of the hot-hardness with the tensile strength of 304 stainless steel to temperatures of 1200°C’, Metallurgical Transactions A, vol. 6, no. 5, pp. 1101–1104, May 1975.